

Hygrothermoelasticity



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Martinus Nijhoff Publishers

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1986 **MARTINUS NIJHOFF PUBLISHERS**
a member of the KLUWER ACADEMIC PUBLISHERS GROUP
DORDRECHT / BOSTON / LANCASTER



Distributors

for the United States and Canada: Kluwer Academic Publishers, 101 Philip Drive, Assinippi Park, Norwell, MA 02061, USA

for the UK and Ireland: Kluwer Academic Publishers, MTP Press Limited, Falcon House, Queen Square, Lancaster LA1 1RN, UK

for all other countries: Kluwer Academic Publishers Group, Distribution Center, P.O. Box 322, 3300 AH Dordrecht, The Netherlands

Library of Congress Cataloging in Publication Data

Hygrothermoelasticity.

I. Hygrothermoelasticity. I. Sih, G. C. (George C.)

II. Michopoulos, J. G. III. Chou, S. C.

QA933.H94 1986 620.1'1232 86-8515

ISBN 90-247-3356-1

ISBN 90-247-3356-1

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Martinus Nijhoff Publishers, P.O. Box 163, 3300 AD Dordrecht,
The Netherlands.

PRINTED IN THE NETHERLANDS

HYGROTHERMOELASTICITY

Preface

This book contains results of more than a decade's effort on coupled deformation and diffusion obtained in research performed at the Institute of Fracture and Solid Mechanics, Lehigh University. Despite the overwhelming number of theories on this subject, little is known on the assessment of *coupling* effects because of the inherent difficulties associated with experimentation. A case in point is couple thermoelasticity, a theory that has remained virtually unused in practice. This is indicative of the inadequacy of conventional approaches.

The interdependence of heat, moisture and deformation arises in many engineering problems of practical interest. Whether these effects are coupled or not depend on the transient character of the boundary conditions. Special attention is given to finding the coupling constants. Invoked is the assumption that the physical parameters should be independent of the specified boundary conditions. They can thus be extracted from known experimental data for situations where coupling effects are relatively weak and then applied to predict strong coupling effects as boundary conditions are altered. This is illustrated for the T300/5208 material commonly used in composites and permits a more reliable evaluation of material behavior under extreme environmental conditions. The lack of this knowledge can often be a major deterrent to the achievement of new technological advances.

The reader will recognize that the material in this book does not follow the main stream of research on moisture-temperature diffusion and deformation. The data measurement approach is not regarded to be informative simply because test results collected from laboratory conditions are not the same as those used in service. A methodology was needed to complete this data transfer process. In this respect, the authors are grateful to two particular individuals of the U.S. government agencies who had the foresight and wisdom to break new grounds and to support ideas that may have been regarded as unorthodox at the time. Many of the results would, no doubt, have remained unknown without their support. To be recognized is the encouragement and direction provided by Mr. W. J. Walker of the Air Force Office of Scientific Research (AFOSR) in Washington, D.C. The project initiated in 1976 was terminated immediately in 1980 upon his departure from AFOSR. The Materials Technology Laboratory* (MTL) in Watertown,

*It was then known as the Army Materials and Mechanics Research Center (AMMRC).

Preface

Massachusetts, under the direction of Mr. J. F. Dignam, continued the support until 1984 when the basic problem of heat, moisture and deformation was understood and reduced to the stage where results can be easily obtained for application.

Chapter 1 gives a brief review of the classical diffusion theories involving temperature and moisture. Defined are the temperature and moisture diffusion coefficients and the methods by which they can be measured experimentally. Typical data are quoted for the T300/5208 graphite/epoxy laminate, a material that will be analyzed throughout the book. Coupling between heat and moisture transfer is shown to be governed by a system of simultaneous partial differential equations whose solutions are dictated by the time-dependent boundary conditions. These are derived in Chapter 2 with the coupling constants determined for the T300/5208 laminate. This was made possible by the assumption that the uncoupled experimental data remain valid for the case of sudden moisture rise as the coupling effects are relatively weak. The same constants are then applied to analyze the situation of sudden temperature rise where coupling between moisture and temperature becomes significant.

Closed form solutions for the hygrothermal stresses in a slab and solid with a spherical cavity are obtained in Chapter 3. Initial and final values of moisture and/or temperature are specified on the boundary and they give rise to stresses that fluctuate with time and space variable. Coupling is again shown to be more pronounced when temperature is prescribed. For more complex geometries and/or boundary conditions, numerical method must be applied. Outlined in Chapter 4 is the development of the finite element procedure in conjunction with Laplace transform. This circumvents the difficulty that arises in selecting the appropriate time and space increment. Solutions to several non-trivial boundary-value problems are presented. They consist of sudden application of temperature and/or moisture on a half-space, circular hole and elliptical cavity. The resulting hygrothermal stresses can achieve high elevation near the boundary, particularly when the local curvature comes into play as in the case of a narrow ellipse. Special attention was also given to reduction of the size of the finite elements in regions where accuracy is needed.

Chapter 5 defines a thermodynamic potential to derive a system of equations that account for the interdependence of heat, moisture and deformation. Because of coupling between diffusion and deformation, the diffusion coefficients will also be affected when approximations are made to reduce the three-dimensional equations to two dimensions. In particular, the coupling constants in plane strain and plane stress must be carefully distinguished. They are derived for the T300/5208 composite. For relatively simple geometries and boundary conditions, the complex variable formulation may be adopted together with conformal mapping to yield close form solutions. This is given in Chapter 6. For a circular region, coupling effects are shown to be more significant when the loads are localized. The same applies to the way with which temperature and/or moisture are specified on the boundary.

The governing equations in Chapter 7 for coupled diffusion and deformation are formally the same as those in Chapter 4 when deformation is not coupled. Therefore, the Laplace transform finite element procedure can again be used to

solve the problems of a slab, circular hole and elliptical cavity. In addition to specifying surface temperature and moisture, boundary stress will also affect the outcome. Large deviations are observed between the coupled and uncoupled results depending on the time and change in the local geometry.

The last chapter derives the strain energy density function for diffusion and deformation coupled problems. This function contains one part that is dominated by mechanical deformation and another by diffusion. Their variations with different boundary conditions are displayed numerically for a crack approximated by a narrow ellipse with a ten-to-one aspect ratio. Possible failure by fracture and/or permanent deformation are also discussed briefly in connection with the strain energy density function.

The final completion of this book was not without a struggle. Many difficulties encountered in the theoretical formulation and numerical calculation had to be overcome. This involved the efforts of a number of individuals. Acknowledgement is due to Professor R. J. Hartranft who jointly developed some of the early formulations. Mr. A. Ogawa of the National Aerospace Laboratory in Japan and Professor I. C. Tsai of the National Taiwan University in the Republic of China also made major contributions in applying Laplace transform to the finite element method. A great deal of expediency was thus gained in the choice of space and time increments. It was unfortunate that Dr. M. T. Shih had to relinquish his co-authorship when he left the Institute of Fracture and Solid Mechanics to take a position with the Bell Telephone Laboratories. Thanks are due to the calculations he made for determining some of the physical constants.

The authors are particularly grateful to Mrs. Barbara DeLazaro and Mrs. Constance Weaver for expertly typing the manuscript that had undergone frequent revisions.

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Classical diffusion theories

1.1 Introduction

Diffusion is the physical process by which atoms and molecules are transported from one position of a system to another. Atoms or molecules in solids occupy definite equilibrium positions. They, however, will migrate if their concentrations are disturbed. The atoms will then diffuse from the region of higher concentration to that of lower concentration. Such a condition exists in solids containing mechanical imperfections and internal stresses. The rate of diffusion increases very rapidly as the temperature is increased. When the temperature becomes very high, thermal agitation can become violent and pull the atoms and molecules apart resulting in melting or disintegration of the solid. Concentration gradient can also be developed by nonuniform distribution of moisture. The process of moisture transfer is fundamentally equivalent to that of heat transfer and each process may or may not be treated independently depending on the physical conditions.

The nonuniform transfer of heat and/or moisture through solids are relevant to the design of high performance structures as their influences can cause dimensional changes of structural elements that, in turn, lead to internal stresses and/or strains. If the physical process is of a more active type, caused by capillary flow of fluids into voids, stresses can occur even when macroscopic deformation is uniform. These fluid or moisture induced stresses can result in the lowering of stiffness and strength. The effect is particularly significant in composites with matrix made of polymeric materials. Moreover, the thermal environment may also interact with moisture. Voids and microscopic defects tend to open as temperature is increased while more fluid is absorbed into the openings. The trapped fluid causes growth of the flaws when temperature is reduced suddenly. Subsequently, the material is capable of absorbing more moisture more quickly than before the thermal gradient was applied. Such a process, if continued, could lead to eventual failure of the solid system.

1.2 Thermal diffusion: heat conduction

Atoms in solids are in constant random motion. If they vibrate symmetrically about their equilibrium positions, then their average spacing would be constant

and there would be no thermal expansion. In the presence of a thermal gradient, heat will be transmitted as a result of elastic vibrations* of the atoms and molecules. The interchange of kinetic energy takes place by means of bonds between neighboring particles in the form of waves. As thermal agitation is more violent at the hot surface, heat tends to travel in the direction of decreasing temperature.

1.2.1 Law of heat conduction

When the temperature at one end of a solid rod is raised higher than the other, experience tells us that the temperature at the other end will also be raised in time. Heat is then said to have travelled along the rod in a direction from the high- to the low-temperature end. This process known as *conduction* is not thoroughly understood but is assumed to occur in an irreversible manner by nature. A useful concept following from the second law of thermodynamics is that heat exchange can be reversible if and only if the process occurs at a vanishingly small rate. The driving force in this process is the temperature gradient. Postulated also from the first law of thermodynamics is that the heat energy must be conserved along the flow path.

Consider a solid whose temperature may vary from point to point. Let $T(x_j, t)$ denote this temperature which is assumed to be a continuous function of the rectangular coordinates x_j and time t . A basic law of heat conduction may be stated as

$$q_i = -k_{ij} \frac{\partial T}{\partial x_j} \quad (1.1)$$

in which q_i are the components of the heat flux vector \mathbf{q} and $\partial T/\partial x_j$ the temperature gradients. The components of the thermal conductivity tensor \mathbf{k} are denoted by k_{ij} and are generally considered to be symmetric†, i.e.,

$$k_{ij} = k_{ji}. \quad (1.2)$$

Hence, heat will not necessarily flow in the direction of the temperature gradient, if the solid is anisotropic. When the solid possesses no preferred direction of heat flow, then k_{ij} take the form

$$k_{ij} = k\delta_{ij}, \quad (1.3)$$

where k does not have to be a constant. It can depend on the space variables and local temperature, i.e., $k = k(x_j, T)$. It was Fourier [5] who first provided the theoretical basis of the heat conduction equations.

1.2.2 Temperature field

The transient temperature field in a three-dimensional isotropic solid may be derived by application of equations (1.1), (1.3) and the first law of thermodynamics.

*The dominant mechanism of heat transfer in nonmetals is elastic vibration of the atoms and molecules which is a slower mechanism than energy transfer by free electrons [1] that occurs more readily in metals.

†The symmetric relation in equation (1.2) is a direct consequence of the assumption that reversal of heat flow results from reversal of temperature gradient [2]. It can also be deduced from the application of Onsager's principle [3].

1.2 Thermal diffusion: heat conduction

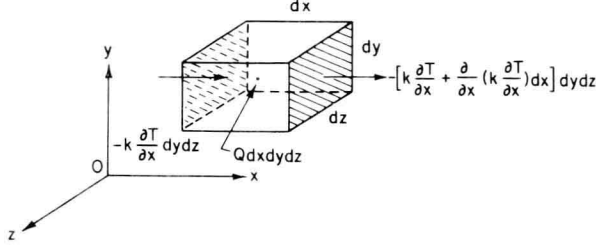


Figure 1.1. Heat conduction in the x -direction for a volume element.

Refer to the differential volume element $dx dy dz$ in which heat is generated at the rate Q per unit time per unit volume, Figure 1.1. The quantity of heat entering and leaving the volume element per unit time is, in general, not the same. Referring to Figure 1.1, the net gain in the x -direction is

$$- \left\{ \left[k \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx \right] - k \frac{\partial T}{\partial x} \right\} dy dz = - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx dy dz. \quad (1.4)$$

Expressions similar to equation (1.4) can be written for the net gain of heat per unit time in the y - and z -direction. The total heat generated in the volume element is $Q dx dy dz$ while the increase in internal energy of the element per unit time is given by $\rho c (\partial T / \partial t) dx dy dz$. The mass density of the solid is ρ and the specific heat* is c . For an homogeneous solid, the isotropic thermal conductivity coefficient k is independent of the space variables and an energy balance gives

$$\nabla(D_h \nabla T) + \frac{Q}{\rho c} = \frac{\partial T}{\partial t} \quad (1.5)$$

in which ∇^2 is the Laplacian operator in three dimensions: $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2$. The *thermal diffusivity* with units of length squared per unit time is defined as

$$D_h = \frac{k}{\rho c}, \quad (1.6)$$

where c is the heat capacity per unit volume. When regarding heat flow as a diffusion process, it should be remembered that the diffusing substance is heat not temperature. Equation (1.5) may be solved by prescribing temperature and/or temperature gradient on the boundary of the solid. Refer to [4] for a host of available solutions to equation (1.5) in heat conduction. The governing equation for an anisotropic and nonhomogeneous medium may also be derived without difficulty.

The thermal diffusivity D_h in equation (1.6) can depend on temperature through the thermal conductivity coefficient k . With $Q = 0$, equation (1.5) in one dimension, say x , becomes

$$D_h \frac{\partial^2 T}{\partial x^2} + \frac{\partial D_h}{\partial T} \left(\frac{\partial T}{\partial x} \right)^2 = \frac{\partial T}{\partial t}. \quad (1.7)$$

*No distinction is made here between the specific heat at constant pressure c_p and specific heat at constant volume c_v .

A simplified method for treating temperature dependent diffusion coefficient prevails when equation (1.7) can be linearized. A mean value of D_h or \bar{D}_h can thus be defined as

$$\bar{D}_h = \frac{\int_{T_1}^{T_2} D_h(T) dT}{\int_{T_1}^{T_2} dT} \quad (1.8)$$

and measured experimentally over sufficiently small intervals of temperature T_1 and T_2 .

1.3 Moisture diffusion

The diffusion of moisture in a solid is basically the same as that of heat. This was recognized by Fick [4] following the work of Fourier in heat conduction [5]. If f is the rate of moisture transfer per unit area and C the moisture concentration, then in one dimension, say along the x -direction, f can be assumed to be proportional to $\partial C/\partial x$, i.e.,

$$f = -D_m \frac{\partial C}{\partial x}. \quad (1.9)$$

The moisture diffusion coefficient is D_m and has units of length squared over time. The negative sign in equation (1.9) follows the physical notion that moisture diffusion occurs in the direction of decreasing concentration. For an anisotropic three-dimensional body, f may be regarded as the moisture flux vector \mathbf{f} with components corresponding to the axes x_j and D_m may be replaced by the components of a second order tensor as in the case of thermal conductivity.

1.3.1 Moisture concentration field

By a direct mathematical analogy,* C can be identified with T and D_m with D_h . Hence, the net gain of moisture transfer across a volume element in the x -direction as shown in Figure 1.1 can be written down immediately as $-\partial/\partial x (D_m \partial C/\partial x) dx dy dz$. In the absence of any moisture source, the conservation energy gives

$$D_m \nabla^2 C = \frac{\partial C}{\partial t}, \quad (1.10)$$

where the operator ∇^2 can depend on x , y and z .

When the moisture diffusion coefficient varies with concentration, equation (1.10) in one-dimension becomes

$$\frac{\partial}{\partial x} \left(D_m \frac{\partial C}{\partial x} \right) = \frac{\partial C}{\partial t}. \quad (1.11)$$

*The flux of moisture f in equation (1.9) would be equivalent to the normalized heat flux q/cq , i.e., $q/cq = -D_h(\partial T/\partial x)$.

Experimental measurement of $D_m(C)$ involves the integration between two moisture concentrations C_1 and C_2 :

$$\bar{D}_m = \frac{\int_{C_1}^{C_2} D_m(C) dC}{\int_{C_1}^{C_2} dC}, \quad (1.12)$$

where \bar{D}_m is interpreted as a mean value found by measuring f . The slopes of a \bar{D}_m versus C plot yield the concentration-dependence of the moisture diffusion coefficient. In what follows, the distinction between D_m and \bar{D}_m will be understood.

1.3.2 Non-steady state moisture distribution

A simple example is that the case of a slab $-h_x < x < h_x$ whose surfaces are maintained at a constant concentration C_∞ . Initially, the slab had a uniform concentration C_0 , i.e.,

$$\begin{aligned} C(x, 0) &= C_0, & \text{for } t = 0, \\ C(0, t) &= C(\pm h_x, t) = C_\infty, & \text{for } t > 0. \end{aligned} \quad (1.13)$$

A possible solution of non-steady state moisture concentration that satisfies the conditions in equations (1.13) can be found from equation (1.11) [6, 7] for constant D_m :

$$\frac{C - C_0}{C - C_\infty} = 1 - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1} \cos \left[\frac{\pi(2n-1)x}{h_x} \right] \exp \left[-\frac{\pi^2(2n-1)^2 t D_m}{4h_x^2} \right]. \quad (1.14)$$

Once $C(x, t)$ is known, the total amount of moisture m_t gained by the slab can be obtained as

$$m_t = \int_0^{h_x} C(x, t) dx. \quad (1.15)$$

If m_0 and m_∞ denote the moisture contents corresponding to $t = 0$ and $t = \infty$, then a normalized form of moisture gained can be written as

$$\frac{m_t - m_0}{m_\infty - m_0} = 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \exp \left[-\frac{\pi^2(2n-1)^2 t D_m}{4h_x^2} \right]. \quad (1.16)$$

The above solution provides a means for the experimental determination of D_m .

1.3.3 Time dependence

One way of finding D_m is to measure the rate of sorption and desorption of moisture in a slab. This is accomplished by monitoring the weight of the slab in a vapor environment under controlled temperature and pressure [8]. The data are then presented on an $m_t - m_0/m_\infty - m_0$ versus \sqrt{t}/h_x plot. The curve is initially linear and consistent with Boltzmann's solution for diffusion in a semi-infinite medium [9], i.e.,

$$\frac{m_t}{m_\infty} = \frac{4}{\sqrt{\pi}} \sqrt{\frac{t D_m}{h_x^2}}. \quad (1.17)$$

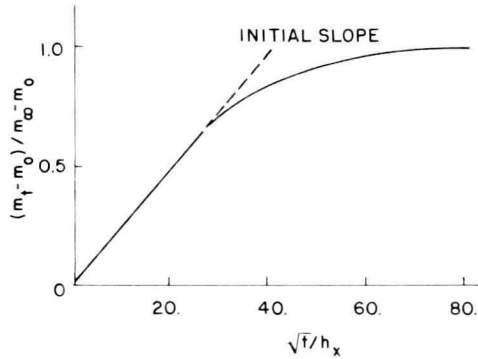


Figure 1.2. Schematic of variations of normalized moisture content with time.

It follows from equations (1.17) and (1.16) that

$$D_m = \frac{\pi}{16} \left[\frac{d(m_t - m_0/m_\infty - m_0)}{d(\sqrt{t}/h_x)} \right]^2 = \frac{\pi}{16} \left[\frac{(m_t - m_0)/(m_\infty - m_0)}{\sqrt{t}/h_x} \right]^2 \quad (1.18)$$

can be deduced from the initial slope of the $(m_t - m_0)/(m_\infty - m_0)$ versus \sqrt{t}/h_x curve. The schematic of a typical curve is shown in Figure 1.2.

1.3.4 Temperature variation

The moisture diffusion coefficient D_m is known to depend on the temperature [10] in the form

$$D_m = D_0 \exp(-E_d/R_g T) \quad (1.19)$$

in which D_0 is a constant, E_d the activation energy, R_g the gas constant, and T the temperature measured on the Kelvin scale. Of particular interest are some typical values of D_0 and E_d for composite laminates and epoxy resins engulfed in liquids and humid air [11–16]. They are shown respectively in Tables 1.1 and 1.2.

Equation (1.19) implies that $\log D_m$ should be a linear function of $1/T$. This relationship is displayed in Figure 1.3 for the T300/5208 laminate [15] in a moisture environment.

Table 1.1. Constants D_0 and E_d for composite systems in distilled water and saturated salt water [11].

Material	D_0 ($\times 10^{-6} \text{ m}^2/\text{s}$)		E_d ($\times 10^4 \text{ KJ/Kgmol}$)	
	Distilled	Salt	Distilled	Salt
T300/1034	16.3	5.85	5.16	5.01
AS/3501-5	768.0	53.8	6.00	5.40
T300/5208	132.0	6.23	5.61	4.92