PRECALCULUS MATHEMATICS

AN ELEMENTARY FUNCTIONS APPROACH

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Precalculus Mathematics

An Elementary Functions Approach

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Precalculus Mathematics:
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To Devora, Lorrie, and Jeannie

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Preface

Mathematics has come to play an increasingly important part in the general curriculum of high schools, colleges, and universities; sets, linear inequalities, graphs, functions, and exponential growth are just some of the concepts often encountered in such diverse subjects as agriculture, biology, economics, psychology, and sociology, not to mention calculus and other areas of higher mathematics. These concepts and others are based on a knowledge of basic high-school algebra.

In setting a goal for this text, I kept in mind that the student will be expected to gain an understanding of the basic concepts and their use, as well as proficiency in certain manipulative skills. For this reason, new concepts, theorems, and definitions are accompanied by many illustrative examples and step-by-step, worked-out problems. These problems also introduce techniques for doing the exercises and serve as models for them. There is a worked-out problem for virtually every type of exercise that the student is expected to do. This integrated treatment of theory and problem solving involves the student in the act of "doing mathematics." It develops his appreciation of mathematical reasoning and methods of proof, while at the same time building up his capabilities for dealing with more and more diversified problems.

To increase the usefulness of the book as an aid for review, each chapter ends with a comprehensive quiz. Also, in addition to numerical tables, the book includes tables of exponents, common logarithms, natural logarithms, and trigonometric functions.

In general, the text follows the CUPM recommended guidelines for Mathematics O (CUPM is the Committee on the Undergraduate Program in Mathematics). The independence of Chapters 7, 8, and 9 offers great flexibility in designing a course of study. It makes the text suitable for a one-semester, one-quarter, or two-quarter course.

I wish to express my thanks to Richard Reed for solving the exercises and suggesting various improvements in the exposition. Barbara Federman prepared excellent sketches for the artwork and in other ways, as well, contributed her many talents and skills to this project. The typing was expertly done by Delores Brannon and Sonía Ospina. Thanks are also due George Telecki, Mathematics Editor, Lois Wernick, Project Editor, and the other members of the excellent staff of Harper & Row.

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Real Numbers

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1.1 RATIONAL NUMBERS

For the purpose of this text, a *rational number* can be thought of as a fraction A/B, where A and B are integers and $B \neq 0$. Rational numbers are associated with points on a straight line as follows.

On a line, taken for convenience to be horizontal, we distinguish an arbitrary point as the *origin* and mark it 0. A second point is chosen to its right and marked 1. The segment $\overline{01}$ is called *unit of length* (Figure 1.1). By means of this unit of length we mark off equally spaced points to the right and left of 0. The point that lies n units to the right of 0 is called n, and the point lying n units to the left of zero is called $-n$ (Figure 1.2). Dividing each segment so created into two equal parts gives the points associated with $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{3}{2}$, $-\frac{3}{2}$, and so on. In general, dividing each segment into n parts gives the points associated with $1/n$, $-1/n$, $2/n$, $-2/n$, $3/n$, $-3/n$, and so on (Figure 1.3).

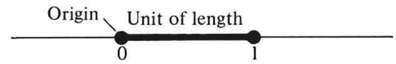

Figure 1.1

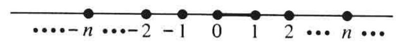

Figure 1.2

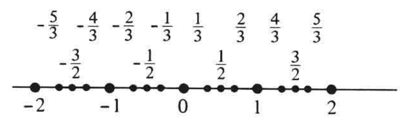

Figure 1.3

Of great significance in mathematics is the fact that there are infinitely many points on the straight line that have no rational number corresponding to them. If P is such a point, then the line segment $\overline{0P}$ has no length associated with it. This is a serious flaw in the rational number system, and we say that it is *incomplete*. To substantiate the claim that certain points on the straight line have no rational number associated with them, we shall exhibit such a point.

Consider a right triangle of base and height of length 1 (see Figure 1.4). If we mark the length of its hypotenuse x, then by the theorem of Pythagoras

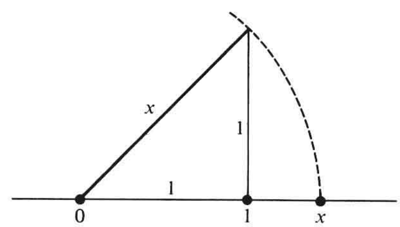

Figure 1.4

$$1^2 + 1^2 = x^2 \quad \text{or} \quad x^2 = 2$$

The positive solution of this equation is designated $\sqrt{2}$; because this number cannot be expressed as the quotient of integers, it is said to be *irrational*.

Theorem 1.1

The number $\sqrt{2}$ is irrational.

REMARK

The proof of this theorem utilizes the following fact: *If a single assumption leads to a false conclusion, then the assumption is likewise false.* We assume, of course, that the steps leading from the assumption to the conclusion are logically correct. Proofs using this fact are called *proof by contradiction*.

Proof

Suppose, on the contrary, that $\sqrt{2}$ is rational. Then there are positive integers A and B such that

$$\sqrt{2} = \frac{A}{B} \tag{1.1}$$

We take the fraction A/B to be in lowest terms, since any common factors of A and B can be eliminated. From the relation

$$2 = \frac{A^2}{B^2}$$

which follows from squaring both sides of Eq. 1.1, we get

$$A^2 = 2B^2 \tag{1.2}$$

This tells us that A^2 is *even*; hence so is A (because if A were odd, A^2 would also be odd). We can therefore write $A = 2C$ for some integer C, and with this substitution Eq. 1.2 becomes $(2C)^2 = 2B^2$, or $4C^2 = 2B^2$. Eliminating the factor 2 gives

$$2C^2 = B^2$$

which shows that B^2 is even, and hence B is even. We have thus arrived at the conclusion that A is even and B is even. Hence A and B have a common factor, 2. This contradicts the fact that A and B have no common factors at all. Therefore the assumption that $\sqrt{2}$ is rational is false, and accordingly $\sqrt{2}$ is irrational.

The reason that led us to regard the rational number system as incomplete also suggests how to "complete" the system: to each point on the line lying to the right of the origin we assign a number which represents its distance from 0; to each point lying to the left of the origin we assign a number representing minus its distance from 0. To represent these nonrational distances we shall use decimals. We shall have more to say about the "completeness" of the real number system in Sections 1.2 and 6.1.

The basic rules for operating with rational numbers follow.

RULES FOR OPERATING WITH RATIONAL NUMBERS

(1) $\dfrac{A}{B} + \dfrac{C}{D} = \dfrac{AD + BC}{BD}$

(2) $\dfrac{A}{B} - \dfrac{C}{D} = \dfrac{AD - BC}{BD}$

(3) $\dfrac{A}{B}\dfrac{C}{D} = \dfrac{AC}{BD}$

(4) $\dfrac{A}{B}\Big/\dfrac{C}{D} = \dfrac{AD}{BC}$

EXAMPLES

(1) $\dfrac{2}{3} + \dfrac{5}{7} = \dfrac{2 \cdot 7 + 5 \cdot 3}{3 \cdot 7} = \dfrac{29}{21}$

(2) $\dfrac{2}{3} - \dfrac{5}{7} = \dfrac{2 \cdot 7 - 5 \cdot 3}{3 \cdot 7} = \dfrac{-1}{21} = -\dfrac{1}{21}$

(3) $\dfrac{2}{3} \cdot \dfrac{5}{7} = \dfrac{2 \cdot 5}{3 \cdot 7} = \dfrac{10}{21}$

(4) $\dfrac{2}{3}\Big/\dfrac{5}{7} = \dfrac{2 \cdot 7}{3 \cdot 5} = \dfrac{14}{15}$

1.1 EXERCISES

1. Insert one of the symbols $=$ or $\neq$ into each box to produce a true statement.

   (a) $1\Big/\dfrac{A}{B} \square \dfrac{B}{A}$

   (b) $\dfrac{A}{B} - C \square \dfrac{BC}{A}$

   (c) $\dfrac{2A + 3B}{4A + 9B} \square \dfrac{A + B}{2A + 3B}$

   (d) $\dfrac{A + B - C}{B} \square 1 + \dfrac{A - C}{B}$

   (e) $\left(\dfrac{A}{B} + C\right)\left(\dfrac{A}{B} - C\right) \square \dfrac{A^2 - B^2C^2}{B^2}$

   (f) $\frac{1}{2} + \frac{1}{3} - \frac{1}{6} \square \frac{2}{3}$

   (g) $\frac{1}{2}/\frac{1}{3} \square 1 + \frac{1}{2}$
2. Use Theorem 1.1 to explain why $\sqrt{4} \neq \sqrt{2} + \sqrt{2}$.
3. Let r be any rational number. Explain why $r + \sqrt{2}$ is irrational and $r \times \sqrt{2}$ is irrational when $r \neq 0$.
4. Follow the routine in the proof of Theorem 1.1 to show that $\sqrt{3}$ is irrational.

5. Can you list five irrational numbers lying between 0 and 1?
6. Can you list five irrational numbers lying between 1 and 10?

Decide which of the numbers in Exercises 7–14 are *rational* when you know that $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{6}$ are irrational.

7. $\sqrt{2} + \sqrt{3}$

   (Hint: Assume $\sqrt{2} + \sqrt{3} = A/B$ for some integers A and B. Write $\sqrt{2} = (A/B) - \sqrt{3}$ and square both sides.)
8. $(2 - \sqrt{2})(2 + \sqrt{2})$
9. $\dfrac{2 - \sqrt{2}}{2 + \sqrt{2}}$

   (Hint: Multiply the numerator and denominator by $2 - \sqrt{2}$.)
10. $\sqrt{2} \times \sqrt{3}$
11. $\sqrt{2} \times \sqrt{3} \times \sqrt{6}$
12. $\dfrac{\sqrt{6}}{2\sqrt{3}} - \dfrac{1}{\sqrt{2}}$
13. $(\sqrt{2} - \sqrt{3})^2 + 2\sqrt{6}$
14. $(\sqrt{2} + \sqrt{3})^2 - (\sqrt{2} - \sqrt{3})^2$
15. Is there a smallest positive rational number? Explain your answer.
16. Is there a largest rational number? Explain your answer.

1.2 DECIMAL EXPANSIONS

Converting fractions to decimals is done by long division. Consider the following examples.

$\frac{1}{5} = 0.2$ $\qquad \frac{25}{8} = 3.125$

$\frac{7}{5} = 1.4$ $\qquad \frac{1}{6} = 0.16666\ 6\ \ldots$

$\frac{9}{14} = 0.6428571428571\ \boxed{428571}\ \ldots$

$\frac{15}{51} = 0.2941176470588235\ \boxed{2941176470588235}\ \ldots$

An example of the fundamental process of long division is

```
   3.125
8 | 25
    24
     10
      8
     20
     16
      40
      40
       0
```

The three dots in the last three decimal expansions indicate that the boxed-in digits repeat over and over again. The first three decimals are called *terminating decimals*, the last three *nonterminating decimals*. The digits in a decimal have a simple geometric interpretation, for they tell us how to locate a given number on the line.

EXAMPLE 1

Consider the number

$$\tfrac{13}{6} = 2.16666\ldots$$

The inequalities

$$2 < \frac{13}{6} < 3$$

$$2 + \frac{1}{10} < \frac{13}{6} < 2 + \frac{2}{10}$$

$$2 + \frac{1}{10} + \frac{6}{10^2} < \frac{13}{6} < 2 + \frac{1}{10} + \frac{7}{10^2}$$

$$\vdots$$

tell us with increasing accuracy where $\frac{13}{6}$ is to be found (see Figure 1.5). Thus decimals are obtained geometrically by successively dividing intervals into 10 equal subintervals.

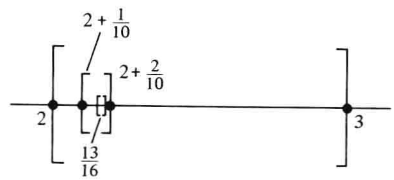

Figure 1.5

REPEATING DECIMALS

The last three decimals above share an interesting feature: They are *repeating decimals*, which is to say that a certain string of digits recurs over and over, without change. The following notation is customary for repeating decimals:

$$\tfrac{1}{6} = 0.1\overline{6}$$
$$\tfrac{9}{14} = 0.6\,\overline{428571}$$
$$\tfrac{15}{51} = 0.\overline{2941176470588235}$$

The bar indicates that from this point on, this part of the decimal repeats over and over. In general,

> a decimal is *repeating* if after a certain number of digits, it consists of repetitions of a fixed string of digits.

A repeating decimal thus has the form

$$a.a_1a_2a_3 \cdots a_n\overline{b_1b_2b_3 \cdots b_m}$$

where each of the symbols $a_1, \ldots, a_n, b_1, \ldots, b_m$ is one of the digits 0, 1, . . . , 9, and the sequence $b_1b_2b_3 \cdots b_m$ is repeated indefinitely. The following fact allows us to convert every terminating decimal to a nonterminating repeating decimal.

EXAMPLE 2

Consider the decimals $a = 0.99999\ldots$ and $1 = 1.00000\ldots$. Then examine the following differences:

$$
\begin{aligned}
1 - 0.9 &= 0.1 \\
1 - 0.99 &= 0.01 \\
1 - 0.999 &= 0.001 \\
1 - 0.9999 &= 0.0001 \\
1 - 0.99999 &= 0.00001 \\
&\vdots
\end{aligned}
$$

We observe that as we locate a with increasing accuracy, it comes closer and closer to 1. Hence, a is indistinguishable from 1, and it turns out that $1 = 0.\overline{9}$. Similarly, we argue that

$$
\begin{aligned}
0.1 &= 0.0\overline{9}, \\
0.01 &= 0.00\overline{9}, \\
0.001 &= 0.000\overline{9}, \text{ etc.}
\end{aligned}
$$

PROBLEM 1

Convert 0.2 to a nonterminating decimal.

Solution

$$0.2 = 0.1 + 0.1 = 0.1 + 0.0\overline{9} = 0.1\overline{9}$$

PROBLEM 2

Convert $\frac{41}{5}$ to a nonterminating decimal.

Solution

$$\frac{41}{5} = 8.2 = 8.1 + 0.1 = 8.1 + 0.0\overline{9} = 8.1\overline{9}$$

PROBLEM 3

Convert 0.12901 to a nonterminating decimal.

Solution

$$
\begin{aligned}
0.12901 &= 0.12900 + 0.00001 \\
&= 0.12900 + 0.00000\overline{9} \\
&= 0.12900\overline{9}
\end{aligned}
$$

PROBLEM 4

Show that 1.257 is rational.

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