

# Laboratory Physics

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Rensselaer Polytechnic Institute

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#### PREFACE

During the last few decades technological advances in all fields of engineering and science and the growth of solid state physics, nuclear and high energy physics, and quantum theory have emphasized the microscopic world, the world of the atom and its parts. This emphasis requires every embryo engineer and scientist to develop his individual initiative to see, to question and, if possible, to find out why. This cannot be done quickly but requires a gradual, directed introduction to the fundamental methods of analysis.

The objectives of the physics laboratory are not, as is often imagined, simple or easy to define. They cannot be generalized in a single sentence. The object of the physics laboratory of today is not the verification of a known law and the blind substitution of data into a formula with the consequent "cranking out" of an answer. The object of a physics laboratory is not to duplicate the engineering laboratory which is concerned with major applications of physical laws and engineering techniques, although occasionally, where repetition as a learning aid is advisable, this is done.

The laboratory is not a separate course in physics. It is an integral part of the theoretical study and must be considered as such. In the laboratory the student comes into actual contact with the fundamental laws and principles which are presented in the theory recitation classes.

Before the student can see, question and find out why, he must master the fundamental tools (or techniques) necessary for the ultimate satisfaction of his curiosity. The degree of mastery depends on the attitude of the student toward his laboratory work.

The objectives of this laboratory are:

- 1. To introduce the student to the significance of the experimental approach through actual experimentation.
- 2. To apply the theory of the recitation class to field problems which help to develop a better understanding of the fundamentals of classical and modern physics.
- 3. To introduce the student to the methods of data analysis used throughout science and engineering.
- 4. To develop an "error conscience" so that the engineer and scientist will at least be aware of the relative worth of his measurements, whatever their type. The method of "precision analysis" presented here should not be thought of as the only way of determining a measurement's relative worth. Precision analysis appears in various guises in all branches of business, government and science. The student should become familiar with its many facets and realize that while learning a fundamental technique, he is also becoming more proficient in applying the calculus.
- 5. To familiarize the student, by direct contact, with a great many basic measuring instruments and their applications.

- 6. To make the student realize that such tools as graphing, difference analysis, the use of calculus, etc., are of fundamental importance.
- 7. To impress on the student that even an experiment which is apparently unimportant to his professional future may contribute directly to his mental development because of the analytics and mathematics involved.
- 8. To introduce the student, from the first day, to the use of references, thereby showing him the value of other viewpoints and of the problem approach. The habit of using references, because their fresh interpretations combat mental inertia, will be invaluable to the student's academic and later professional progress.
- 9. To improve the student's ability of self expression through report presentation.
- 10. To give the student direct contact with his instructor, and thus the advantages of close direction and personal discussion of ideas and methods.

How some of these objectives are accomplished, depends on the type of laboratory course offered. In recent years there has been talk of various types of introductory physics experiments, such as the "traditional" laboratory, the "free" laboratory, the "diverging" laboratory or the "Berkeley" laboratory. In the opinion of the authors not any one approach is necessarily more successful than others. In this text we have tried to combine as many different ways of doing things as possible. This gives the instructor or the student the chance of picking the type of experiment they prefer.

Some of the experiments contain rather detailed instructions while others are - on purpose - very vague, leaving the details up to the student. The experiments differ greatly in difficulty as well as in length. Some use very simple equipment while in others the apparatus is rather sophisticated. Most of the experiments have a number of different parts and it is not expected that the student finishes all parts; very often he may have a choice.

This laboratory text is not to be regarded as an instruction book to be followed word by word; it is meant to be a general guide to various experiments in physics. It is hoped that the student will deviate from the directions given and will add his own parts to the experiments. Student creativeness and initiative are stressed by suggesting various projects without going into details. Some individual experiments as well as the introduction to chapters often suggest further work. Some of the equipment can easily be used for "free" experiments not described; examples are the low-friction devices discussed in Chapter 5, the oscilloscope with accessories, the microwave apparatus and the laser.

Although no specific mention has been made in any experiment of the role of computers in the physics laboratory, it is obvious that their possible use should not be overlooked. The extent of using the computer for lengthy calculations depends on the availability of facilities and the experience of the students.

The computer may add interest and motivation as far as the student is concerned. Programs for many of the experiments have been written and used by students; they were able to carry out their calculations in much more detail , especially when errors are evaluated. Many of the sophomores taking our physics laboratory have used their data to practice programming or fulfill requirements of a computer programming course.

Report writing has traditionally been a time - consuming activity in connection with physics laboratories. We do not want to minimize the importance of well presented experimental results, but this can be accomplished without too much busy work. The authors do not suggest how the laboratory should be organized and what type of reports, if any, the student should present. This is left up to the instructor in charge of the course. We have had very good results with "oral reports" - students report to their class or just their instructor the results obtained in an experiment. Students should be encouraged to record all data - including qualitative observations - in a laboratory notebook.

The first chapter on errors must be studied in some detail - at some time during the first physics course the material should be assigned in a formal manner. The ideas of Chapter 2 on graphing will be applied throughout the experiments and should be referred to at the appropriate time. Chapter 3 includes a very brief description of some important pieces of equipment - students should read these sections before handling the apparatus involved.

Although this book is based on the text "Analytical Laboratory Physics" published by the same authors in 1956, a great many experiments have been added. Most of the old experiments have been changed to be less formal, to include more optional parts and to make use of newly available equipment. Air tracks and air tables, for instance, did not exist and the laser had not yet been invented in 1956. Both are used in many of the present experiments.

The authors wish to express their appreciation to the thousands of Freshmen and Sophomores at Rensselaer Polytechnic Institute who have tried out all of the experiments and have helped us greatly by their constructive criticism. They are also grateful for the many comments they have received from the graduate assistants teaching in our introductory laboratories as well as from faculty members. We also wish to express our appreciation to the manufacturers of scientific equipment who have supplied us with photographs and literature relating to their products. The authors are also indepted to Mrs. Walter Eppenstein for her typing of the final manuscript.

May 1972 Troy, New York The Authors

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#### Chapter 1

#### MEASUREMENT AND ERROR

The content and level of the material of this chapter have been designed to make it appropriate and usable from the beginning of a college basic physics sequence. For this reason, no developments involving more than quite elementary calculus have been included. While relations which are derived from statistical theory are presented without proof, every effort has been made to present the basic ideas in logical and understandable fashion.

Students in the latter part of multi-semester general physics sequences or in second year or intermediate level courses will find it desirable to consult or purchase one or more of the following more extensive treatments (all concerned with <a href="https://physical.org/physical.org/">physical.org/</a> applications):

- 1. Lyman G. Paratt, <u>Probability and Experimental Errors in Science</u> (John Wiley & Sons, Inc., New York, 1961, Paperback 1966) p. 255.
- 2. J. Topping, <u>Errors of Observation and Their Treatment</u> (Reinhold, New York, 1955 et seq., British Inst. Phys. Monograph for Students, Paperback) p. 119.
- 3. D. C. Baird, Experimentation an Introduction to Measurement Theory and Experimental Design (Prentice-Hall, Inc., Englewood, New Jersey, 1962) p. 198.
- 4. E. M. Pugh and G. H. Winslow, <u>The Analysis of Physical Measurements</u> (Addison Wesley Publishing Co., Inc., Reading, Massachusetts, 1966, Paperback) p. 248.

There are also available many excellent introductory and intermediate books and texts concerned with mathematics statistics, per se.

#### 1.1 Introduction

Lord Kelvin once summed up the importance of measurement as part of the very essence of a science in a statement which will bear re-quotation:

'T often say that when you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be."

It is familiar knowledge that the specification of a physically measurable quantity requires at least two items:

- l. a number
- 2. a unit

(Plus, when necessary, a statement of direction for vector quantities and certain tensor quantities.)

Frequently neglected is a third item of virtually equal importance:

3. an indication of the reliability or degree to which we can place confidence in the value stated. This is usually done by specification of a 'precision index'.

As a science develops and matures, its experiments tend to become more exact and/or sophisticated, but there are always areas or particular experiments in which only relatively low precision is obtainable and others in which moderate or relatively low precision is quite adequate or even desirable. At the frontiers of a developing branch (nuclear or space science as examples), extremely high precision may be possible and necessary in one instance; in another, to obtain a value reliable within 100% or even to an order of magnitude may be a great break-through.

Every scientist and engineer should be able to assess the relative worth of a measurement, direct or calculated. He should develop an "error conscience", awake at all times even when not in full operation. This is fully as important in dealing with quantities of low precision as in dealing with very exact ones. To help attain this goal, this chapter will serve as an elementary introduction to measurement and error theory as applied to direct measurements of a single entity and to results calculated from them (indirect measurements).

## 1.2 Limit of Error, Confidence Intervals and Calculated Risk

To understand and appreciate the methods of measurement analysis, it is important to realize that the numerical value of a measurement is not a unique number like the integer 3274 or even the irrational numbers 2/3 or  $\pi$ . On the contrary, whether it results from a single trial or from repeated observations, a measurement is only a sample from the population of all possible observations. It is subject to statistical fluctuations due to environmental and other agencies; is obtained by the use of instruments which cannot be made completely free of error; and involves the observer, a fallible human, to a greater or lesser degree, as the final element in the measurement process. Even the choice of unit may introduce small errors due to uncertainty associated with the very specification or definition of the unit itself. It is truly remarkable that present day measurements have attained, in a few instances, an accuracy and precision in the order of one part of uncertainty in  $10^{11}$  parts of measured quantity!

The simplest indication of relative reliability is proper attention to significant figures or digits. This is quite adequate for many routine purposes, and is certainly to be borne in mind consistently and used when insufficient data or time makes more detailed analysis impossible. Significant digits will not be discussed here. Experience in high school laboratories and in General Chemistry can be supplemented by the summary in Art.1.24.

Let us suppose that one encounters a measurement reported as  $28.4_6 \pm 0.2_5$  km/sec. Simple attention to significant figures would round the value to 28.5 km/sec (3 significant digits). It is obvious that  $\pm 0.2_5$  km/sec (a trifle less than 1% of the measured  $28.4_6$  km/sec) plays the role of a precision index or error measure of some sort. Unfortunately, as we shall see, there are a number of calculable quantities which may be used, and it is important that it be made clear which one is stated.

One accurate and complete statement could be given as:

(a)  $28.4_6$  km/sec, with a limit of error of  $\pm 0.2_5$  km/sec or  $28.4_6$  km/sec, with a limit of error of  $\pm 0.9\%$ .

This means that to the best of his ability, the experimenter guarantees<sup>1</sup> that the error is not greater than the limiting value given. To rephrase the result: - one can be sure (confidence level 100%, i.e., certainty) that the value is between 28.2 and 28.7 km/sec.

lA guarantee for an automobile is only as good as the manufacturer and dealer make it. Guarantee of experimental data is based on the integrity of instrument makers and experimenters.

Limits of error, while safe, are conservative, sometimes overly so. When replication of experimental values is possible, as it is more often than not, the powerful tools of modern statistics can be used to narrow the confidence interval or error range quoted, but at the expense of a lowering of the confidence <u>level</u>. For comparison, checking or establishing criteria, this is the equivalent of taking a calculated risk, with fairly stated odds. Several examples follow, to illustrate the general idea. The indices mentioned are discussed later.

- (b) 28.46 km/sec, probable error  $^{\pm}$  0.05 $_4$  km/sec. This is a poor name, retained by inertia. It implies a confidence level of (approximately) 50%, i.e., it is about "even-steven" whether we know the value within  $^{\pm}$  0.54 km/sec or not. This index tends to make values look better than they really are, and is likely to confuse the unwary. Formerly very popular with physicists, its use is now becoming much less frequent.
- (c) 28.46 km/sec, standard error (S.E.)  $\pm$  0.081 km/sec. Again, not a good name, but a very sound statistical quantity; the index probably used more than any other today. It implies a confidence level of about 68%; that infinite replication will produce values within  $\pm$  0.81 km/sec of 28.46 km/sec about 2/3 of the time -- or that the odds are 2:1 that the confidence interval 28.38 28.54 km/sec brackets the "true" value.
- (d) Other frequently used indices are  $\pm 2$  S.E., confidence level 95%, and  $\pm 3$  S. E., confidence level 99.7%; a rather commonly used measure of a statistical <u>limit</u> of error, since the chances of exceeding it are so small. There are only about 3 chances in a thousand of exceeding 3 S. E.

It should be clear at this point why the precision index quoted should be identified, either for a single quantity, or consistently throughout a series of measurement procedures.

#### 1.3 Types of Measurement

Measurements may be either (1) DIRECT or (2) INDIRECT:

(1) <u>Direct measurements</u> are the result of direct comparison, usually with the aid of instruments, of an unknown amount of a physical entity x with a known or standarized amount of the same entity s. Three of the most important types of direct comparison are:

#### (a) Balanced, equality or null measurements

Here, the standard value s is selected or adjusted to be equal to x and the balance value is recorded. Measurements of this sort can give extremely precise results.

An obvious example is weighing with the old-fashioned, equal-arm balance. Another is the measurement of electrical resistance by the use of a 1.1 ratio Wheatstone Bridge.

#### (b) Small difference measurements

If the difference

$$x - s = \Delta$$
 is determined, then  $x = s + \Delta$ . ( $\Delta$  may be + or -)

When  $\Delta$  is small, relatively large uncertainties in its value may be tolerated (on a percentage basis), without introducing large errors in x. As an example, consider the measurement of a length a trifle greater than 0.5 inch, using a 0.5 inch gage block as a standard (s) and a difference detector for  $\Delta$ . Suppose that a dial indicator is set to read ZERO, using a gage block for which s = 0.500 000 inch  $\pm$  5 microinches ( $\pm$ 0.000 005 inch). Now substitute x for s and suppose that the dial gage indicates the difference  $\Delta$  = 98  $\pm$  2 microinches.

It is clear that x = 0.500 098 inch. It will also be noted that the  $\pm 2$  microinch uncertainty in  $\Delta$ , which is  $\pm 2\%$  of the value  $\Delta$ , contributed less to the uncertainty in x than did the  $\pm 5$  microinch (0.001%) tolerance in the 0.500 000 inch value of s. This is a rather extreme example but illustrated the possibilities.

#### (c) Ratio measurements

An unknown x is compared with a known s in terms of some fraction or multiple (R).

then x = Rs

(R is to be operationally determined)

An excellent example is the linear potentiometer, as used in measuring devices and in analog computers. In this case a standardized electrical potential difference s is applied across a resistance. The latter is usually a long, uniform wire (which may be wrapped up or doubled back) arranged so that a sliding contact can pick off a determinable fraction of its length (R), and therefore the same fraction of its resistance. For uniform wire, this also means that the potential difference picked off also has the ratio (R) to that of s.

The precision with which R is determined (on a fractional or percentage basis) will be a major factor in the reliability of the measured value of x (See Fig. 1-1).

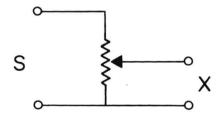


Fig. 1-1.

(2) <u>Indirect</u> or <u>derived measurements</u> result from calculation of a value as a function of one or more direct measurements.

A very simple example of an indirect measurement based on a single direct measurement is the determination of the volume of a sphere from a direct measurement of its radius, or better, its diameter:

$$V = \frac{1}{6} \pi D^3.$$

The diameter D, a length, can be measured directly with sufficient accuracy to suit most purposes. The volume V can then be calculated, --- any unreliability in D being thrice compounded.

Again, the volume of a right circular cylinder of diameter D and length L is

$$V = \frac{\pi}{4} D^2 L.$$

If D and L are measured by unbiased observers using different instruments, errors in D and L will not necessarily compound. There will, of course, be a compounding effect in  $D^2$ .

IT IS OUR PURPOSE TO MAKE AVAILABLE A SIMPLE BUT RELIABLE PROCEDURE FOR EVALUATING THE RELIABILITY OF DIRECT MEASUREMENTS, AND FOR FOLLOWING UNCERTAINTIES THROUGH THE CALCULATIONS INVOLVED IN AN INDIRECT MEASUREMENT.

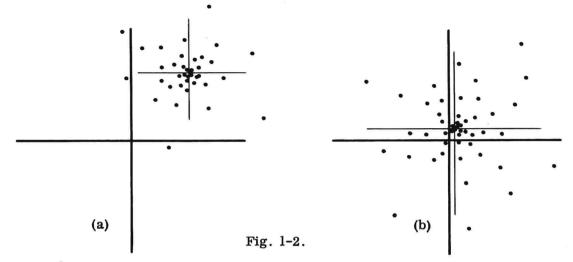
#### 1.4 Experimental Errors

Gross mistakes are not considered part and parcel of experimental errors. The misuse of instruments, mis-reading of scales, mis-handling of data, numerical and mathematical mistakes and other similar blunders can be discounted as unnecessary when adequate training, supervision, experience and cooperation are available. Unfortunately, the same cannot be said of experimental errors, which, though they may be reduced, are always with us. The dividing line between minor mistakes and unavoidable errors becomes rather tenuous.

It is customary and convenient to divide experimental error into two classes because of difference in character and of methods of treatment.

- (a) <u>Systematic errors</u> are due to assignable causes and are, at least in principle, determinable or correctable if enough is known about the physics of the process. They are called systematic because they result in consistent effects -- values which are consistently too high or too low. Methods for dealing with systematic errors and typical examples are presented in Articles 1.5 through 1.11.
- (b) Accidental (erratic or random) errors are due to the summation of large numbers of small, fluctuating individual disturbances which combine to give results which are too high at one time (or place) and too low at another. The individual causes may be known or only suspected. While accidental errors are usually reducible in over-all effect, they can never be completely eliminated and cannot be individually evaluated. In general, the greater the sensitivity of a measuring process, the more important the role of erratic variations. An introduction to the statistical methods used in their analysis begins in Article 1.12.
- A third class, <u>residual errors</u>, is sometimes introduced to describe remanent but indeterminate systematic errors still present or suspected after as much correction as feasible has been carried out. Since they are indeterminate, they can be included with accidental errors though they must be treated slightly differently.

A nice analogy for systematic and random deviations is provided by the shot patterns obtained when a rifle is fired at a target from a machine rest. The rifle is aimed at the center of the 'true" target just as we try to secure experimental data which will provide a 'true" value. The patterns shown in Fig. 1-2 represent moderately sized sampled of shots from two different rifles, each set up and aimed at identical targets in the same manner.



In both cases, the center of fire is systematically displaced from the target center, less in pattern (b) than in (a). It has been proposed but not universally accepted that the word accuracy be associated with systematic displacements ("errors") of this sort. On this basis, pattern (b) is more accurate than (a). The spread or dispersion of the individual "values" (whose individual locations vary randomly and cannot be predicted in advance) is less in (a) than in (b). The word precision has been recommended as an inverse measure of dispersion or scatter. On this basis, the precision of pattern (a) exceeds that of (b). It is well to spell out what one means in error analysis, when universally accepted terms are not available.

#### 1.5 Systematic Errors

The behavior of <u>individual</u> fundamental particles, such as electrons, photons etc., - even of <u>individual</u> atoms or molecules cannot be accurately predicted. This is not just a matter of lack of the proper instruments or theories, or of not knowing enough, but is due to the innate quantum mechanical properties of the particles. It is comforting that in the macroscopic universe in which we live and make measurements and most of our observations, the over-all, statistical behavior of the enormous number of particles involved leads to phenomena in which effect follows cause as night follows day. It is not surprising that one tends to feel more or less at home with the business if dealing with errors which are due to systematic disturbing agencies. This does not mean that in every experiment we immediately mount a campaign to eliminate systematics or make them fantastically small. Such procedures, involving considerable expenditures of money and effort, are often unnecessary and sometimes even undesirable. A very large share of measurements are quite satisfactory if the uncertainty is in the range of a few tenths of a percent to a few percent.

Most systematic errors fit neatly into one of four main categories: -

1. Theoretical

3. Environmental

2. Instrumental

4. Observational

#### 1.6 Theoretical Errors

Theoretical errors are concerned with the equations or relationships used in designing or calibrating instruments or used in the determination of indirect measurements. In the limit, one may allow for a small buffer or residual uncertainty for unknown theoretical errors, but, for the most part: