Radiation Heat Transfer

A Statistical Approach

Includes CD-ROM

J. Robert Mahan

RADIATION HEAT TRANSFER

A Statistical Approach

J. Robert Mahan

Department of Mechanical Engineering
Virginia Polytechnic Institute and State University

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RADIATION HEAT TRANSFER

To Bea

PREFACE

This book is a result of the author's thirty years of experience teaching and directing research in radiation heat transfer at Virginia Tech. As is often the case, the book evolved from class notes distributed to critical graduate students. Therefore, it bears the brand not only of the author but also of a generation of bright young scholars who continuously challenged the author to get it right and make it relevant. For better or for worse, the result is a book written for students rather than for professors.

The material in this book is divided into three parts:

Part I: Fundamentals of Thermal Radiation

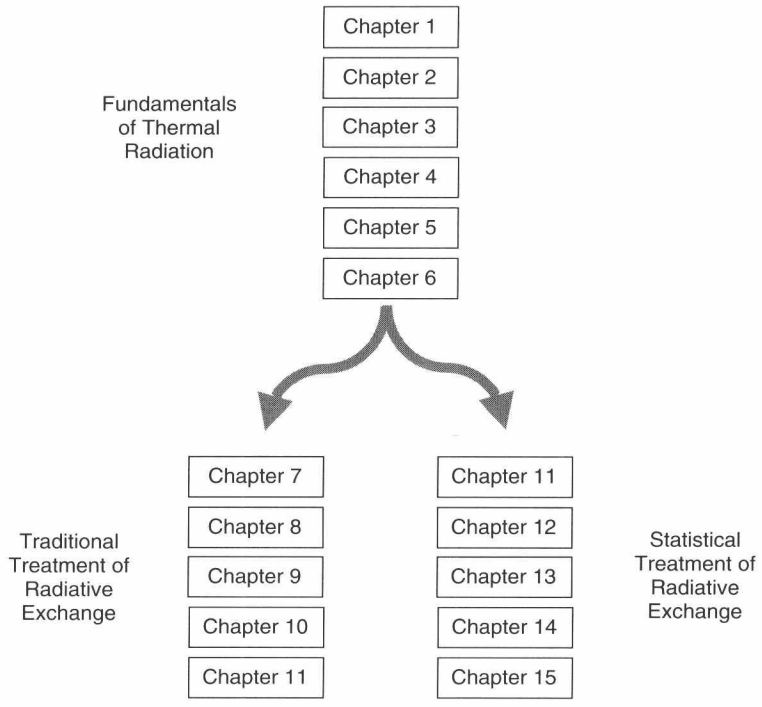
Part II: Traditional Methods of Radiation Heat Transfer Analysis

Part III: The Monte Carlo Ray-Trace Method

If the book is to be used in a one-semester course it is recommended that one of the two options indicated in the figure on page xvi be followed. Both options would use the first six chapters, which present the fundamentals of thermal radiation. A one-semester course emphasizing the traditional methods of radiation heat transfer, which includes the net-exchange formulation, would be based on the first six chapters plus Chapters 7, 8, 9, 10, and 11; while a one-semester course emphasizing the statistical formulation (the Monte Carlo ray-trace method) would use the first six chapters plus Chapters 11, 12, 13, 14, and 15. Chapter 11, "Introduction to the Monte Carlo Ray-Trace Method," is included in both of these options. In addition to these two options, the book is ideally suited for a two-semester (or three-quarter) sequence that covers all of the material.

While authors of recent radiation heat transfer textbooks have included the MCRT method as a viable option, it has usually been presented as an option of secondary importance. In this textbook the method has been promoted to its rightful position as an equal partner in radiation heat transfer modeling. The goal of this book is to present the subject at a level of detail and nuance that will allow the uninitiated practitioner to begin formulating accurate models of complex radiative systems without first assuming away all of the complexity.

If the MCRT method has been criticized in the past for its excessive demand on computer resources, such criticism stands without merit today in a world inundated by a virtual tidal wave of inexpensive computing power. Software tools such as the MCRT-based Program FELIX, the student version of which is packaged with this



book, are now fully capable of free interaction with a wide range of CAD and spreadsheet systems. It seems that this trend must inevitably lead to a future that places increased value on the material in Part III of this book at the expense of the material in Part II. Still, change comes slowly. The traditional methods of Part II are well entrenched in our technical culture and are likely to remain influential in the foreseeable future.

J. R. MAHAN, PhD, PE
Blacksburg, Virginia

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In the mid 1960s one of the most remarkable heat transfer faculties ever assembled came together at the University of Kentucky under the leadership of Dean R. M. Drake, Jr. As a result, from September 1966 through December 1970 I had the privilege of studying heat transfer under the tutelage of Professors Drake, R. C. Birkebac, C. J. Cremers, Roger Eichhorn, and J. H. Lienhard IV. The education I received from these outstanding contributors to heat transfer knowledge, and especially the guidance I obtained from my advisor, Cliff Cremers, largely account for any success I have enjoyed in my subsequent career as an engineering educator.

My good fortune continued in the fall of 1970 when I began my teaching career at Virginia Tech under the direction of J. B. Jones, to whom I am greatly indebted for any acquired teaching skills I may now possess. After my first year in Blacksburg Professor Jones packed me off to NASA's Langley Research Center, where I spent the summer of 1971 under the mentorship of the late George E. Sweet, a founding father of earth radiation budget research. It was this experience as a NASA/ASEE Summer Faculty Research Fellow that determined the course of my future career in radiation heat transfer. In recent years my NASA-sponsored research has been ably monitored by Robert B. Lee III, whose support and encouragement have made it possible for me to work with a steady stream of outstanding graduate students. Over the course of a long career it has been my privilege to direct the doctoral research of a number of exceptional young engineers and scientists. Farshad Kowsary, Thomas H. Fronk, Nour E. Tira, Douglas A. Wirth, Edward L. Nelson, Jeffrey A. Turk, Pierre V. Villeneuve, Martial P. A. Haeffelin, Kory J. Priestley, Félix J. Nevárez, María Cristina Sánchez, Ira J. Sorensen, Dwight E. Smith, and Amie S. Nester have all made important contributions to this book. In addition, the book has benefited from the thesis research of more than forty master-of-science students, including Leo D. Eskin, who in 1980 became the first of my students to study the Monte Carlo ray-trace method.

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A recent semester spent as a visiting professor at the United States Naval Academy permitted me the time to pull the final manuscript together and send it off to my publisher, while at the same time exposing me to a whole other way of doing

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Finally, and most of all, I owe more to my wife, Bea, than I will ever be able to adequately express or repay. Her proofreading, encouragement, support, companionship, and love have made all the difference.

J. R. MAHAN, PhD, PE
Blacksburg, Virginia

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