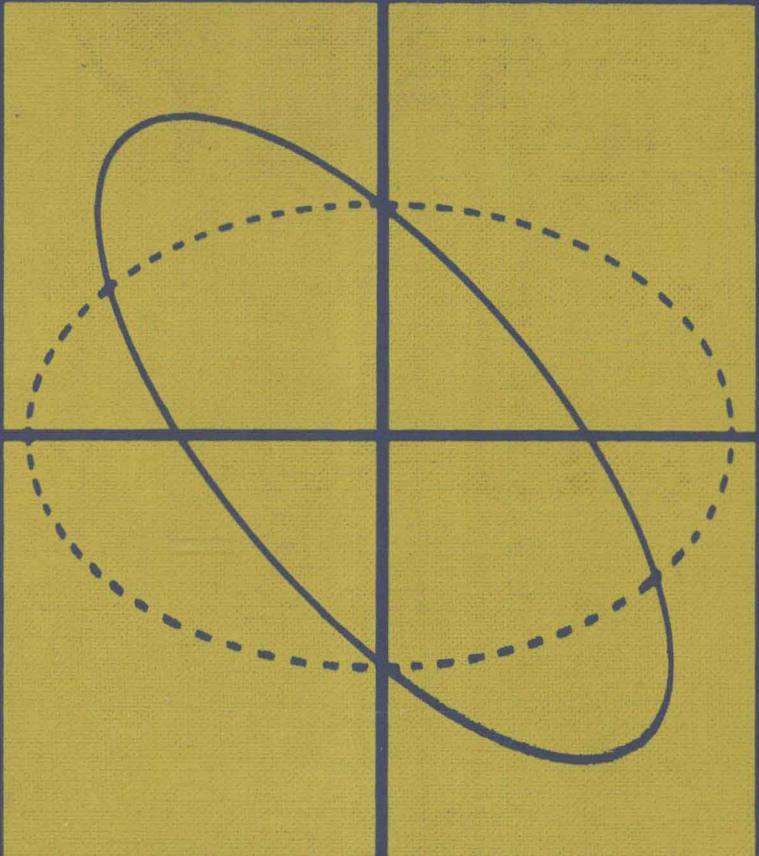
COLLEGE ALGEBRA AND TRIGONOMETRY



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PREFACE

Mathematicians nearly always associate mathematical ideas with pictures; beginners seldom do. This is unfortunate, because it is almost impossible to study many important mathematical functions without also studying their graphs.

Since the main purpose of this book is to prepare the reader for the study of calculus, the analysis of *functions and their graphs* is emphasized throughout. The reader will find that to come up with the correct picture he will not only exercise his skills in arithmetic and algebra, but will also acquire and strengthen his intuition of *continuity*—so indispensable for a further study of mathematics. To help the growth of this intuition, many problems in this book simply ask for the graph of a given function, and the answer (a picture) is given in the answer section.

The basic review (Chapter 1) provides the less well prepared reader with an opportunity to strengthen his techniques in arithmetic and algebra. However, it should be remembered that this material is *preliminary* to the subject matter of the book.

The early occurrence of the chapter on sequences makes it possible to discuss the completeness of the real number system at the right place: *before* the topics of continuity and irrational exponents. Furthermore, the brief exposition of *limits* in this chapter is good preparation for the discussion of asymptotic behavior in rational functions.

The core of the book is contained in Parts Two and Three. Part Two deals in detail with *algebraic* (polynomial and rational) functions, Part Three with *transcendental* (exponential, logarithmic, and trigonometric) functions. Part Four is a self-contained treatment of *linear systems, matrices, and determinants;* however, Chapter 10 (Analytic Geometry) provides the geometric background for the study of linear systems as well as an introduction to conics and three-dimensional coordinate geometry. Lastly, in Part Five the reader will find an introduction to several important areas of modern mathematics: *sets, combinatorics,* and *probability.*

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CONTENTS

PART ONE

NUMBERS AND FUNCTIONS

The four chapters of Part I develop the basic concepts and tools for the analysis of functions in Parts II and III.

Chapter 1 is a review of the properties of real numbers. Chapter 2 introduces the coordinate plane and graphs, thus providing the geometric background for the study of functions.

In Chapter 3 functions and their graphs are introduced as well as some aspects of the algebra of functions including sums, products, shifts, and stretches.

Chapter 4 deals with sequences and progressions, and then introduces limits, convergence, and the completeness of the real number system. A short section on induction is included.

CHAPTER 1

BASIC REVIEW

1-1 Introduction

In addition to providing a review of the properties of real numbers, this chapter is designed to help the reader increase the skills he needs in studying algebra, and to indicate without undue emphasis how the subject of algebra is developed from just a few fundamental properties (axioms).

By the end of the chapter the reader will have acquired competence in solving problems that involve order, powers, roots, fractions, factoring, and related equations.

1-2 Basic Properties

We shall assume that the reader is familiar with the interpretation of real numbers as points on a line (see Figure 1-1) and with the usual computations of arithmetic. In this section, we shall show how these computations, as well as certain much-used theorems, are derived from basic properties (or axioms) of the real number system.

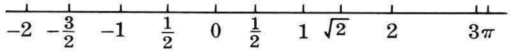

Figure 1-1

While the results of this section are merely a review, the proofs may be new to the reader; and although the remainder of the book does not depend on these proofs, some students do benefit from this type of activity (following a proof step by step), since it provides a secure basis for deciding whether or not a given computation is valid.

In the following list of basic properties, and in the remainder of this section, a, b, c, and d are *any* real numbers, unless otherwise specified. Furthermore, a statement such as "$a = b$" means "a *is* b" or, equivalently, that "a" and "b" are merely different symbols for the *same* number (just as H_2O and water are merely different names—i.e., symbols—for the same thing).

Name of property	*Addition*	*Multiplication*
1. **Commutative:**	$a + b = b + a$	$ab = ba$
2. **Associative:**	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
3. **Distributive:**	$a(b + c) = ab + ac$ and, in view of the commutative property, $(b + c)a = ba + ca$	
4. **The property of identities:***	$a + 0 = 0 + a = a$	$a \cdot 1 = 1 \cdot a = a$
	(i.e., 0 and 1 are, respectively, *additive* and *multiplicative identities*)	
5. **The property of inverses:†**	$a + (-a) = (-a) + a = 0$	$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$ (if $a \neq 0$)
	(i.e., a has $-a$ as *additive inverse* and, if $a \neq 0$, $1/a$ as *multiplicative inverse*)	

* It turns out that 0 and 1 are *the only* (i.e., *unique*) identities, respectively, for addition and multiplication: for if $0'$ is an *additive identity*, then $0 + 0' = 0$; and we also have $0 + 0' = 0'$ because of the property of 0. Hence, $0' = 0$. An analogous argument shows that 1 is the only *multiplicative identity*.

† The *uniqueness* of inverses will be shown shortly.

Recall at this point that, *by definition,*

$$\boldsymbol{a - c = a + (-c)}$$

and, for $c \neq 0$,

$$\boldsymbol{\frac{a}{c} = a \cdot \frac{1}{c}}$$

That is, *to subtract c means to add* $-c$, and *to divide by c means to multiply by* $1/c$. Note also that, since $a = b$ means that a and b are the same number, it follows that for *any* number c, if

$$\boldsymbol{a = b}$$

then

$$\boldsymbol{a + c = b + c}$$

and

$$\boldsymbol{ac = bc}$$

That is, addition and multiplication (and hence also subtraction and division) "preserve equality."* Conversely,

if $\boldsymbol{a + c = b + c}$, **then** $\boldsymbol{a = b}$

and, for $c \neq 0$, if $\boldsymbol{ac = bc}$, **then** $\boldsymbol{a = b}$

To see this, suppose that $a + c = b + c$; then, since addition preserves equality, $(a + c) + (-c) = (b + c) + (-c)$, and hence $a + [c + (-c)] = b + [c + (-c)]$. Since $c + (-c) = 0$, it follows that $a = b$. A similar argument shows that if $ac = bc$ (with $c \neq 0$) then $a = b$.

Summarizing the foregoing observations, we see that

$$\boldsymbol{a = b} \quad \textbf{if and only if} \quad \boldsymbol{a + c = b + c}$$

and for $c \neq 0$,

$$\boldsymbol{a = b} \quad \textbf{if and only if} \quad \boldsymbol{ac = bc}$$

Remark When "*if and only if*" connects two statements, it means that these two statements are *logically equivalent;* that is, if either one of them is true, then so is the other.

* It is now possible to show why *inverses are unique.* Suppose x is an additive inverse of a: $a + x = 0$; then, since addition preserves equality,

$$(-a) + (a + x) = (-a) + 0$$

$$(-a + a) + x = (-a) + 0 \qquad \textbf{[by associativity]}$$

$$0 + x = (-a) + 0 \qquad \textbf{[since } \boldsymbol{-a + a = 0}\textbf{]}$$

and finally

$$x = -a \qquad \textbf{[since } \boldsymbol{0 + x = x} \textbf{ and } \boldsymbol{(-a) + 0 = -a}\textbf{]}$$

The uniqueness of multiplicative inverses is shown analogously.

We shall now prove several important theorems, using the properties discussed above.

Theorem 1

> *For any real number a,* $-(-a) = a$; *and if* $a \neq 0$, *then* $\dfrac{1}{1/a} = a$.

Proof Note that (by definition) $-(-a)$ is the additive inverse of $-a$; and that a is the additive inverse of $-a$ [since $(-a) + a = 0$]. Since $-a$ has *only one* additive inverse, it follows that $-(-a) = a$. An analogous proof shows that if $a \neq 0$, then $\dfrac{1}{1/a} = a$, and is left as an exercise.

Theorem 2

> *For any real numbers a and b,* $-(a + b) = -a - b$; and if $a \neq 0$ and $b \neq 0$, *then* $\dfrac{1}{ab} = \dfrac{1}{a} \cdot \dfrac{1}{b}$.

Proof Note that

$$\begin{aligned}(-a - b) + (a + b) &= (-a) + (-b) + a + b \\ &= (-a) + a + (-b) + b = 0 + 0 = 0\end{aligned}$$

while by definition we also have $-(a + b) + (a + b) = 0$. Since $a + b$ has only one additive inverse, it follows that $-(a + b) = -a - b$. The proof for multiplicative inverses is similar, and is left as an exercise.

Theorem 3

> *For any real number a,* $a \cdot 0 = 0$.

Proof We have

$$\begin{aligned} a \cdot 0 &= a(0 + 0) && \text{[since } 0 + 0 = 0\text{]} \\ &= a \cdot 0 + a \cdot 0 && \text{[by the distributive property]}\end{aligned}$$

Hence,

$$a \cdot 0 - a \cdot 0 = a \cdot 0 + a \cdot 0 - a \cdot 0 \qquad \text{[since subtraction preserves equality]}$$

and therefore $\quad 0 = a \cdot 0$