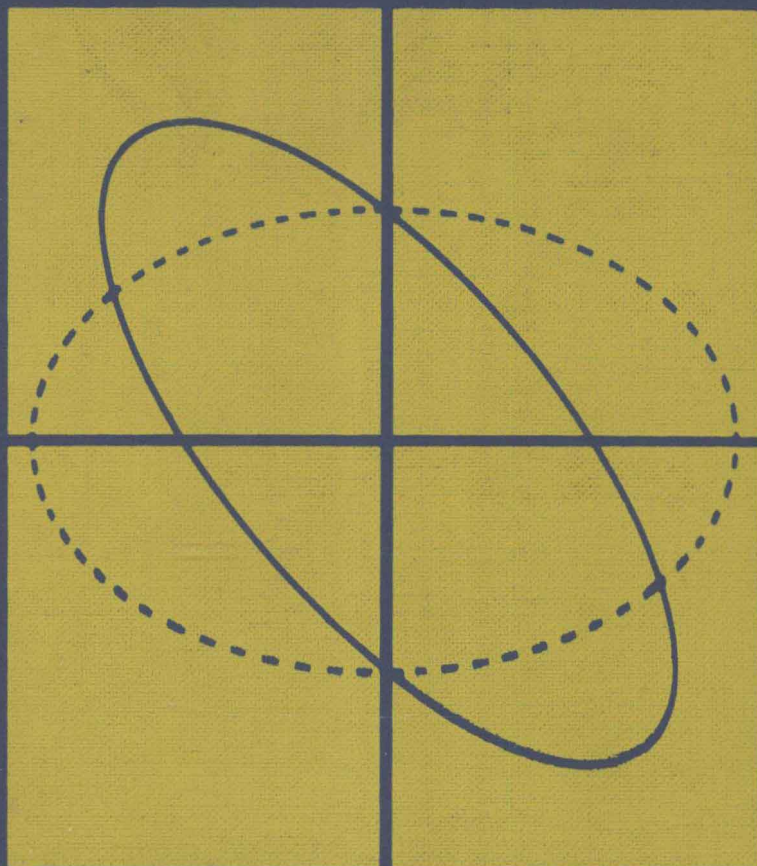


COLLEGE ALGEBRA AND TRIGONOMETRY



**STEVEN BRYANT
JACK KARUSH
LEON NOWER
DANIEL SALTZ**

COLLEGE ALGEBRA AND TRIGONOMETRY

STEVEN J. BRYANT

San Diego State College

JACK KARUSH

LEON NOWER

San Diego State College

DANIEL SALTZ

San Diego State College



GOODYEAR PUBLISHING COMPANY, INC.
Pacific Palisades, California

© 1971 by Goodyear Publishing Company, Inc.
Pacific Palisades, California

*All rights reserved. No part of this book
may be reproduced in any form or by any
means without permission in writing from
the publisher.*

Current printing (last digit):

10 9 8 7 6 5 4 3

ISBN 0-87620-191-5

Library of Congress Catalog Card Number 75-104833

Y-1915-1

Printed in the United States of America

**COLLEGE ALGEBRA
AND TRIGONOMETRY**

PREFACE

Mathematicians nearly always associate mathematical ideas with pictures; beginners seldom do. This is unfortunate, because it is almost impossible to study many important mathematical functions without also studying their graphs.

Since the main purpose of this book is to prepare the reader for the study of calculus, the analysis of *functions and their graphs* is emphasized throughout. The reader will find that to come up with the correct picture he will not only exercise his skills in arithmetic and algebra, but will also acquire and strengthen his intuition of *continuity*—so indispensable for a further study of mathematics. To help the growth of this intuition, many problems in this book simply ask for the graph of a given function, and the answer (a picture) is given in the answer section.

The basic review (Chapter 1) provides the less well prepared reader with an opportunity to strengthen his techniques in arithmetic and algebra. However, it should be remembered that this material is *preliminary* to the subject matter of the book.

The early occurrence of the chapter on sequences makes it possible to discuss the completeness of the real number system at the right place: *before* the topics of continuity and irrational exponents. Furthermore, the brief exposition of *limits* in this chapter is good preparation for the discussion of asymptotic behavior in rational functions.

The core of the book is contained in Parts Two and Three. Part Two deals in detail with *algebraic* (polynomial and rational) functions, Part Three with *transcendental* (exponential, logarithmic, and trigonometric) functions. Part Four is a self-contained treatment of *linear systems, matrices, and determinants*; however, Chapter 10 (Analytic Geometry) provides the geometric background for the study of linear systems as well as an introduction to conics and three-dimensional coordinate geometry. Lastly, in Part Five the reader will find an introduction to several important areas of modern mathematics: *sets, combinatorics, and probability*.

STEVEN BRYANT
JACK KARUSH
LEON NOWER
DANIEL SALTZ

**COLLEGE ALGEBRA
AND TRIGONOMETRY**

CONTENTS

PART ONE	NUMBERS AND FUNCTIONS	1
1	Basic Review	3
1-1	Introduction	3
1-2	Basic Properties	3
1-3	Order	14
1-4	Integer Exponents	20
1-5	Roots and Absolute Value	24
1-6	Rational Exponents	29
1-7	Factoring and Special Products	31
1-8	Fractions	34
1-9	Solving Equations	37
1-10	Completing the Square. The Quadratic Formula	39
2	The Coordinate Plane	44
2-1	Introduction	44
2-2	Rectangular Coordinates and Distance	44
		vii

2-3	Graphs of Relations	49
2-4	The Complex Number System	58
3	Functions and Their Graphs	70
3-1	Introduction	70
3-2	Notation. Graphs	75
3-3	Operations on Functions	83
4	Sequences	93
4-1	Introduction	93
4-2	The Sigma Notation	96
4-3	Arithmetic Progression	101
4-4	Geometric Progression	106
4-5	Convergence. Infinite Series	111
4-6	The Completeness Property of the Real Number System	120
4-7	Mathematical Induction	124
PART TWO POLYNOMIAL AND RATIONAL FUNCTIONS		129
5	Polynomial Functions	131
5-1	Introduction	131
5-2	Lines and First Degree Polynomial Functions	133
5-3	Quadratic Functions	139
5-4	The Remainder and Factor Theorems	147
5-5	Zeros of Polynomial Functions	153
5-6	The Fundamental Theorem of Algebra. Complex Zeros	156
5-7	Bounds and Rational Zeros	161
5-8	Graphs of Polynomial Functions	166
6	Rational Functions	170
6-1	Rational Functions	170
6-2	Division	180

PART THREE	TRANSCENDENTAL FUNCTIONS	183
7	Exponential and Logarithmic Functions	185
7-1	Inverse Functions	185
7-2	The Exponential Functions	190
7-3	The Logarithmic Functions	194
8	Trigonometry	203
8-1	The Measurement of Angles	203
8-2	Sine and Cosine	208
8-3	Periodic Functions. Graphs of Sine and Cosine	220
8-4	The Addition Formulas	228
8-5	Other Trigonometric Functions. Applications	237
8-6	Inverses of Trigonometric Functions	250
8-7	The Laws of Sines and Cosines. Solving Triangles	260
8-8	Polar Coordinates	267
8-9	Complex Numbers (Continued)	275
PART FOUR	LINEAR SYSTEMS	283
9	Vectors	285
9-1	Vectors: Introduction	285
9-2	Vectors with Coordinates	296
10	Analytic Geometry	307
10-1	Introduction	307
10-2	Parabolas	308
10-3	The Ellipse and the Hyperbola	312
10-4	Translation	318
10-5	Rotation	322
10-6	Distance. Spheres. Planes	326

11	Linear Systems. Matrices	336
11-1	Introduction. Echelon Forms	336
11-2	Echelon Matrices	344
11-3	Rank and Augmented Matrices	348
11-4	Determinants and Cramer's Rule	352
11-5	Properties of Determinants. Larger Linear Systems	355
11-6	Operations on Matrices	359
11-7	Determinants, Inverses, and Cramer's Rule	366
	 PART FIVE ADDITIONAL TOPICS	 375
12	Sets and Counting	377
12-1	Introduction: Sets and Operating on Sets	377
12-2	The Fundamental Principle of Counting	387
12-3	Permutations and Subsets	390
12-4	The Binomial Theorem	394
13	Probability	397
13-1	Introduction. Events	397
13-2	Probability Functions	398
13-3	Dependent and Independent Trials	407
	 Answers to Odd-Numbered Exercises	 416
	 Tables	 455
1	Common Logarithms of Numbers	454
2	The Trigonometric Functions (at Angles, in Radians)	456
3	Values of Trigonometric Functions at Angles	460
4	Powers and Roots	461
	Formulas from Trigonometry	462
	 Index	 463

PART ONE

NUMBERS AND FUNCTIONS

The four chapters of Part I develop the basic concepts and tools for the analysis of functions in Parts II and III.

Chapter 1 is a review of the properties of real numbers. Chapter 2 introduces the coordinate plane and graphs, thus providing the geometric background for the study of functions.

In Chapter 3 functions and their graphs are introduced as well as some aspects of the algebra of functions including sums, products, shifts, and stretches.

Chapter 4 deals with sequences and progressions, and then introduces limits, convergence, and the completeness of the real number system. A short section on induction is included.

CHAPTER 1

BASIC REVIEW

1-1 Introduction

In addition to providing a review of the properties of real numbers, this chapter is designed to help the reader increase the skills he needs in studying algebra, and to indicate without undue emphasis how the subject of algebra is developed from just a few fundamental properties (axioms).

By the end of the chapter the reader will have acquired competence in solving problems that involve order, powers, roots, fractions, factoring, and related equations.

1-2 Basic Properties

We shall assume that the reader is familiar with the interpretation of real numbers as points on a line (see Figure 1-1) and with the usual computations of arithmetic. In this section, we shall show how these computations, as well as certain much-used theorems, are derived from basic properties (or axioms) of the real number system.

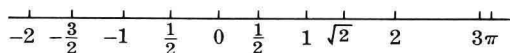


Figure 1-1

While the results of this section are merely a review, the proofs may be new to the reader; and although the remainder of the book does not depend on these proofs, some students do benefit from this type of activity (following a proof step by step), since it provides a secure basis for deciding whether or not a given computation is valid.

In the following list of basic properties, and in the remainder of this section, a , b , c , and d are *any* real numbers, unless otherwise specified. Furthermore, a statement such as " $a = b$ " means " a is b " or, equivalently, that " a " and " b " are merely different symbols for the *same* number (just as H_2O and water are merely different names—i.e., symbols—for the same thing).

<i>Name of property</i>	<i>Addition</i>	<i>Multiplication</i>
1. Commutative:	$a + b = b + a$	$ab = ba$
2. Associative:	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
3. Distributive:	$a(b + c) = ab + ac$	
	and, in view of the commutative property,	
	$(b + c)a = ba + ca$	
4. The property of identities:*	$a + 0 = 0 + a = a$ (i.e., 0 and 1 are, respectively, <i>additive</i> and <i>multiplicative identities</i>)	$a \cdot 1 = 1 \cdot a = a$
5. The property of inverses:†	$a + (-a) = (-a) + a = 0$ (i.e., a has $-a$ as <i>additive inverse</i> and, if $a \neq 0$, $1/a$ as <i>multiplicative inverse</i>)	$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$ (if $a \neq 0$)

* It turns out that 0 and 1 are the *only* (i.e., *unique*) identities, respectively, for addition and multiplication: for if $0'$ is an *additive identity*, then $0 + 0' = 0$; and we also have $0 + 0' = 0'$ because of the property of 0. Hence, $0' = 0$. An analogous argument shows that 1 is the *only multiplicative identity*.

† The *uniqueness* of inverses will be shown shortly.

Recall at this point that, *by definition*,

$$a - c = a + (-c)$$

and, for $c \neq 0$,

$$\frac{a}{c} = a \cdot \frac{1}{c}$$

That is, *to subtract c means to add $-c$* , and *to divide by c means to multiply by $1/c$* . Note also that, since $a = b$ means that a and b are the same number, it follows that for *any* number c , if

$$a = b$$

then

$$a + c = b + c$$

and

$$ac = bc$$

That is, addition and multiplication (and hence also subtraction and division) “preserve equality.”* Conversely,

if

$$a + c = b + c, \quad \text{then } a = b$$

and, for $c \neq 0$, if

$$ac = bc, \quad \text{then } a = b$$

To see this, suppose that $a + c = b + c$; then, since addition preserves equality, $(a + c) + (-c) = (b + c) + (-c)$, and hence $a + [c + (-c)] = b + [c + (-c)]$. Since $c + (-c) = 0$, it follows that $a = b$. A similar argument shows that if $ac = bc$ (with $c \neq 0$) then $a = b$.

Summarizing the foregoing observations, we see that

$$a = b \quad \text{if and only if} \quad a + c = b + c$$

and for $c \neq 0$,

$$a = b \quad \text{if and only if} \quad ac = bc$$

Remark When “*if and only if*” connects two statements, it means that these two statements are *logically equivalent*; that is, if either one of them is true, then so is the other.

*It is now possible to show why *inverses are unique*. Suppose x is an additive inverse of a : $a + x = 0$; then, since addition preserves equality,

$$(-a) + (a + x) = (-a) + 0$$

$$(-a + a) + x = (-a) + 0 \quad [\text{by associativity}]$$

$$0 + x = (-a) + 0 \quad [\text{since } -a + a = 0]$$

and finally

$$x = -a \quad [\text{since } 0 + x = x \text{ and } (-a) + 0 = -a]$$

The uniqueness of multiplicative inverses is shown analogously.

We shall now prove several important theorems, using the properties discussed above.

Theorem 1

For any real number a , $-(-a) = a$; and if $a \neq 0$, then $\frac{1}{1/a} = a$.

Proof Note that (by definition) $-(-a)$ is the additive inverse of $-a$; and that a is the additive inverse of $-a$ [since $(-a) + a = 0$]. Since $-a$ has *only one* additive inverse, it follows that $-(-a) = a$. An analogous proof shows that if $a \neq 0$, then $\frac{1}{1/a} = a$, and is left as an exercise.

Theorem 2

For any real numbers a and b , $-(a + b) = -a - b$; and if $a \neq 0$ and $b \neq 0$, then $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$.

Proof Note that

$$\begin{aligned} (-a - b) + (a + b) &= (-a) + (-b) + a + b \\ &= (-a) + a + (-b) + b = 0 + 0 = 0 \end{aligned}$$

while by definition we also have $-(a + b) + (a + b) = 0$. Since $a + b$ has only one additive inverse, it follows that $-(a + b) = -a - b$. The proof for multiplicative inverses is similar, and is left as an exercise.

Theorem 3

For any real number a , $a \cdot 0 = 0$.

Proof We have

$$\begin{aligned} a \cdot 0 &= a(0 + 0) && \text{[since } 0 + 0 = 0\text{]} \\ &= a \cdot 0 + a \cdot 0 && \text{[by the distributive property]} \end{aligned}$$

Hence,

$$a \cdot 0 - a \cdot 0 = a \cdot 0 + a \cdot 0 - a \cdot 0 \quad \text{[since subtraction preserves equality]}$$

and therefore $0 = a \cdot 0$