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Basic Algebra

Robert A. Carman **A Guided Approach** Marilyn J. Carman

BASIC ALGEBRA
A GUIDED APPROACH
Second Edition

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A teacher who can arouse a feeling for one single good action . . . accomplishes more than he who fills our memory with rows on rows of natural objects, classified with name and form. **—Goethe**

This book is dedicated to our first teachers, our mothers, Nellie and Mary

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PREFACE

This book is intended for use in a first course in elementary algebra. Specifically, it is designed for the many students with little or no background in algebra and a very limited general ability in mathematics. Most of these students have had little success in mathematics, some openly fear it, and all need a very carefully guided approach with emphasis on understanding and explanation rather than abstraction and formalism. They need instructional materials designed to meet them at their own level of ability and to develop gradually the concepts and skills of elementary algebra. This book is intended for such students.

Those who have difficulty with mathematics will find in this book several special features designed to make it most effective for them. These include the following.

- Careful attention has been given to readability, and reading specialists have helped plan both the written text and the visual organization.
- An optional diagnostic pretest and performance objectives keyed to the text are given at the beginning of each unit. These clearly indicate the content of each unit and provide the student with a sense of direction.
- Each unit ends with a self-test covering the work of the unit.
- The format is clear and easy to follow. It respects the individual needs of each reader, providing immediate feedback at each step to assure understanding and continued attention.
- The *why* as well as the *how* of every concept and operation are carefully explained. Algebra concepts are presented as natural extensions of arithmetic concepts and operations.
- The emphasis is on *explaining* algebra concepts rather than *presenting* them. This book is not a bag of equations or a set of formal proofs.
- Throughout the book special attention has been given to word problems, including specific instruction in an effective strategy for solving such problems. This is the most difficult part of elementary algebra for most students and therefore receives our most careful attention.
- Both routine drill and more imaginative and challenging problems are included. Answers to all of these problems are given in the back of the book.
- Supplementary problem sets are included at the end of each unit, and additional problem sets are available in the accompanying teacher's manual. Answers to all supplementary problem sets are given in the teacher's manual.
- Arithmetic reviews are provided for those students who need to work through the arithmetic concepts and operations generally assumed as prerequisite to the course.
- A light, lively conversational style of writing and a pleasant, easy-to-understand visual approach are used. The use of humor and historical topics are designed to appeal to students who have in the past found mathematics to be dry and uninteresting.

This book has been used in both classroom and individualized instruction settings and carefully field-tested with hundreds of students representing a wide range of interests and ability levels, from junior high school to college graduates. Students who have used the book tell us it is helpful, interesting, even fun to work through. More important, it works—they learn algebra, many of them experiencing success in mathematics for the first time.

Flexibility of use was a major criterion in the design of the book, and field-testing indicates that the book can be used successfully in a variety of course formats. It can be used as a textbook in traditional lecture-oriented courses. It is very effective in situations where an instructor wishes to modify a traditional course by devoting a portion of class time to independent study. The book is especially useful in programs of individualized or self-paced instruction, whether in a learning lab situation, with tutors, with audiotapes, or in totally independent study.

An accompanying teacher's resource book and test manual provides

- information on a variety of self-paced and individualized course formats that may be used;
- multiple forms of all unit tests, brief quizzes, and final examinations; additional problem sets;
- answers to all problems in the unit tests, quizzes, exams, and supplementary problem sets;
- additional references and information on material included in the book.

During the writing of preliminary versions of this book and in its revision we had the good fortune of having it tested under classroom conditions by a very perceptive and capable teacher. To Susanne Culler goes our gratitude for her generous help and encouragement.

This revised second edition includes many improvements suggested by students and teachers who used the first edition. We are especially grateful to the following teachers for their generous advice and valuable assistance: Colin Godfrey, University of Massachusetts; Ronald A. Stoltenberg, Sam Houston State University; Dena Patterson, Santa Monica College; Teri Y. H. Chiang, Mission College; Grace DeVelbiss, Sinclair Community College; Frank Hammons, Sinclair Community College; Sue Myers, Sinclair Community College; John Pfetzing, Sinclair Community College; A. H. Tellez, Glendale Community College; Bryn Gary, Oscar Rose Junior College; Leonard Orman, University of Southern Colorado; and John Loughlin, Lane Community College; and William Whicher, Parks College, St. Louis University. We have been particularly fortunate to have had the expert assistance of Laurie Carman at every stage of writing and production of this second edition, and we are grateful.

A century ago Ralph Waldo Emerson wrote "Our chief want in life is somebody who shall make us do what we can." For the authors, this want has been capably filled by Gary W. Ostedt, Mathematics Editor at Wiley. His confidence, warm support, and creative enthusiasm turned insoluble problems into radiant opportunities. We are grateful for his help.

It is a pleasure to acknowledge the valuable assistance of Robert W. Pirtle and Claire E. Egielski of John Wiley & Sons and Richard C. Spangler of Tacoma Community College during the planning and production of this revision.

Finally, at every step of the seemingly endless sequence of writing, testing, and rewriting that makes a textbook, we have benefited from the active participation, suggestions, help, proofreading, concern, kibitzing, and curiosity of our children: Patty, Laurie, Mary, and Eric. They not only made it worth doing, they made it fun, and in the process they made it a better book than we could have produced without them.

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ABOUT THIS BOOK

Everyone knows that it is easy to find a book that they can read and read and read again, and never understand at all. This is especially true of textbooks, and mathematics textbooks are often the worst culprits of all. We have tried to make this book one you will understand. It is not algebra taught in a vacuum of theory and abstractions, with pages of symbols and strange roundabout ways of saying things that seem obvious, but algebra you can understand and use.

This book is designed so that you can:

★ start at the beginning or where you need to start,
★ work on only what you need to know,
★ move as fast or as slow as you wish,
★ skip material you already understand,
★ do as many practice problems as you need,
★ take self-tests to measure your progress.

If you are worried that algebra may be difficult, and you want a book you will understand, this book is designed for you.

This is no ordinary book. It is not designed for browsing or casual reading. You *work* your way through it. The ideas are arranged step-by-step in short portions or *frames*. Each frame contains information, careful explanations, examples, and questions to test your understanding. Read the material in each frame carefully, follow the examples, and answer the questions that lead to the next frame. Correct answers move you quickly through the book. Further explanation is provided when it is needed. You move through this book frame by frame. Because we know that every person is different and has different needs, each major section of the book starts with an optional preview test that will help you determine the parts on which you need to work.

As you move through the book you will notice that material not directly connected to the frames appears in boxes and cartoons. Read these at your leisure. They contain information that you may find useful, interesting, and even fun.

Because we know that word problems are a very difficult part of algebra for most students, we have taken special care throughout the book to explain how to do such problems. You'll be guided through many carefully worked examples and given helpful hints designed to make you an algebra user rather than an algebra memorizer.

Most students hesitate to ask questions. They would rather risk failure than look foolish by asking "dumb questions." To relieve you of worry over dumb questions (or DAQs), we'll ask and answer them for you. Thousands of students have taught us that "dumb questions" can produce smart students. Watch for DAQ.

More than thirty-six centuries ago, Ahmes, an Egyptian scribe copying an algebra text, wrote wonderingly that it contained "rules for inquiring into nature, and for knowing all that exists, every mystery, . . . every secret." No one today would make such a claim for an algebra textbook or even for all of mathematics. In this book we will show you how to do simple algebra, how to use it and understand it. Old Ahmes would be fascinated.

Now, turn to page 1 and let's begin.

CONTENTS

1 Introduction to Algebra

PREVIEW 1

Objective	Sample Problems		Where To Go for Help: Page	Frame
Upon successful completion of this program you will be able to:	Identify each of the following as a term, coefficient, factor, expression, variable, constant, or equation.		Page	Frame
1. Understand and use basic algebraic words such as *term, expression, factor, variable, coefficient,* and *equation.*	(a) $2x$	__________	3	1
	(b) The 3 in $3x^2$	__________		
	(c) The x in $2x + 3x^2$	__________		
	(d) $3x + 5$	__________		
	(e) $x + 4 = 9$	__________		
2. Add, subtract, multiply, and divide signed numbers.	(a) $4 - (-7)$	= __________	15	14
	(b) $8 - 14$	= __________		
	(c) $(-6.4) \times (-3.1)$	= __________		
	(d) $(-12.2) \div (-4.0)$	= __________		
3. Work with numbers in exponential notation.	(a) 2^3	= __________	29	25
	(b) $(-0.4)^2$	= __________		
	(c) Write in exponent form $3 \cdot a \cdot a \cdot b \cdot b \cdot b$	= __________		
	(d) Multiply $a^2 \cdot a^3$	= __________		
	(e) Calculate $(a^3)^2$	= __________		
	(f) Divide $\frac{a^6}{a^4}$	= __________		
	Simplify:			
4. Perform the basic algebraic operations.	(a) $3a^2b + ab^2 - a^2b - 4ab^2 + ab$	= __________	43	37
	(b) $-3(xy)(2xy^2)(x^3z)$	= __________		
	(c) $\frac{8ab^2c^4}{2ab^3c}$	= __________		
	(d) $-2(3x - 4)$	= __________		

Date

Name

Course/Section

Objective	Sample Problems		Where To Go for Help	
	Write as an algebraic expression:		59	48
5. Translate English phrases and sentences to algebraic expressions and equations.	(a) Seven less than twice a given number.	= ________		
	(b) The product of some number and 16 equals that number plus six.	= ________		
	If $x = 2$, $a = 5$, and $b = 6$, find the value of		69	56
6. Calculate the numerical value of literal expressions.	(a) $\frac{2b - x}{a}$	= ________		
	(b) $x^2 + 3x$	= ________		
	(c) $3abx^2$	= ________		

(Answers to these problems are at the bottom of this page.)

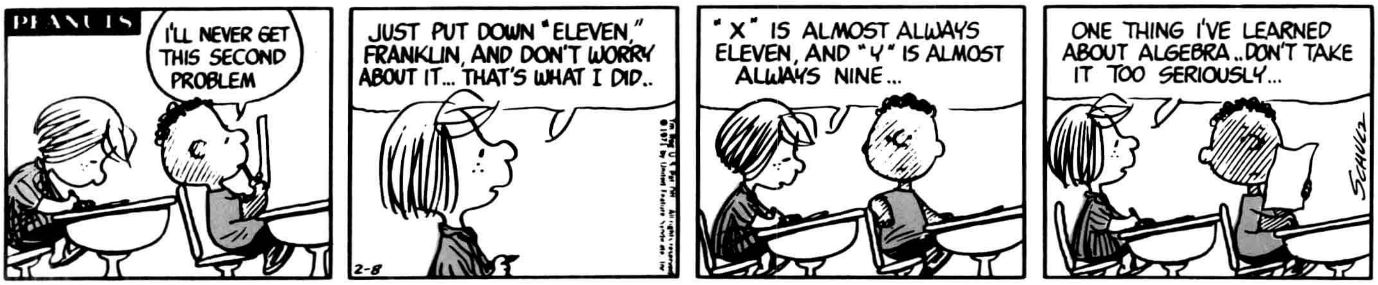
If you are certain you can work all of these problems correctly, turn to page 79 for a self-test. If you want help with any of these objectives or if you cannot work one of the sample problems, turn to the page indicated. Super-students who want to be certain they learn all of this will turn to frame 1 and begin work there.

PREVIEW 1

Answers to Sample Problems

1. (a) Term (b) Coefficient or factor (c) Variable
 (d) Expression (e) Equation
2. (a) 11 (b) -6 (c) 19.84 (d) 3.05
3. (a) 8 (b) 0.16 (c) $3a^2b^3$ (d) a^5 (e) a^6 (f) a^2
4. (a) $2a^2b - 3ab^2 + ab$ (b) $-6x^5y^3z$ (c) $\frac{4c^3}{b}$ (d) $8 - 6x$
5. (a) $2x - 7$ (b) $16x = x + 6$
6. (a) 2 (b) 10 (c) 360

Introduction to Algebra



1-1 THE LANGUAGE OF ALGEBRA

1 Before you decide whether or not algebra is a serious business, you should learn what it is and why people bother to learn it. What is algebra? The simplest answer is that algebra is arithmetic in disguise—generalized arithmetic.

Arithmetic statements and operations always involve specific numbers: $2 + 3 = 5$, $3 \times 4 = 12$, $28 \div 4 = 7$, and so on. Algebra statements can tell us something about many numbers, or even all numbers. For example, what do the following arithmetic statements have in common?

$$\begin{aligned} 3 - 2 &= 1 \\ 4 - 2 &= 2 \\ 5 - 2 &= 3 \\ 10 - 2 &= 8 \\ 101 - 2 &= 99 \end{aligned}$$

The answer is that they are all statements of the form "some number minus 2 equals some other number," or, in symbols,

$$\square - 2 = \triangle$$

Where the symbols $\square$ and $\triangle$ represent numbers.

This is an algebraic equation and we can use it to study and reason about *all* arithmetic statements like those listed above. A mathematician is more interested in this general equation than in any particular example of it. Arithmetic teaches you how to make calculations relating to special one-shot situations. With algebra you begin to study mathematics as a tool for general abstract thought.

Ready to learn about algebra? Then turn to 2 to continue.

2 Many of the ideas behind algebra started thousands of years ago, but the kind of notation you see in a modern algebra book dates from the sixteenth century when François Vieta, a French lawyer and amateur mathematician, began the systematic use of letters to represent numbers. A mathematical statement in which letters are used to represent numbers is called a *literal* expression. Letters can represent single numbers or entire sets of numbers. Algebra is the arithmetic of literal expressions. It is a kind of symbolic arithmetic that enables us to find answers to problems by simple operations with letters rather than by repeated and difficult arithmetic with numbers.

Literal Expression

Any letters will do. In mathematics, English, Greek, or even Hebrew alphabets are used, including lowercase, capital, and even script letters. The letters

a, A, $\mathbf{A}$, $\mathfrak{A}$, or $\mathscr{A}$

are all different and each represents a *different* quantity in algebra even though all are the same letter of the alphabet. If you want to represent the distance a car travels by the letter d, you should be consistent and use the symbol d and not confuse it with D, $\mathbf{D}$, $\mathfrak{D}$, or $\mathscr{D}$.

People who use algebra often choose the symbols they use on the basis of their memory-jogging value. Time is represented by t, distance by d, cost by c, area by A, and so on. The symbol is chosen to remind you of its meaning.

For no special reason, other than habit, mathematicians usually reserve the last six letters of the alphabet, u, v, w, x, y, and z to represent unknown quantities.

What letters would you use for algebra symbols to represent the following quantities:

(a) Radius of a circle (b) Interest on a loan
(c) Speed of motion (d) Work done
(e) Rate of change (f) Volume of a box

Check your answers in 3.

---

3 (a) R or r (b) i or I
(c) s or sometimes v for velocity (d) W
(e) R or r (f) V

The Numbers of Algebra

Numbers are the basic stuff of both arithmetic and algebra, and it is important that you have a clear understanding of the various kinds of numbers that appear in mathematics. Fortunately, we have a simple, easy to understand scheme for classifying all numbers. Look at the diagram on page 5. The familiar numbers we use to count objects, 1, 2, 3, 4, and so on, are called the ***counting numbers*** or the ***natural numbers.*** These are shown at the bottom of the diagram. The set of numbers known as ***integers*** includes the positive natural numbers, 1, 2, 3, . . ., their negative counterparts, -1, -2, -3, . . ., and zero.

Counting or Natural Numbers
Integers

The set of ***rational numbers*** includes any numbers that can be written as a fraction $\frac{a}{b}$ where a and b can be any integers except that b cannot equal zero. All fractions, such as $\frac{1}{2}$, $\frac{3}{2}$, or $-\frac{7}{8}$, are rational numbers and of course any integer can be written as a fraction:

Rational Numbers

$$4 = \frac{4}{1},\ 7 = \frac{7}{1},\ \text{or}\ -12 = \frac{-12}{1}.$$

Any ordinary decimal number can be written as a fraction and they are therefore rational numbers.

$0.4 = \frac{4}{10} \qquad 1.5 = \frac{3}{2} \qquad 2.375 = 2\frac{3}{8}$

We build the integers from the natural numbers and the rational numbers from the integers.

Real Numbers

Irrational Numbers

All of the numbers that appear in elementary algebra and in most scientific or technical applications of mathematics are included in the set of *real numbers*. The real numbers include the rational numbers, that is, all positive and negative integers and fractions, and the *irrational numbers*. Irrational numbers are quantities such as $\sqrt{2}$, $\sqrt[3]{10}$, or $2 + \sqrt{5}$ that are not integers and cannot be written as fractions or finite decimals.

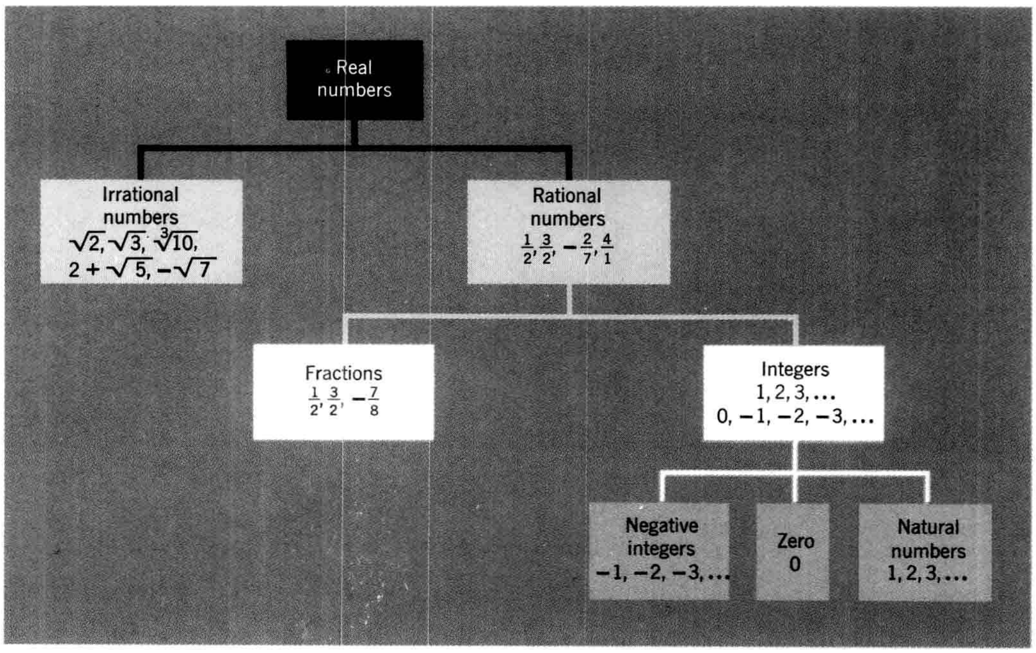
To test your understanding of this number classification, examine the following list and (a) draw a circle around each integer, (b) place a check $\surd$ over each rational number, (c) place an **x** by each natural number.

$2, \quad -4, \quad 0, \quad 1.3, \quad \frac{1}{2}, \quad \frac{-2}{3}, \quad \sqrt{3}, \quad 10, \quad -1, \quad \frac{3}{2}, \quad 1 - \sqrt{5}$

$4 + 1, \quad 5 - 7, \quad 7 \div 2, \quad 3 \times 1\frac{1}{3}, \quad -8, \quad -2.4$

Careful, some numbers may require more than one mark.

Check your answers in frame 4.



4

x $\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$		x $\surd$	$\surd$	$\surd$
2	-4	0	1.3	$\frac{1}{2}$	$-\frac{2}{3}$	$\sqrt{3}$	10	-1	$\frac{3}{2}$

	x $\surd$	$\surd$	$\surd$	x $\surd$	$\surd$	$\surd$
$1 - \sqrt{5}$	$4 + 1$	$5 - 7$	$7 \div 2$	$3 \times 1\frac{1}{3}$	-8	-2.4

Algebraic Symbols

Most of the usual arithmetic symbols have the same meaning in algebra that they have in arithmetic. For example, the addition (+) and subtraction (−) signs are used in exactly the same way.

$a + b \qquad 3 + x \qquad 1 - y \qquad Q - 5 \qquad d - e$

Multiplication

However, the *multiplication* sign ($\times$) of arithmetic looks like the letter x and to avoid confusion we show multiplication in algebra in other ways. The product of two algebraic quantities a and b, "a times b," may be written using

A raised dot	$a \cdot b$				
Parentheses	$a(b)$	or	$(a)b$	or	$(a)(b)$
Or with no symbol at all	ab				

Obviously this last way of showing multiplication won't do in arithmetic; we cannot write "two times four" as "24"—it looks like twenty-four. But it is a quick and easy way to indicate multiplication in algebra.

Placing two quantities side by side to show multiplication is not new and is not only an algebra gimmick; we use it every time we write 20¢ or 4 feet.

20¢ $=$ 20 $\times$ 1¢
4 feet $=$ 4 $\times$ 1 foot

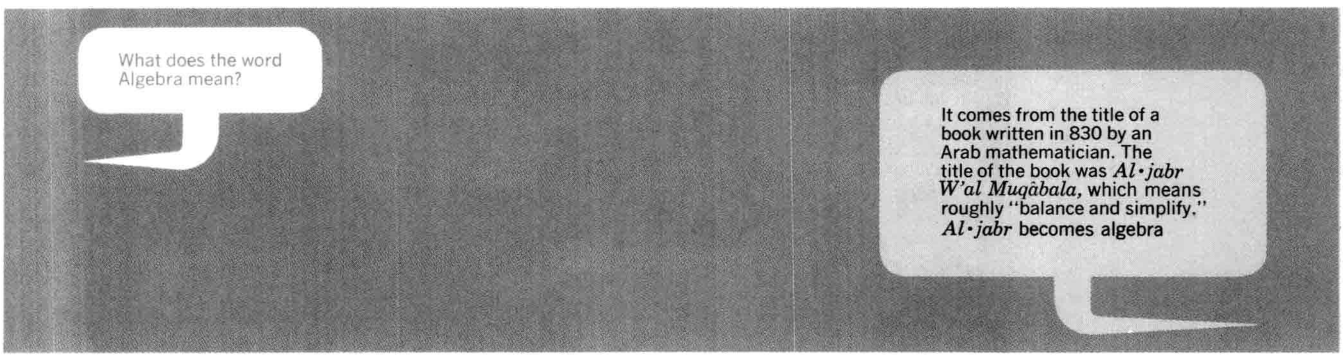
Write the following multiplications using no multiplication symbols:

(a) 8 times b = ______ (b) a times h = ______

(c) 2 times s times t = ______ (d) 3 times x times y = ______

(e) 4 times y times a times k = ______

Check your answers in 5.



5 (a) 8 times $b = 8b$ (b) a times $h = ah$ (c) 2 times s times $t = 2st$

(d) 3 times x times $y = 3xy$ (e) 4 times y times a times $k = 4yak$

We pronounce "$8b$" as "eight bee" and not "eight bees."

Parentheses

Parentheses are used in arithmetic to show that some complicated quantity is to be treated as a single number. For example,

$2 \cdot (3 + 4)$ means that the number 2 multiplies *all* of the quantity in the parentheses.

$2 \cdot (3 + 4) = 2 \cdot 7 = 14$

Examples:

$5 + (11 - 7) = 5 + 4 = 9$

$8 - (2 + 5) = 8 - 7 = 1$

$3 \cdot (9 - 5) = 3 \cdot 4 = 12$

Practice using parentheses by evaluating the following arithmetic quantities.

(a) $4 \cdot (8 - 1) =$ ______ (b) $12 - (3 - 1 + 2) =$ ______

(c) $(2 + 4 - 1) + (6 - 3 - 2) =$ ______ (d) $5 \cdot (9 - 7 + 1) =$ ______

(e) $4 \cdot \left(2 + \frac{1}{2}\right) =$ ______ (f) $\frac{2}{3} + \left(4 - 2\frac{1}{2}\right) =$ ______

Check your work in 6.

---

6 (a) $4 \cdot (8 - 1) = 4 \cdot 7 = 28$

(b) $12 - (3 - 1 + 2) = 12 - 4 = 8$

(c) $(2 + 4 - 1) + (6 - 3 - 2) = 5 + 1 = 6$

(d) $5 \cdot (9 - 7 + 1) = 5 \cdot 3 = 15$

(e) $4 \cdot \left(2 + \frac{1}{2}\right) = 4 \cdot \frac{5}{2} = 10$

(f) $\frac{2}{3} + \left(4 - 2\frac{1}{2}\right) = \frac{2}{3} + \frac{3}{2} = \frac{13}{6} = 2\frac{1}{6}$

If you had any trouble working with the fractions in problems (e) and (f) you should take time out to review the arithmetic of fractions beginning on page 521.

In algebra, as in arithmetic, parentheses ( ), brackets [ ], or braces { }, indicate that whatever is enclosed in them should be treated as a single quantity. An expression such as $(3x^2 - 4ax + 2by^2)^2$ should be thought of as $(\text{something})^2$. The expression $(2x + 3a - 4) - (x^2 - 2a)$ should be thought of as (first quantity) − (second quantity).

$\frac{3x - 4y}{7 - x}$ should be thought of as $\frac{\text{first quantity}}{\text{second quantity}}$.

It is a division of two quantities.

Parentheses are the punctuation marks of algebra. Like the period, comma, or semicolon in regular sentences, they tell you how to read an equation and get its correct meaning.

Write the following using algebraic notation:

(a) 8 times $(2a + b)$ = ______ (b) $(a + b)$ times $(a - b)$ = ______

(c) $(x + y)$ times $(x + y)$ = ______ (d) 3 times x times $(1 - y)$ = ______

(e) Add x to $(y - 2)$ and multiply the sum by $(x^2 + 1)$ = ______

Check your work in 7.

---

7 (a) $8(2a + b)$ (b) $(a + b)(a - b)$
(c) $(x + y)^2$ (d) $3x(1 - y)$
(e) $[x + (y - 2)](x^2 + 1)$ or $(x + y - 2)(x^2 + 1)$