Introductory Mathematics

A Prelude to Calculus

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To
Arnold E. Ross

PREFACE

This text is designed to be used by students with various backgrounds and abilities. The main body of the text is a thorough, yet concise, presentation of the essential precalculus topics. The supplements after each section contain solved and partially solved examples and additional problems for the students to work. Students who need to study more examples and work more problems have that opportunity. The advanced students can skip many or all of these supplements without loss of continuity.

The text has been classroom-tested for five years at The Ohio State University. The precalculus classes normally cover the entire book in one term, but the text can be covered successfully at a more relaxed pace in two terms. A spiral approach is used in treating difficult concepts such as absolute value and inverse functions, and geometrical interpretations are liberally interspersed. In addition, an increasing level of sophistication is used in developing the topics as the students gain experience and confidence with the material.

The Q-questions scattered throughout the text check the students' understanding of the immediately preceding material and actively involve them in the learning process. The students also are actively involved in solving problems in the specially designed supplements. An achievement test at the end of each section serves as an additional check on the students' progress. Altogether, about 600 solved examples and over 1100 problems with answers are provided.

Chapter 1 begins with a review of algebra by introducing numbers via sets and the number line. The rules for algebraic manipulations are reviewed as the axioms for real numbers are mentioned briefly. (A more axiomatic treatment appears in Appendix A.) The remainder of the chapter is devoted to overcoming the common stumbling blocks of (fractional) exponents, inequalities, and absolute value.

Chapter 2 introduces the concept of functions and deals primarily with polynomial functions and their graphs. Translations are introduced in the chapter, although the graphs, not the axes, are moved. In addition, other "mappings" are introduced to facilitate graphing. This unique geometric approach has been very effective in extending students' abilities to sketch graphs quickly. These techniques, used throughout the remainder of the text in graphing transcendental functions, have proved quite useful during and after the study of calculus.

Chapter 3 begins with the study of inverse functions, and immediately uses these concepts to develop exponential and logarithmic functions. Properties of logarithmic functions are reinforced by computational work, which could be omitted, considering the increasing popularity of electronic calculators. The last section of the chapter (also optional) deals with summation notations, geometric series, and problems involving compound interest.

Chapters 4 and 5 provide a concise, yet thorough, coverage of trigonometry. The concepts of functions and inverse functions as well as graphing techniques are reinforced here. Chapter 4 introduces the trigonometric functions, covers the usual identities, and includes applications to right triangles. Chapter 5, which could be omitted all or in part, develops trigonometry further, and, in addition, introduces complex numbers. Complex numbers are introduced to provide an introduction to polar coordinates and to further reinforce certain facts about the trigonometric functions.

This book has served as the text for two precalculus courses at The Ohio State University. Administrative science majors cover Chapters 1 through 3, while engineering, physical science, and mathematics majors also cover Chapters 4 and 5.

Finally, we wish to thank all the individuals who have contributed to this book. Literally thousands of students and many teachers have contributed greatly to its present form. Our colleagues Larry Elbrink, Dave Mader, Nick Moore, and Jim Schultz have been especially helpful. But certainly our highest distinguished service award must go to Dodie Huffman, who faithfully deciphered horrendous scribbling in order to type and retype the manuscript in its several versions.

CONTENTS

INTRODUCTION

Before you begin to use this book, you should be aware of a number of special features incorporated in it. Interspersed in the usual text material are a number of Q-questions which appear in boxes. As soon as you encounter a Q-question, you should answer it and write down the answer. The Q-questions are designed to test your understanding of the immediately preceding material. The answers appear at the top of the next page you *turn*. If you cannot answer a Q-question correctly, you should reread the preceding paragraph or two.

Q0: Where do you find the answer to Q0?

At the end of each section is a problem set containing two types of problems. The early problems are basic and direct applications of the material in the section, while those following the double line are more difficult. You should work all the basic problems and check your answers with those provided immediately after the problem set. Additional basic problems are found in the supplement, a section which immediately follows each text section and which provides more work in that section, if you feel that you need it. The supplement consists of solved problems of the

A0: Here!

type encountered in the immediately preceding section, followed by partially solved problems to which you are to contribute. There are drill problems for each major topic in the supplement, and, finally, a set of supplementary problems. After completing the supplement, you should rework the problems that gave you difficulty at the end of the text section. Of course, you can use the supplement to obtain additional practice and to improve understanding, even if you have successfully completed a problem set.

The process of learning mathematics is one of internalizing it. While a good teacher can provide guidelines to make the process easier, in the end it is the work and effort of the student that accomplishes the internalization. It is our hope that the format and style of this text will help you to learn in a way that is more personal and independent.

1 REAL NUMBERS

1.0 INTRODUCTION

Real numbers are the most useful mathematical objects that man has created. In this first chapter, some basic properties of the system of real numbers will be considered, including some review of exponential and radical notation. We begin with some work involving sets to develop our ability to express ourselves clearly and concisely. We introduce the number line in order to picture sets of real numbers and to study the concepts of order and absolute value.

While it is not our intention to provide a rigorous development of the real number system, we do want to point out that a few basic rules and their logical consequences govern the algebra of real numbers. It is important that you be able to use this algebra easily and confidently in your study of mathematics and its applications.

1.1 SETS, NUMBER LINES, AND INTERVALS

The language of sets and set notation is used widely in mathematics and its applications. The word *set* is used to denote a collection of objects. This usage is consistent with the use of the word set in some common phrases such as "a set of dishes" or "a tool set."

It is possible to describe a set by listing its members or elements; for example, $\{1, 2, 3, 6\}$ is the set whose members are the numbers 1, 2, 3, and 6. Frequently, braces are used as above in set notation. At other times a capital letter may be used to denote a set. As a matter of convenience, in this text we will consistently designate certain sets of numbers by particular letters. Among the conventions we will use are the following:

N is the set of natural numbers $\{1, 2, 3, \ldots\}$.
Z is the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
R is the set of real numbers.

If x is a member of set A, we write

$$x \in A$$

and read this statement as "x belongs to A," or "x is an element of A," or "x is a member of A." For example, 2/3 is a real number so we can write $2/3 \in$ R. On the other hand, 2/3 is not a natural number, indicated by $2/3 \notin N$.