The Fourier Transform and Its Applications

Second Edition, Revised

Ronald N. Bracewell

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The Fourier Transform and Its Applications

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Special symbols

Function	*Notation*
Rectangle	$\Pi(x) = \begin{cases} 1 & \lvert x\rvert < \frac{1}{2} \\ 0 & \lvert x\rvert > \frac{1}{2} \end{cases}$
Triangle	$\Lambda(x) = \begin{cases} 1 - \lvert x\rvert & \lvert x\rvert < 1 \\ 0 & \lvert x\rvert > 1 \end{cases}$
Heaviside unit step	$H(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$
Sign (signum)	$\operatorname{sgn} x = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$
Impulse symbol*	$\delta(x)$
Sampling or replicating symbol*	$\text{III}(x) = \sum_{-\infty}^{\infty} \delta(x - n)$
Even impulse pair	$\text{II}(x) = \frac{1}{2}\delta(x + \frac{1}{2}) + \frac{1}{2}\delta(x - \frac{1}{2})$
Odd impulse pair	${}^{\text{I}}{}_{\text{I}}(x) = \frac{1}{2}\delta(x + \frac{1}{2}) - \frac{1}{2}\delta(x - \frac{1}{2})$
Filtering or interpolating	$\operatorname{sinc} x = \frac{\sin \pi x}{\pi x}$
Asterisk notation for convolution	$f(x) * g(x) \triangleq \int_{-\infty}^{\infty} f(u)g(x - u)\, du$
Asterisk notation for serial products	$\{f_i\} * \{g_i\} \triangleq \left\{\sum_j f_j g_{i-j}\right\}$
Pentagram notation	$f(x) \star g(x) \triangleq \int_{-\infty}^{\infty} f(u)g(x + u)\, du$
Various two-dimensional functions	${}^2\Pi(x,y) = \Pi(x)\Pi(y)$ ${}^2\delta(x,y) = \delta(x)\,\delta(y)$ ${}^2\text{III}(x,y) = \text{III}(x)\text{III}(y)$ ${}^2\operatorname{sinc}(x,y) = \operatorname{sinc} x \operatorname{sinc} y$

Guide to reference data

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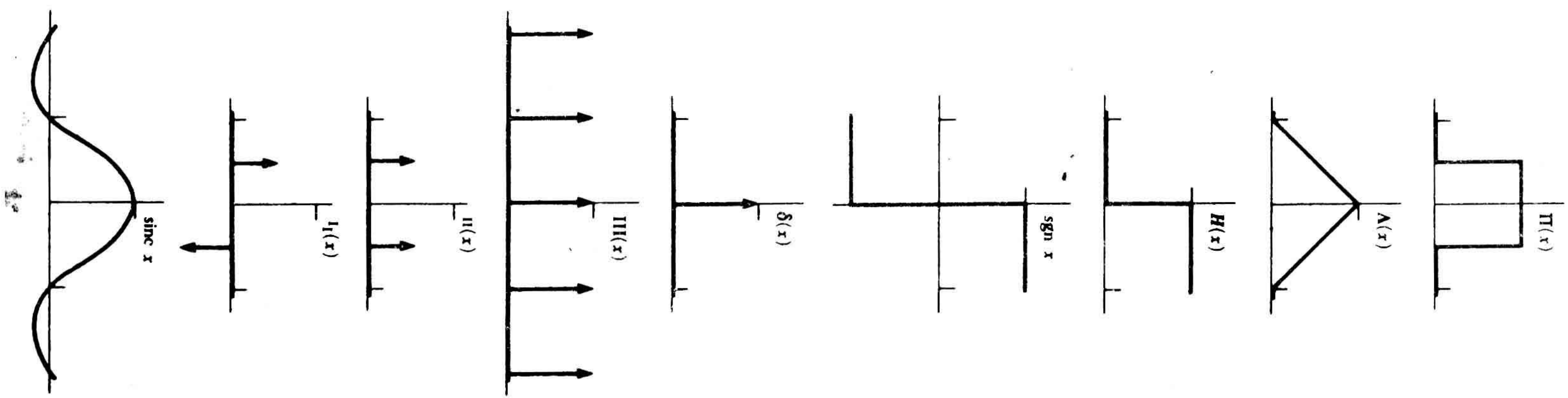

CIRCUITS AND SYSTEMS

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Antoniou: DIGITAL FILTERS: Analysis and Design
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Bracewell: THE FOURIER TRANSFORM AND ITS APPLICATIONS
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Truxal: INTRODUCTORY SYSTEM ENGINEERING

Joseph Fourier, 21 March 1768–16 May 1830. (*By permission of the Bibliothèque Municipale de Grenoble.*)

About the Author

Ronald N. Bracewell was born in Australia, received his B.Sc., B.E., and M.E. degrees from the University of Sydney, and earned a Ph.D. from Cambridge University. Currently Professor of Electrical Engineering at Stanford University, Dr. Bracewell has an impressive roster of professional affiliations, awards, and publications to his credit. He is a Fellow of the Royal Astronomical Society, a Fellow of the Institute of Electrical and Electronic Engineers, a Life Member of the Astronomical Society of the Pacific, and a past Councilor of the American Astronomical Society. He has served as an advisor to several government organizations, including the National Science Foundation and the National Academy of Sciences, and in 1952 received the Duddell Premium of the Institute of Electrical Engineers for his contributions to the study of the ionosphere. Among his many achievements, Dr. Bracewell constructed a microwave spectroheliograph which automatically produced in printed form daily temperature maps of the sun, and was used by NASA for support of its first manned landing on the moon.

Fourier's theorem is not only one of the most beautiful results of modern analysis, but it may be said to furnish an indispensable instrument in the treatment of nearly every recondite question in modern physics.

Lord Kelvin

Note on 1986 Revision

Developments based on Hartley's formulas (page 179) have made it possible to dispense with imaginaries in the computation of Fourier transforms. In fact, in any application where Fourier analysis is practiced it is now possible in numerical work to proceed elegantly and simply using the real Hartley formalism. In computing, a speed advantage, or reduction in the space allocated to memory, results from the fact that multiplying complex numbers takes four times longer than multiplying real numbers; which leads to a net advantage of a factor of 2 in computing with real data when account is taken of the nonredundancy of the Hartley transform. The new ideas and a demonstration computer program are now introduced in Chapters 19 and 20. For a more detailed treatment, see R. N. Bracewell, "The Hartley Transform," Oxford University Press, 1986. For historical interest a chapter on the fascinating life of Joseph Fourier has also been added.

Ronald N. Bracewell

Preface to the Second Edition

The unifying role played by the Fourier transform in linking together the diverse fields mentioned in the original preface has now firmly established transform methods at the heart of the electrical engineering curriculum. Computing and data processing, which have emerged as large curricular segments, though rather different in character from circuits, electronics, and waves, nevertheless do share a common bond through the Fourier transform. Consequently, the subject matter has easily moved into the pivotal role foreseen for it, and faculty members from various specialties have found themselves comfortable teaching it. The course is taken by first-year graduate students, especially students arriving from other universities, and increasingly by students in the last year of their bachelor's degree.

Introduction of the fast Fourier transform (FFT) algorithm has greatly broadened the scope of application of the Fourier transform to data handling and has brought prominence to the discrete Fourier transform (DFT). The technological revolution brought about by these topics, now treated in a new Chapter 18, is only beginning to be felt, but will make an understanding of Fourier notions (such as aliasing, which only aficionados knew about) indispensable to any engineer who handles masses of data. In the future this will mean nearly everyone.

Transforms presented graphically in the Pictorial Dictionary proved to be a useful reference feature and have been added to. Graphical presentation adds a new dimension to the published compilations of integral transforms where it is sometimes frustrating to seek commonly needed entires among the profusion of rare cases. In addition, simple functions that are impulsive, discontinuous, or defined piecewise may not be included or may be hard to recognize.

A good problem assigned at the right stage can be extremely valuable for the student, but a good problem is hard to compose. Among the collection of supplementary problems now included at the end of the book are several that go beyond being mathematical exercises by inclusion of technical background or by asking for opinions.

Notation is a vital adjunct to thinking and I am happy to report that the *sinc function*, which we learned from P. M. Woodward's book, is alive and well and surviving erosion by occasional authors who do not know that "sine x over x" is not the sinc function. The unit rectangle function (unit height and width) $\Pi(x)$, the transform of the sinc function, has also proved extremely useful, especially for blackboard work. In typescript or other circumstances where the Greek letter is less desirable, $\Pi(x)$ may be written "rect x," and it is convenient in any case to pronounce it *rect*. The shah function $\text{III}(x)$ has caught on. It is easy to type and is twice as useful as you might think because it is its own transform. The asterisk for convolution, which was in use a long time ago by Volterra and perhaps earlier, is now in wide use and I recommend ** to denote two-dimensional convolution, which has come into extensive use as a result of the explosive growth of image processing.

Early emphasis on convolution in a text on Fourier transforms turned out to be exactly the right way to go. Convolution has changed in a few years from being presented as a rather advanced concept to one that can easily be explained at an early stage as is fitting for an operation that describes all those systems that respond sinusoidally when you shake them sinusoidally.

Ronald N. Bracewell

Preface to the First Edition

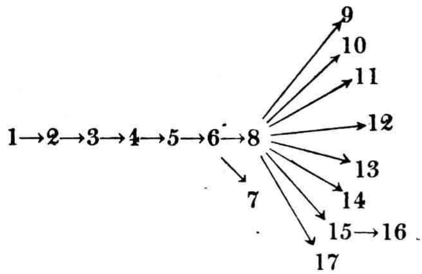
Transform methods provide a unifying mathematical approach to the study of electrical networks, devices for energy conversion and control, antennas, and other components of electrical systems, as well as to complete linear systems whether electrical or not. These same methods apply equally to the subjects of electrical communication, radio propagation, and ionized media—which are concerned in the interconnection of electrical systems—and to information theory which, among other things, relates to the acquisition, processing, and presentation of data. Other theoretical techniques are used in handling these basic fields of electrical engineering, but transform methods are virtually indispensable in all of them.

A course on transforms and their applications has formed part of the electrical engineering curriculum at Stanford University for some years, and has been given with no prerequisites which the holder of a bachelor's degree does not normally possess. One objective has been to develop a pivotal course to be taken at an early stage by all graduates, so that in later, more specialized courses, the student would be spared encountering the same basic material over and over again, and the instructor would be able to proceed more directly to his special subject matter.

It is clearly not feasible to give the whole of linear mathematics in a single course, and the choice of material must necessarily remain a matter of judgment. The choice must, however, be sharply defined if later instructors are to rely on it.

An early-level course should be simple, but not trivial; the objective of this book is to simplify the presentation of many key topics that are ordinarily dealt with in advanced contexts by making use of suitable notation and an approach through convolution.

The organization of chapters is as follows:



One way of working from the book is to take the chapters in numerical order. This sequence is feasible for students who can read the first half unassisted, or who can be taken through it rapidly in a few lectures, but if the material is approached at a more normal pace, then as a practical teaching matter, it is desirable to illustrate the theorems and concepts by dealing simultaneously with a physical topic, such as waveforms and their spectra (Chapter 9), for which the student already has some feeling.

The amount of material is suitable for one semester, or for one quarter, according to how many of the later chapters on applications are included.

Many fine mathematical texts on the Fourier transform have been published. This book differs in that it is intended for those who are concerned with applying Fourier transforms to physical situations rather than with furthering the mathematical subject as such. The connections of the Fourier transform with other transforms are also explored, and the text has been purposely enriched with condensed information that will suit it for use as a reference source for transform pairs and theorems pertaining to transforms.

My interest in the subject was fired when I was studying analysis from Carslaw's "Fourier Series and Integrals" at the University of Sydney in 1939; more recently I have applied transform methods to various problems arising in connection with directive antennas, a subject that is touched on only briefly in this book, but which may be followed up by reference to the "Encyclopedia of Physics" (vol. 54, S. Flügge, ed., Springer-Verlag, Berlin, 1962). In solving these problems I benefited from the physical approach to the Fourier transformation that I learned from J. A. Ratcliffe at the Cavendish Laboratory, Cambridge.

Ronald N. Bracewell

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