Beginning Algebra

SECOND EDITION

BEGINNING ALGEBRA

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PREFACE TO THE SECOND EDITION

PURPOSE: This textbook was written to provide every possible assistance to students who are first being introduced to the principles of algebra and to students who need to review some of the topics covered in high school algebra. In this second edition some topics have been added, others rearranged, and others have been completely rewritten. The inspiration for these improvements came not only from the authors' continued teaching of the course since publication of the first edition, but also from the many excellent recommendations received from teachers throughout the world who have provided us with the insights gained from their classroom experience with the book.

PREREQUISITES: No previous background in algebra is required. Some familiarity with the basic concepts of arithmetic is assumed.

OBJECTIVES: The basic objectives of the first edition have been maintained. First we build a good foundation in the basic operations of arithmetic. Then we carefully begin to introduce algebraic notation and format in the course of demonstrating the properties of real numbers using illustrative examples from arithmetic. In this way, the student is prepared for the transition from arithmetic concepts to the generalized algebraic form. This sort of careful progression is used throughout the book, in examples, explanations, and problems. By making the transition between topics gradual, so that no difficult hurdles remain, we believe that student interest can be kept from flagging. All the additions, deletions, and refinements in the second edition have been made simply because they seemed to us to contribute to the realization of these basic objectives. Carried over from the first edition are certain features which have won a uniformly favorable reception:

Throughout, students are carefully led to conclusions by the use of meaningful illustrations and are not merely given a set of rules to be memorized;

The extensive review problem sets at the end of each chapter are designed to help students gain confidence in their newfound abilities, or to indicate those topics in which additional study is required;

Several worked-out examples are given to illustrate the solution to each type of problem contained in the problem sets;

Every effort has been made (1) to enable the average student to learn to solve routine problems and develop computational skills with little, if any, assistance from the instructor, (2) to provide the average student with an appreciation of the concepts and logical reasoning involved in this level of mathematics, and (3) to enhance the ability of the average student to apply these concepts and this orderly way of thinking to new situations and new problems.

NEW FEATURES: Color is used to highlight definitions, properties, important statements, and parts of graphs. There are more examples and problems in this edition. Examples have been reworked and reorganized so that every concept is immediately illustrated by many examples. Definitions and properties are carefully stated and in each case they are clarified and justified with specific examples. The problems are related more closely to the examples. The problem sets have been reviewed and revised as experience with the first edition dictated. The grading of the problems, from the very elementary to the more challenging, has been done with greater care. Problem sets are now arranged so that the odd-numbered problems strive for the level of understanding desired by most users while the even-numbered problems probe for deeper understanding of the concepts. This arrangement should simplify the task of making assignments. The answers to odd-numbered problems are given at the back of the book, and the even-numbered answers are provided for the instructor in a supplemental booklet.

CONTENTS: The presentation of topics has been improved in several respects. Much of the material has been rewritten to increase clarity. Because the operations of signed numbers are closely related to the properties of real numbers, Chapters 1 and 2 of the first edition have been combined into Chapter 1 of this edition. Chapter 1 also contains a section on order of operations. Chapter 2 has been rearranged so that it includes all operations of polynomials. Because fractions (Chapter 4) require a knowledge of factoring, Chapter 3 has been reworked to include all types of factoring and special products. Chapter 5 has been reworked and rearranged. The appendixes now contain the metric system and formulas from geometry.

PACE: The pace of a course and the selection of topics depends on the curriculum of the individual school and on the academic calendar. The following suggested pace, based on the authors' and their colleagues' classroom experience, is meant only as a general guide for a three-credit, one-semester course:

Chapter 1:	4 lectures	Chapter 6:	3 lectures
Chapter 2:	5 lectures	Chapter 7:	3 lectures
Chapter 3:	5 lectures	Chapter 8:	4 lectures
Chapter 4:	6 lectures	Chapter 9:	5 lectures
Chapter 5:	5 lectures	Chapter 10:	3 lectures

If students are well prepared, Chapters 1, 2, and 3 can be reviewed briefly in three lectures so that topics in Chapters 9 and 10 can be covered more extensively.

ADDITIONAL AIDS: A *Study Guide* is available for students who need more drill and assistance. The *Study Guide* is written in a semiprogrammed format and conforms to the arrangement of topics in the textbook. It contains fill-in statements and problems, true-or-false statements, and a test for each chapter. All answers are provided in the *Study Guide* to encourage self-testing.

ACKNOWLEDGMENTS: By the time a book reaches its second edition, a great many people have contributed to its development. Of course there would be no second edition if the book's first edition had not been so widely adopted. For this reason, we have asked our publisher to reprint below our acknowledgments to the first edition. In addition, we wish to thank those who were particularly helpful in the writing of this edition: Professors Dolores L. Matthews of Robert Morris College; William G. Cunningham, Walter R. Rogers, and James H. Soles of DeKalb Community College; and Joseph Buggan of the Community College of Allegheny County (Allegheny Campus).

We again would like to express our thanks to our colleagues at Macomb College and to the staff of Worth Publishers for their many contributions.

ACKNOWLEDGMENTS TO THE FIRST EDITION: Discussions with colleagues at many two-year colleges about the problems they face in teaching similar courses have greatly influenced the content and the pace of this text. The guidance of the Committee of Undergraduate Programs in Mathematics (CUPM) was also very helpful.

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Warren, Michigan
January 1977

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CONTENTS

CHAPTER 1

Fundamental Operations of Real Numbers

1 FUNDAMENTAL OPERATIONS OF REAL NUMBERS

Introduction

The study of arithmetic requires a high degree of mastery of the fundamental operations of numbers—adding, subtracting, multiplying, and dividing. Algebra is simply a generalized form of arithmetic that uses variables, usually letters of the alphabet, to represent numbers. Just as arithmetic has to do with the operations of numbers, in a like manner the study of algebra concerns itself with the operations of algebraic expressions. It is worthwhile, then, to become thoroughly familiar with the operations of real numbers before we begin the study of algebra. The objective of this chapter is to study the operations of real numbers and to introduce the basic language and symbols of sets.

1.1 Symbols and Variables

An understanding of *symbols* is essential in the study of algebra. Symbols are the language of mathematics and are used to express mathematical statements in simple and short form. For example, using symbols the statement "six plus four" is written in the form $6 + 4$. A *numeral* is a symbol with a name used to represent the idea of a number. For example, the number "ten" is denoted by the numeral 10.

Four symbols are used for the basic operations in arithmetic:

1 $+$ is used for addition.
2 $-$ is used for subtraction.
3 $\times$ is used for multiplication.
4 $\div$ or —— is used for division.

Additional symbols are used to convey different ideas. For instance, the symbol $=$ is used to represent the idea of *equality*, whereas the symbol $\neq$ is used for not equal.

EXAMPLES

Write the following statements in mathematical symbols.

1 Seven equals four plus three.

SOLUTION. The statement is written in symbols as

$$7 = 4 + 3$$

2 Twenty-one equals seven times three.

SOLUTION. The statement is written in symbols as

$$21 = 7 \times 3$$

3 Nine is not equal to five.

SOLUTION. The statement is written in symbols as

$$9 \neq 5$$

In Example 2, in the statement $21 = 7 \times 3$, the number 21 is called the *product* of 3 and 7. This product can also be written using parentheses:

$$21 = 3(7) \quad \text{or} \quad 21 = (3)7 \quad \text{or} \quad 21 = (3)(7)$$

Order of Operations

Some numerals include a combination of operational symbols. For example, to find the value of $5 \times 2 + 3$, we have the option of first multiplying 5 and 2, then adding 3; that is,

$$5 \times 2 + 3 = 10 + 3 = 13 \qquad \text{which is correct}$$

or, first adding 2 and 3, then multiplying by 5:

$$5 \times 2 + 3 = 5 \times 5 = 25 \qquad \text{which is not correct}$$

To avoid such ambiguous situations and to assure that everyone who works such a problem always gets the same result, we shall adopt the following *order of operations*:

1 Perform all operations inside parentheses.
2 Perform multiplications or divisions from left to right, and then
3 Perform all additions or subtractions from left to right.
4 In the case of problems involving fractions, perform the operations above and below the fraction bar separately; then simplify, if possible, to obtain the final result.

The following examples will illustrate these operations.

EXAMPLES

Find the value of each expression. Write the answer in the simplest possible form.

1 $5 + 7 \times 4$

SOLUTION. First, we perform multiplication, working from left to right, and then add:

$$\begin{aligned} 5 + 7 \times 4 &= 5 + 28 && \text{Multiplication} \\ &= 33 && \text{Addition} \end{aligned}$$

2 $5(4 + 7) - 8 + 2$

SOLUTION. First, we perform the operation inside the parentheses:

$$\begin{aligned} 5(4 + 7) - 8 + 2 &= 5(11) - 8 + 2 && \text{Working inside parentheses} \\ &= 55 - 8 + 2 && \text{Multiplication} \\ &= 49 && \text{Addition or subtraction} \end{aligned}$$

3 $\dfrac{5(7 - 3) + 4}{3 \times 16 \div 8}$

SOLUTION. In this case, we simplify the top and bottom of the fraction separately. To simplify the top, we have

$$\begin{aligned} 5(7-3)+4 &= 5(4)+4 && \text{(Why?)} \\ &= 20+4 && \text{Multiplication} \\ &= 24 && \text{(Why?)} \end{aligned}$$

The bottom will be simplified as follows:

$$\begin{aligned} 3 \times 16 \div 8 &= 48 \div 8 && \text{(Why?)} \\ &= 6 && \text{Division} \end{aligned}$$

Therefore,

$$\frac{5(7-3)+4}{3 \times 16 \div 8} = \frac{24}{6} = 4$$

Concept of a Variable

Up to this point, we have encountered no difficulty in evaluating and simplifying numerical expressions. However, in dealing with expressions such as $5x + 3$, it is not clear which number the letter x represents or, for that matter, which number the expression $5x + 3$ represents. Because these situations occur frequently in algebra, we shall establish a vocabulary to cover such occurrences. A *variable* is a letter, such as x, y, or z, that represents an unknown number. For example, to translate statements such as "the sum of an unknown number and seven" we could write "the sum of x and 7" or, using symbols, express the statement as "$x + 7$." In this case, we definitely do not know the unknown number, so we use the letter x to stand for it until we can find out the number.

In algebra, we indicate the multiplication of x and y by a raised dot or by using no sign of operation, simply writing x and y next to each other. Thus, the multiplication of x and y is written

$$x \cdot y \quad \text{or} \quad xy$$

Also, we indicate the multiplication of 5 and x by

$$5 \cdot x \quad \text{or} \quad 5x$$

In this book, we shall frequently use the notation illustrated by xy and $5x$ to indicate the preceding products.

A combination of variables, numbers, and symbols for operations is called an *algebraic expression*. Thus, $5 + x$, $x - 2$, $12x + 13$, and $8x + 7x$ are algebraic expressions. Each of the expressions above has different numerical values for different values of x. For example, if x represents the number 5, the numerical value of each expression can be found as follows:

$$\begin{array}{rll} 5 + x & \text{becomes} & 5 + 5 = 10 \\ x - 2 & \text{becomes} & 5 - 2 = 3 \\ 12x + 13 & \text{becomes} & 12(5) + 13 = 60 + 13 = 73 \\ 8x + 7x & \text{becomes} & 8(5) + 7(5) = 40 + 35 = 75 \end{array}$$

However, if we replace x by 3, then

$$\begin{array}{rll} 5 + x & \text{becomes} & 5 + 3 = 8 \\ x - 2 & \text{becomes} & 3 - 2 = 1 \\ 12x + 13 & \text{becomes} & 12(3) + 13 = 36 + 13 = 49 \\ 8x + 7x & \text{becomes} & 8(3) + 7(3) = 24 + 21 = 45 \end{array}$$

It should be noted that the number represented by an algebraic expression depends on the replacement chosen for the letter that appears in it. Both letters and numerals are symbols that take the place of numbers. When a letter or numeral can represent only a single number, we shall call it a *constant*. For example, 5, 7, and $\frac{2}{3}$ are constants; x is also a constant if only one number is allowed as a replacement for it.

EXAMPLES

Find the numerical value of each of the following expressions for the indicated value of each variable.

1 $5x + 2$, when $x = 3$.

SOLUTION

$$5x + 2 = 5(3) + 2 = 15 + 2 = 17$$

2 $3x + 7y - 3$, when $x = 5$ and $y = 4$.

SOLUTION

$$\begin{aligned} 3x + 7y - 3 &= 3(5) + 7(4) - 3 \\ &= 15 + 28 - 3 \\ &= 40 \end{aligned}$$

3 $\dfrac{3x - 5y}{x + y}$, when $x = 7$ and $y = 1$.