

# Grundlagen der exakten Naturwissenschaften

Band 6

## Recent Developments in Quantum Logic

Edited by  
P. Mittelstaedt and  
E.-W. Stachow



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exakten Naturwissenschaften  
Band 6**

# **Grundlagen der exakten Naturwissenschaften**

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### **Recent Developments on Quantum Logic**

Edited by P. Mittelstaedt and  
E.-W. Stachow

# Recent Developments in Quantum Logic

Proceedings of the International Symposium  
on Quantum Logic  
Cologne, Germany, June 13-16, 1984

Edited by  
Peter Mittelstaedt and  
Ernst-Walther Stachow  
Universität zu Köln



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## FOREWORD

This volume contains the proceedings of the "International Symposium on Quantum Logic" which was held at the Institute for Theoretical Physics of the University of Cologne, Germany, from June 13-16, 1984.

The Symposium was concerned with the formal language of quantum logic, its logic and semantics, the relation between these linguistic structures and mathematical models and the interpretation of quantum physics. In order to best treat the wide range of problems involved, scientists from the fields of physics, mathematics, and philosophy were invited to participate. The Cologne Symposium was host to approximately thirty participants, the names and institutions of whom are listed at the end of this volume.

The program of the Conference was divided into the following sections:

Opening Session

Problems of the Interpretation of Quantum Theory

Formal Language of Quantum Physics

Mathematical Structures of Quantum Logic

Post-Symposium Discussion Meeting

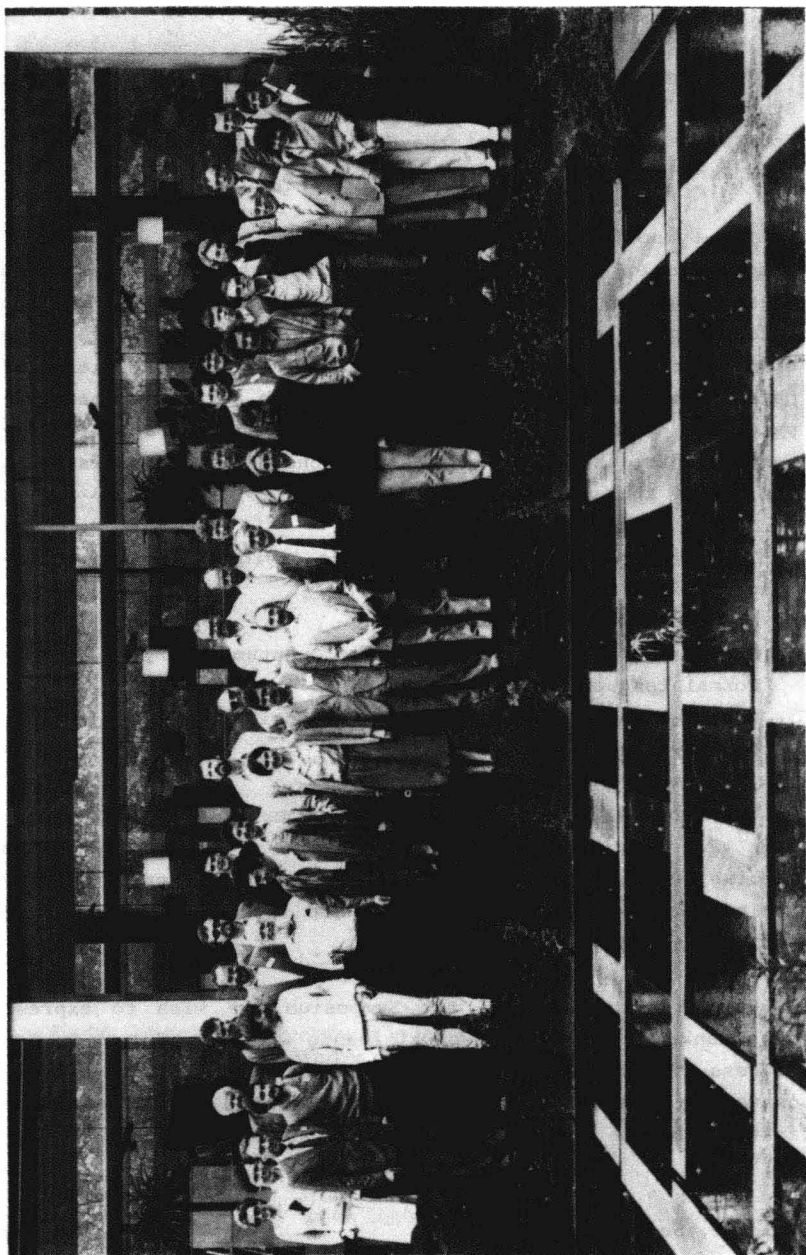
This volume contains the invited paper of the opening session and twenty-three contributed papers. The post-symposium meeting, which was devoted to the question "Is there a Quantum Logic", was however, not recorded and therefore could not be included in the present volume.

As the organizers of the Symposium, we wish to express our thanks to all of the participants who contributed to its success. The Conference was supported by the Stiftung Volkswagenwerk and by the University of Cologne. To both of these institutions we are indebted and express our gratitude.

Cologne, September 1984

Peter Mittelstaedt

Ernst-Walter Stachow



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**Part 1: OPENING SESSION**



## RECENT FACTS IN QUANTUM LOGIC AND SURROUNDINGS

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I.N.F.N. - Sezione di Genova

### 1. INTRODUCTION

This symposium follows the "Workshop on Quantum Logic" that took place in Erice (Sicily) about four years ago, and that left in the participants a feeling of great vitality of research in the area of quantum logic. I shall not make any attempt of summarizing the outcomes of that workshop, but it is perhaps interesting to remind G.C. Rota's review of the volume<sup>(1)</sup> that collected the proceedings. It reads<sup>(2)</sup>: "At the beginning, it was difficult to take quantum logic seriously. A malicious algebraist dubbed it contemptuously, 'poor man's von Neumann algebras'. The lattice-theoretic background made some people suspicious, what with the bad press lattice theory had for a long time. But on leafing through this book, it is hard to dismiss the notion that something very new and very beautiful has come into the midst of mathematics. Advocates of so-called mainstream mathematics, a name which is given to the mathematics that more fittingly belongs on Sunset Boulevard, should take the trouble of studying some of the papers in this book, and to realize that the world is bigger and wider than they thought"

I believe that the present symposium will outline that vitality is still a pertinent attribute of research in the area of quantum logic, and, may be, that the world is indeed bigger and wider than often thought.

The title of my talk should be made a little bit more precise for I shall be concerned only with some recent facts picked up mainly from the surroundings of quantum logic. Indeed, the recent facts in the very area of quantum logic will naturally emerge by

their own from this symposium, and, on the other hand, what is going on in the neighbouring areas may influence the development of quantum logic itself.

## 2. SOME NEW EXPERIMENTAL TESTS OF QUANTUM EFFECTS

In the last years there has been a significant growing of experimental work on basic, text-book effects of non-relativistic quantum mechanics whose empirical evidence was not as direct as desirable<sup>(3)</sup>. Full, sometimes spectacular, agreement with quantum theory is the general conclusion of this experimental work. Before coming to the crucial experiments on two-photon polarization correlations, that touch upon the completeness issue of quantum mechanics, let me quote a couple of other examples.

Remarkable progress in neutron interferometry come in the seventies along with the invention of "perfect crystal interferometry" that uses as interferometer a suitably-shaped single (silicon) crystal, free of dislocations and other defects<sup>(4)</sup>. The typical shape is as in Figure 1. When a beam of neutrons strikes the first ear

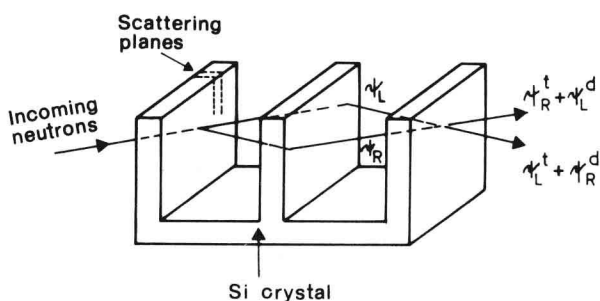


Fig. 1

at some non-zero angle  $\vartheta$  (in practice  $20^\circ$ - $30^\circ$ ) from the normal to the surface, it splits into two paths due to scattering from the planes of atoms perpendicular to the face: a transmitted path and

a Bragg-diffracted one at the same angle but opposite side relative to the scattering planes. Similarly on the subsequent ears. The crystal is made to have the two relevant paths converging at the same point on the third ear thus giving rise to interference phenomena, with superpositions of the transmitted and deflected components. Typical path lengths are  $\sim 10$  cm with path separation  $\sim 5$  cm. The experimental conditions on the incoming-beam intensity are such that the average time spacing between two subsequent neutrons is much larger than the time-of-flight through the apparatus: one is thus in presence of the text-book self-interference. (With photons it was observed in the fifties with a very delicate experiment<sup>(5)</sup>). The interference is sensitive to changes of the relative phase of the wave functions  $\psi_L, \psi_R$  associated with the two paths. A series of nice effects (ranging through gravitational, electromagnetic and strong interactions) have been observed<sup>(6)</sup>. Let me mention, in particular, the effect of a magnetic field on one neutron-path causing a spin precession: it has been explicitly observed that a  $2\pi$ -rotation (of the spin) is not physically equivalent to no rotation, but only a  $4\pi$ -rotation is such. This result is expected by quantum theory, for spin-1/2 wave functions suffer a change of sign under a  $2\pi$ -rotation. Notice, however, that the question of whether this fact is genuinely quantum is not so obvious: there is an old ingenious construction suggested by P.A.M. Dirac to argue that the lack of invariance under a  $2\pi$ -rotation is already present at the classical level, as a mysterious underlying property of space. It consists of remarking that, considering a solid body connected by flexible strings to a fixed reference frame as in Figure 2, a  $2\pi$ -rotation of the solid body makes the strings hopelessly tangled up, while a  $4\pi$ -rotation allows the disentangling of the strings, recovering the initial configuration precisely<sup>(7)</sup>.



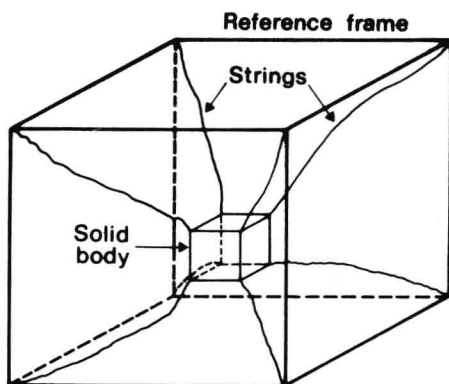


Fig. 2

Remarkable are also the experiments on the quasibound states of ultracold neutrons, trapped into a macroscopic potential well, that provide direct and precise demonstration of predictions of standard quantum mechanics about reflection and tunnelling properties of potential barriers<sup>(8)</sup>.

But let me come now to the experimental study of polarization correlations in the (Bohm's version of the) Einstein-Podolski-Rosen issue. Here the possibility of a hidden variable theory underlying quantum mechanics is called into play. After the long series of no-go theorems for hidden-variable theories the discovery of Bell's inequalities opened the way to a series of no-go experiments. The last of the series, and the most accurate, is by A. Aspect and coworkers<sup>(9)</sup>. The experimental situation is conceptually the same as in the usual, but unfeasable, thought experiment with an electron-positron pair in the singlet state, but one has to do with a pair of photons,  $\gamma_1, \gamma_2$ , from the 0-1-0 cascade of calcium. The source consists of an atomic calcium beam on a point of which converge a krypton laser and a tunable dye laser so as to excite the cascade by non-linear two-photon absorption (about  $4 \cdot 10^7$  cascades/sec are produced). The higher signal-to-noise ratio and