Brief Calculus

Applications + Technology

Tomastik

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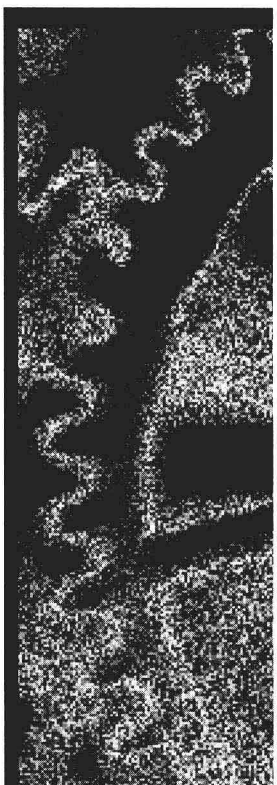
by

Edmond C. Tomastik

University of Connecticut



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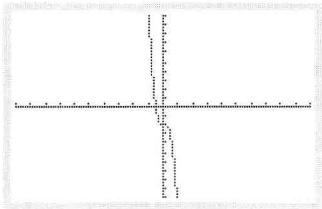
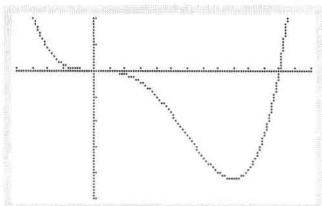


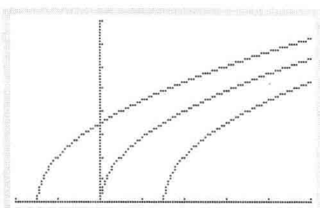
$[-10, 10]$ by $[-10, 10]$

A graph of $y = x^4 - 12x^3 + x^2 - 2$ in the standard viewing window. How do we know this gives the complete picture?



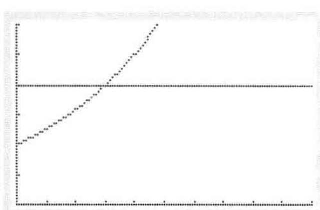
$[-5, 14]$ by $[-2500, 1000]$

Another graph of $y = x^4 - 12x^3 + x^2 - 2$, showing behavior missed on the previous screen. Does this give the complete picture? Only by using calculus can we be sure that this is the complete picture.



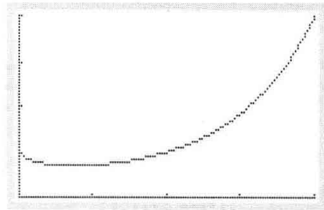
$[-4, 10]$ by $[0, 4]$

The graph of $y = f(x) = \sqrt{x}$ is in the middle. The graph of $y = f(x + 3) = \sqrt{x + 3}$ is the graph of $y = f(x)$ shifted 3 units to the left. The graph of $y = f(x - 3) = \sqrt{x - 3}$ is the graph of $y = f(x)$ shifted 3 units to the right.



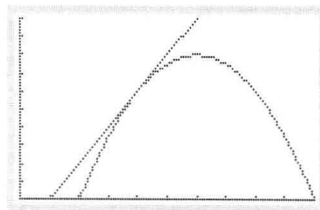
$[0, 1]$ by $[0, 3]$

The solution of $10^x = 2$ can be found by finding the x-coordinate of the point of intersection of the graphs of the functions $y_1 = 10^x$ and $y_2 = 2$. The solution is called log 2.



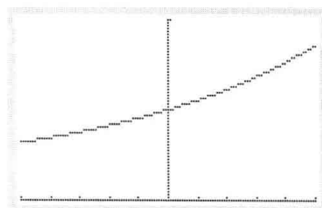
$[0, 2]$ by $[0, 4]$

Determining $\lim_{x \to 0^+} x^x$ is difficult. From the graph on the graphing calculator, it appears this limit is 1.



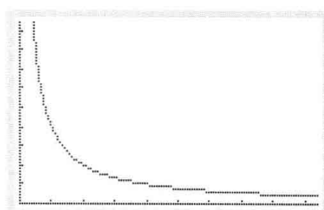
$[0, 10]$ by $[0, 2000]$

It is easy to draw tangent lines using our graphing calculators.



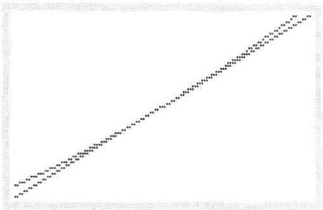
$[-1, 1]$ by $[0, 2]$

A graph of $y = \dfrac{e^x - 1}{x}$ indicates that $\lim_{x \to 0} \dfrac{e^x - 1}{x} = 1$. This is an important limit.



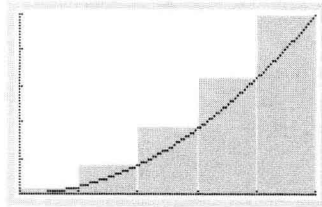
$[0, 4.7]$ by $[0, 4.7]$

Let $f(x) = \ln x$. Then $f'(x) \approx \dfrac{\ln(x + 0.001) - \ln x}{0.001} = g(x)$. Shown is the graph of $y = g(x)$. We can readily see that $g(x) \approx 1/x$. Thus, we suspect that $f'(x) = 1/x$.



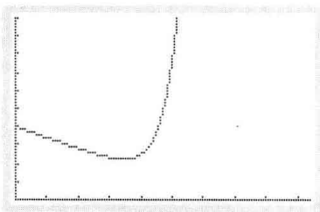
[0.75, 1.25] by [0.5, 1.5]

The graph of $y_1 = x^2$ is nearly the same as the graph of the line tangent to $y_1 = x^2$ at $x = 1$. Thus, we see that near a point where the function is differentiable, the graph of the function is approximately the same as the graph of the tangent line.



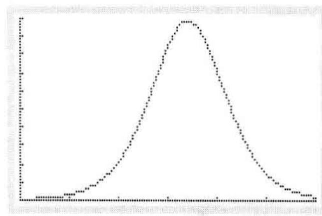
[0, 1] by [0, 1]

Using the program RECT we can easily draw a graph of $y = x^2$ and the rectangles associated with the right-hand sums.



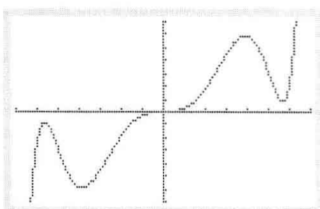
[0, 18.8] by [0, 5000]

A graph of marginal cost indicates marginal cost initially decreases, as the firm gains efficiency through increased production, but then marginal cost increases as too much production overwhelms the firm's capacity.



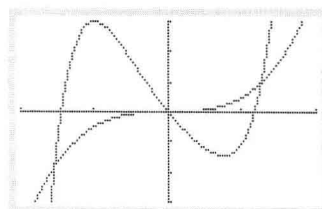
[0, 30] by [0, 1000]

The graph of $y = 3960(e^{0.2x-3.4} + e^{-0.2x+3.4})^{-2}$ models the spread of the Bombay plaque of 1905.



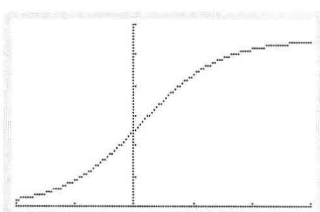
[−1.4, 1.4] by [−1.4, 1.4]

A graph of the complicated function $y = 1.6x^9 - 8x^5 + 7x^3$ is shown. Without using calculus, however, we will never know if this is the complete picture.



[−2, 2] by [−3, 3]

We want to find the area between the graphs of $y = x^5 + x^2 - 3x$ and $y = 0.5x^3$.



[−100, 150] by [0, 12]

A graph of the logistic curve $y = \dfrac{11.5}{1 + 1.2e^{-0.03x}}$ giving the population of the world over the last 200 years and projecting the population over the next 200 years. Logistic curves model population and sales growth and the spread of a technology. There is a rapid, exponential, increase at first, and then a leveling off later.

Brief Calculus

Applications + Technology

To Nancy

PREFACE

Brief Calculus With Technology and Applications is designed to be used in a one semester calculus course aimed at students majoring in business, management, economics, or life/social sciences. The text is written for students with two years of high school algebra. A wide range of topics is included, giving the instructor considerable flexibility in designing a course.

Since the text uses technology as a major tool, the reader is required to use a graphing calculator. The Technology Resource Manual, available with the text, gives all the details in user-friendly terms needed to use a graphing calculator in conjunction with the text. Instructors and students unfamiliar with modern graphing calculators will be surprised at how easy they are to use, and how powerful they have become. This text, together with the accompanying Technology Resource Manual, constitutes a completely organized, self-contained, and easy to use set of material, even for those without any knowledge of graphing calculators.

PHILOSOPHY

The writing of this text has been guided by four basic principles, all of which are consistent with the movement by national mathematics organizations for reform in calculus teaching and learning.

1. **The Rule of Three:** Every topic should be presented graphically, numerically, and algebraically.
2. **Technology:** Incorporate technology into the calculus instruction.
3. **The Way of Archimedes:** Formal definitions and procedures should evolve from the investigation of practical problems.
4. **Teaching Method:** Teach calculus using the investigative, exploratory approach.

The Rule of Three

By always bringing graphical and numerical, as well as algebraic, viewpoints to bear on each topic, the text presents a conceptual understanding of the calculus that is deep and useful in accommodating diverse applications. Sometimes a problem is done algebraically, then *supported* numerically and/or graphically (with a graphing calculator). Sometimes a problem is done numerically and/or graphically (with a graphing calculator), then *confirmed* algebraically. Other times a problem is done numerically or graphically because the algebra is too time consuming or impossible.

Technology

Technology permits more time to be spent on concepts, problem solving, and applications. The technology is used to assist the student to think about the geometric and numerical meaning of the calculus, without undermining the algebraic aspects. In this process, a balanced approach is presented. The text clearly points out that the graphing calculator may not give the whole story, motivating the need to learn the calculus. On the other hand, the text also stresses common situations where exact solutions are impossible, requiring an approximation technique using the graphing calculator. Thus, the graphing calculator is just another tool needed, along with the calculus, if we are to solve a variety of problems in the applications.

Applications and the Way of Archimedes

The text is written for *users* of mathematics. Applications play a central role and are woven into the development of the material. Practical problems are always investigated first, then used to motivate, to maintain interest, and to use as a basis for developing definitions and procedures. Here too, technology plays a natural role, allowing the forbidding and time-consuming difficulties associated with real applications to be overcome.

The Investigative, Exploratory Approach

The text also emphasizes an investigative and exploratory approach to teaching. Whenever practical, the text gives students the opportunity to explore and discover for themselves the basic calculus concepts. Again, technology plays an important role. For example, using their graphing calculators, students discover for themselves the derivatives of x^2, x^3, and x^4, and then generalize to x^n. They also discover the derivatives of $\ln x$ and e^x. None of this is realistically possible without technology.

Student response in the classroom has been exciting. My students enjoy using their graphing calculators in class and feel engaged and part of the learning process. I find students much more receptive to answering questions concerning their observations and more ready to ask questions.

A particularly effective technique is to take 15 or 20 minutes of class time and have students work in small groups to do an exploration or make a discovery. By walking around the classroom and talking with each group, lively discussions arise, even from students who do not normally participate. After such a minilab, the whole class is ready to discuss the insights gained.

Fully in sync with current goals in teaching and learning mathematics, every section in the text includes an Enrichment Exercise Set that encourages exploration, investigation, critical thinking, writing, and verbalization.

Which Graphing Calculator?

Any user of this text faces the immediate problem of what "technology" to use. The TI-82 graphing calculator made by Texas Instruments is highly recommended. The text does not require the additional functions found on the Texas Instruments TI-85, which is, in general, more difficult to use. The student should spend a minimum of time mastering the technology, leaving a maximum of time to learn calculus. In this regard the TI-82 strikes a good balance, powerful while still being user-friendly.

Other graphing calculators can be used. The Technology Resource Manual available with this text covers the TI-81, TI-82, TI-85, the Casio fx-7700G and fx-8700G, and the Sharp EL-9200C and EL-9300C. The TI-82, TI-85, and EL-9300C have a significant advantage in this group. Programs used in the text can be readily transferred electronically from calculator to calculator, eliminating time consuming and error prone programming by hand. About 10 or 15 seconds is required to link two of these calculators, and another second or two to transfer all the needed programs. The TI-82 and TI-85 can communicate with a personal computer and transfer programs. (A special cable and software are needed.) To make this process easier there are two disks available to adopters of the text, one for IBM compatible computers and one for Macintoshes, that contain the many programs used in this text. For more information see your Saunders' sales representative.

Why Graphing Calculators?

The modern graphing calculator is a more effective practical tool than computers in one-dimensional calculus. Computers, unfortunately, are stuck in a laboratory, graphing calculators are completely mobile. Every student can have one instantly ready for use at any time in the classroom. The graphing calculator can be used at the precise moment in the course when needed, with minilabs of 10 minutes being very practical.

Computers are expensive to purchase and maintain, and become obsolete all too soon. Computers also require rooms and monitors. On the other hand, the expense of graphing calculators can be shouldered by the student, not the institution.

IMPORTANT FEATURES

Cost. We understand the financial burden of buying a text and a graphing calculator. To help ease the student burden, the price of this text is less than half the usual price. Costs have been cut by printing with one color and using a soft cover. We believe that the creative layout of the text makes it visually appealing and user-friendly.

Style. The text is designed to implement the philosophy stated earlier. Every section opens by posing an interesting and relevant applied problem using familiar vocabulary, which is later solved in the section after the appropriate mathematics has been developed. Concepts are always introduced intuitively, evolve gradually from the investigation of practical problems or particular cases, and culminate in a definition or result. Students are given the opportunity to investigate and discover concepts for themselves, by using the graphing calculator to create the screens in the text or by doing the Explorations. Topics are presented graphically, numerically, and algebraically to give the reader a deep and conceptual understanding. Scattered throughout the text are historical and anecdotal comments. The historical comments are not only interesting in themselves, but also indicate that mathematics is a continually developing subject. The anecdotal comments relate the material to contemporary real life situations.

Applications. The text includes many meaningful applications drawn from a variety of fields, including numerous referenced examples extracted from current journals. Applications are given for all the mathematics that is presented and are used to motivate the student.

Worked Examples. About 300 worked examples, including self-help examples mentioned below, have been carefully selected to take the reader progressively from the simplest idea to the most complex. All the steps needed for the complete solutions are included.

Screens. There are nearly 100 screens shown in the text. In almost all cases, they represent opportunities for the instructor to have the students reproduce these on their graphing calculators at the point in the lecture when they are needed. This allows the student to be an active partner in the learning process, emphasizes the point being made, and makes the classroom more exciting. A majority of the other graphs can also be done on the graphing calculator.

Explorations. These explorations are designed to further make the student an active partner in the learning process. Some of these explorations can be done in class, some can be done outside class, as group or individual projects. Not all of these explorations use the graphing calculator, some ask to solve a problem or make a discovery using pencil and paper.

Self-Help Exercises. Immediately preceding each exercise set is a set of Self-Help Exercises. These exercises have been very carefully selected to bridge the gap between the exposition in the chapter and the regular exercise set. By doing these exercises and checking the complete solutions provided, students will be able to test or check their comprehension of the material. This, in turn, will better prepare them to do the exercises in the regular exercise set.

Exercises. The book contains over 2500 exercises. The exercises in each set gradually increase in difficulty, concluding with the Enrichment Exercises mentioned below. The exercise sets also include an extensive array of realistic applications from diverse disciplines, including numerous referenced examples extracted from current journals.

Enrichment Exercises. Fully in line with current goals in teaching and learning mathematics, every section in the text includes an Enrichment Exercise Set that encourages exploration, investigation, critical thinking, writing, and verbalization.

End-of-Chapter Projects. These projects, found at the end of each chapter, are especially good for group assignments. These projects are interesting and will serve to motivate the mathematics student.

STUDENT AIDS

- **Boldface** is used when defining new terms.
- **Boxes** are used to highlight definitions, theorems, results, and procedures.
- **Remarks** are used to draw attention to important points that might otherwise be overlooked.
- **Warnings** alert students against making common mistakes.
- **Titles** for worked examples help to identify the subject.
- **Chapter summary outlines**, at the end of each chapter, conveniently summarize all the definitions, theorems, and procedures in one place.
- **Review exercises** are found at the end of each chapter.
- **Chapter projects** are found at the end of each chapter.
- **Answers** to odd-numbered exercises and to all the review exercises are provided in an appendix.
- The **Technology Resource Manual** available with this text has all the details, in user-friendly terms, on how to carry out any of the graphing calculator operations used in the text.
- A **Student's Solution Manual** that contains completely worked solutions to all odd-numbered exercises and to all chapter review exercises is available.

INSTRUCTOR AIDS

- An **Instructor's Solution Manual** with completely worked solutions to the even-numbered exercises and to all the Explorations is available free to adopters. The **Student's Solution Manual** is free to adopters and contains the completely worked solutions to all odd-numbered and to all chapter review exercises. Between the two manuals all exercises are covered.

- The **Technology Resource Manual**, available with this text has all the details, in user-friendly terms, on how to carry out any of the graphing calculator operations used in the text. The manual includes the Texas Instrument TI-81, TI-82, and TI-85, the Casio fx-7700G and fx-8700G, and the Sharp EL-9200C and EL-9300C.
- **A TI Graphing Calculator Program Disk** is available free to all adopters and contains all the programs found in the text. This disk allows you to download the programs from your PC or Macintosh to a Texas Instrument TI-82 or TI-85, with the proper hardware.
- A **Test Bank** written by Joan Van Glabek (Collier County College) contains 100 questions per chapter and is set up like the exercise sets.
- A **Computerized Test Bank** allows instructors to quickly create, edit and print tests or different versions of tests from the set of test questions accompanying the text. It is available free to adopters and is available in IBM or Mac versions.
- **Graph 2D/3D**, a software package by George Bergeman, Northern Virginia Community College, is available free to users. This software graphs functions in one variable and graphs surfaces of functions in two variables. It also provides computational support for solving calculus problems and investigating concepts. It is available for IBM (or IBM-compatible) computers.

Custom Publishing

Courses in business calculus are structured in various ways, differing in length, content, and organization. To cater to these differences, Saunders College Publishing is offering **Brief Calculus with Technology and Applications** in a custom-publishing format. Instructors can rearrange, add, or cut chapters to produce a text that best meets their needs.

The diagram below shows chapter dependencies in Brief Calculus which instructors should consider. Beyond these dependencies, instructors, with custom publishing, are free to choose the topics they want to cover in the order they want to cover them, thereby creating a text that follows their course syllabi.

Saunders College Publishing is working hard to provide the highest quality service and product for your courses. If you have any questions about custom publishing, please contact your local Saunders sales representative.

CONTENT OVERVIEW

Chapter 1. Section 1.0 contains some examples that clearly indicate instances when the graphing calculator fails to tell the whole story, and therefore motivates the need to learn the calculus. Chapter 4 features examples for which our current mathematical knowledge is inadequate to find the exact values of critical points, requiring us to use some approximation technique on our graphing calculators. This theme of needing both mathematical analysis and technology to solve important problems continues throughout the text. The rest of Chapter 1 presents a review of algebra topics. Depending on the preparation of the students, some of the material can be omitted. The first section presents standard coverage (with technology) of lines, the second presents linear models, including an introduction to the theory of the firm with some necessary economics background. The third section introduces functions, and the remainder of the chapter considers a variety of functions, together with graphing techniques and combinations of functions.

Chapter 2. Chapter 2 begins the study of calculus. The first section introduces limits intuitively, lending support with many geometric and numerical examples. The second section continues with continuity, limits at infinity, and a derivation of the natural exponential function from the idea of continuous compounding. The next two sections cover rates of change, slope of the tangent line to a curve, and the derivative. Using a program provided, students can use their graphing calculators to see secant lines converging to a tangent line to a curve. In the section on derivatives, graphing calculators are used to find the derivative of $f(x) = \ln x$. From the limit definition of derivative we know that for h small, $f'(x) \approx \frac{f(x + h) - f(x)}{h}$. We then take $h = 0.001$ and graph the function $g(x) = \frac{\ln(x + 0.001) - \ln x}{0.001}$. We see on our graphing calculator screens that $g(x) \approx 1/x$. Since $f'(x) \approx g(x)$, we then have strong evidence that $f'(x) = 1/x$. This is confirmed algebraically later in Chapter 3. The chapter ends with the tangent line approximation and local linearity near a point where the derivative exists.

Chapter 3. The first section of the chapter begins with some rules for derivatives. In this section we also discover the derivatives of a number of functions using graphing calculators. Just as we found the derivative of $\ln x$ in the preceding chapter, we graph $g(x) = \frac{f(x + 0.001) - f(x)}{0.001}$ for the functions $f(x) = x^2$, x^3, and x^4, and then discover from our graphing calculator screens what particular function $g(x)$ is in each case. Since $f'(x) \approx g(x)$, we then discover $f'(x)$. We then generalize to x^n. In the same way we find the derivative of $f(x) = e^x$. This is an exciting and innovative way for students to find these derivatives. Now that the derivatives of $\ln x$ and e^x are known, these functions can be used in conjunction with the product and quotient rules found in the second section, making this material more interesting and compelling. The third section covers the chain rule, and the fourth section derives the derivatives of the exponential and logarithmic functions in the standard fashion. The last section presents applications to business and economics.

Chapter 4. Graphing and curve sketching are introduced in this chapter. Section 1 describes the importance of the first derivative in graphing. We show clearly that our graphing calculator can fail to give a complete picture of the graph of a function, demonstrating the need for the calculus. We also consider examples for which the exact values of the critical points cannot be determined, and thus need to resort to using an approximation technique on our graphing calculators. Section 2 considers important and timely applications to Laffer curves in tax policy and to the harvesting of renewable natural resources. Section 3 presents the second derivative and its use in graphing, while Section 4 continues with more curve sketching. Extensive applications are given, including population growth, radioactive decay, and the logistic equation with derived estimates of the limiting human population of the earth. The chapter ends with optimization, implicit differentiation, related rates, and Newton's method.

Chapter 5. The first two sections of this chapter present antiderivatives and substitution. Section 3 lays the groundwork for the definite integral by considering left- and right-hand Riemann sums. Here again the graphing calculator plays a vital role. Using programs provided, students can easily graph the rectangles associated with these Riemann sums, and see graphically and numerically what happens as $n \to \infty$. The chapter continues with the definite integral, fundamental theorem of calculus, area between curves, and presents a number of additional applications of the integral, including average value, density, consumer's and producer's surplus, Lorentz's curves, and money flow.

Chapter 6. This chapter contains material on integration by parts, integration using tables, numerical integration, and improper integrals.

Chapter 7. The first sections presents an introduction to functions of several variables, including cost and revenue curves, Cobb-Douglas production functions, and level curves. The second section then introduces partial derivatives with applications that include competitive and complementary demand relations. The third section gives the second derivative test for functions of several variables and applied application on optimization. The fourth section covers Lagrange multipliers and carefully avoids algebraic complications. The method of least squares and correlation is presented in Section 5 and the tangent plane approximation is presented in the sixth section. The last section on double integrals covers double integrals over general domains, Riemann sums, and applications to average value and density. A program is given for the graphing calculator to compute Riemann sums over rectangular regions.

Chapter 8. This chapter is a brief introduction to differential equations and includes the technique of separation of variables, approximate solutions using Euler's method, some qualitative analysis, and mathematical problems involving the harvesting of a renewable natural resoruce. The graphing calculator is used to graph approximate solutions and to do some experimentation.

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