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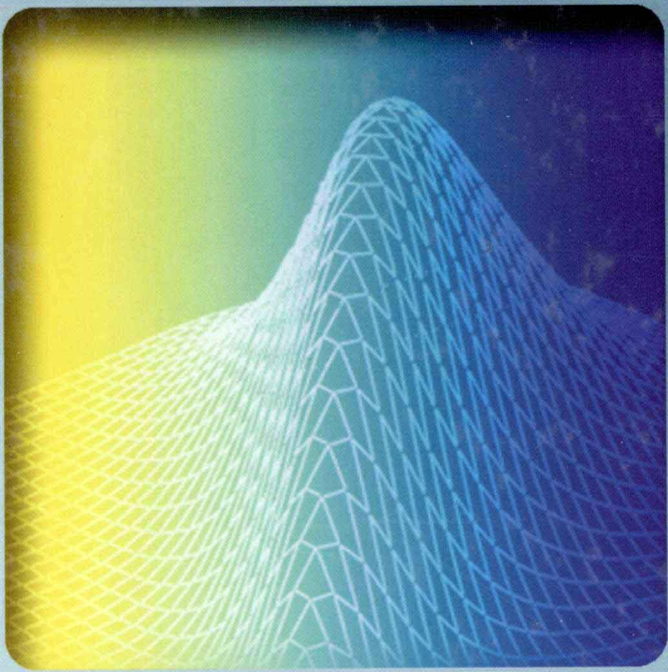

Actuarial Theory for DEPENDENT RISKS

Measures, Orders and Models

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Actuarial Theory for Dependent Risks

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M. Denuit
Université Catholique de Louvain, Belgium

J. Dhaene
Katholieke Universiteit Leuven, Belgium and Universiteit van Amsterdam, The Netherlands

M. Goovaerts
Katholieke Universiteit Leuven, Belgium and Universiteit van Amsterdam, The Netherlands

R. Kaas
Universiteit van Amsterdam, The Netherlands

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Jossey-Bass, 989 Market Street, San Francisco, CA 94103-1741, USA

Wiley-VCH Verlag GmbH, Boschstr. 12, D-69469 Weinheim, Germany

John Wiley & Sons Australia Ltd, 42 McDougall Street, Milton, Queensland 4064, Australia

John Wiley & Sons (Asia) Pte Ltd, 2 Clementi Loop #02-01, Jin Xing Distripark, Singapore 129809

John Wiley & Sons Canada Ltd, 22 Worcester Road, Etobicoke, Ontario, Canada M9W 1L1

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Library of Congress Cataloging in Publication Data

Actuarial theory for dependent risks / M. Denuit … [et al.].
p. cm.
Includes bibliographical references and index.
ISBN 0-470-01492-X (acid-free paper)
1. Risk (Insurance)—Mathematical models. I. Denuit, M. (Michel)

HG8781.A35 2005
368′.001′51—dc22

2005043709

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

ISBN-13 978-0-470-01492-9 (HB)
ISBN-10 0-470-01492-X (HB)

Typeset in 10/12pt Times by Integra Software Services Pvt. Ltd, Pondicherry, India
Printed and bound in Great Britain by Antony Rowe Ltd, Chippenham, Wiltshire
This book is printed on acid-free paper responsibly manufactured from sustainable forestry in which at least two trees are planted for each one used for paper production.

Foreword

Dependence is beginning to play an increasingly important role in the world of risk, with its strong embedment in areas like insurance, financial activities, safety engineering, etc. While independence can be defined in only one way, dependence can be formulated in an unlimited number of ways. Therefore, the assumption of independence prevails as it makes the technical treatment easy and transparent. Nevertheless, in applications dependence is the rule, independence the exception. Dependence quickly leads to an intricate and a far less convenient development.

The authors have accepted the challenge to offer their readership a survey of the rapidly expanding topic of dependence in risk theory. They have brought together the most significant results on dependence available up to now. The breadth of coverage provides an almost full-scale picture of the impact of dependence in risk theory, in particular in actuarial science. Nevertheless, the treatment is not encyclopaedic. In their treatment of risk, the emphasis is more on the ideas than on the mathematical development, more on concrete cases than on the most general situation, more on actuarial applications than on abstract theoretical constructions.

The first three chapters provide in-depth explorations of risk: after dealing with the concept of risk, its measurement is covered via a plethora of different risk measures; its relative position with respect to other risks is then treated using different forms of stochastic orderings. The next three chapters give a similar treatment of dependence as such: modelling of dependence is followed by its measurement and its relative position within other dependence concepts. While illustrations come mainly from the actuarial world, these first two parts of the book have much broader applicability; they make the book also useful for other areas of risk analysis like reliability and engineering. The last three chapters show a stronger focus on applications to insurance: credibility theory is followed by a thorough study of bounds for dependent risks; the text ends with a treatment of risk comparison by using integral orderings and probability metrics. An asset of the book is that a wealth of additional material is covered in exercises that accompany each chapter.

This succinct text provides a thorough treatment of dependence within a risk context and develops a coherent theoretical and empirical framework. The authors illustrate how this theory can be used in a variety of actuarial areas including among others: value-at-risk, ALAE-modelling, bonus-malus scales, annuities, portfolio construction, etc.

Jozef L. Teugels

Katholieke Universiteit Leuven, Belgium

Preface

Traditionally, insurance has been built on the assumption of independence, and the law of large numbers has governed the determination of premiums. But these days, the increasing complexity of insurance and reinsurance products has led to increased actuarial interest in the modelling of dependent risks.

In many situations, insured risks tend to behave alike. For instance, in group life insurance the remaining lifetimes of husband and wife can be shown to possess a certain degree of 'positive dependence'. The emergence of catastrophes and the interplay between insurance and finance also offer good examples in which dependence plays an important role in pricing and reserving.

Several concepts of bivariate and multivariate positive dependence have appeared in the mathematical literature. Undoubtedly, the most commonly encountered dependence property is actually 'lack of dependence', in other words mutual independence. Actuaries have so far mostly been interested in positive dependence properties expressing the notion that 'large' (or 'small') values of the random variables tend to occur together. Negative dependence properties express the notion that 'large' values of one variable tend to occur together with small values of the others. Instances of this phenomenon naturally arise in life insurance (think, for instance, of the death and survival benefits after year k in an endowment insurance, which are mutually exclusive and hence negatively correlated), or for the purpose of competitive pricing. Note that, in general, a negative dependence results in more predictable losses for the insurance company than mutual independence. The independence assumption is thus conservative in such a case. Moreover, assuming independence is mathematically convenient, and also obviates the need for elaborate models to be devised and statistics to be kept on mutual dependence of claims.

There is only one way for risks to be independent, but there are of course infinitely many ways for them to be correlated. For efficient risk management, actuaries need to be able to answer the fundamental question: is the correlation structure dangerous? And if it is, how dangerous is the situation? Therefore, tools to quantify, to compare and to model the strength of dependence between different risks have now become essential.

The purpose of this book is to provide its readership with methods to:

- measure risk
- compare risks
- measure the strength of dependence
- compare dependence structures
- model the dependence structure.

To illustrate the theoretical concepts, we will give many applications in actuarial science.

This book is innovative in many respects. It integrates the theory of stochastic orders, one of the methodological cornerstones of risk theory, the theory of risk measures, the very foundation of risk management, and the theory of stochastic dependence, which has become increasingly important as new types of risks emerge.

More specifically, risk measures will be used to generate stochastic orderings, by identifying pairs of risks about which a class of risk measures agree. Stochastic orderings are then used to define positive dependence relationships.

The copula concept is examined in detail. Apart from the well-known correlation coefficient, other measures of dependence are presented, as well as multivariate stochastic orderings, to evaluate the strength of dependence between risks. We also emphasize the numerous connections existing between multivariate and univariate stochastic orders.

In the third part of the book, we discuss some applications in actuarial mathematics. We first review credibility models. In these models, past claims history not only of the risk itself, but also of related risks, is used to determine the future premium. This method is based on the serial correlation among the annual claim characteristics (frequencies or severities) induced by their sharing a common random effect, and on the correlation between 'related' risks caused by a similar effect. We describe the kind of dependence induced by credibility models, and establish numerous stochastic inequalities showing that the classical credibility construction pioneered by Bühlmann produces very intuitive results.

Secondly, we will derive bounds on actuarial quantities involving correlated risks whose joint distribution is (partially) unknown or too cumbersome to work with. Our focus will be on stop-loss premiums and Value-at-Risk.

Next, we will present probabilistic distances, and show the close connection between this theoretical tool and stochastic orderings. In particular, the relevance of probabilistic distances for the analysis of dependent risks will be demonstrated.

This book complements our *Modern Actuarial Risk Theory* (Kaas *et al.* 2001), which only scratches the surface of the material found here. Since the traditional actuarial risk theory assumes independence between the different random variables of interest, the present book may be thought of as an advanced course on risk theory dropping this hypothesis.

The target audience of this book consists of academics and practitioners who are eager to master modern modelling tools for dependent risks. The inclusion of many exercises also makes the book suitable as the basis for advanced courses on risk management in incomplete markets, as a complement to Kaas *et al.* (2001).

Sometimes, we will give proofs only under simplifying assumptions, in order to help the reader understand the underlying reasoning, bringing out the main ideas without obscuring them with mathematical technicalities. Some proofs are omitted. Appropriate references to the literature will guide the readers interested in a more thorough mathematical treatment of the topic.

Insurance markets are prominent examples of incomplete markets, since the products sold by insurance companies cannot be replicated by some financial trading strategy. We firmly believe that this book should be of interest not only to actuaries but more generally to traders aware that perfect hedges do not exist in reality. The main effect of accounting for market incompleteness has indeed been to bring utility theory back into pricing. More generally, it should bridge quantitative finance and actuarial science.

We would like to thank the Committee on Knowledge Extension Research of the American Society of Actuaries for financial support, under grant 'Actuarial Aspects of Dependencies in Insurance Portfolios'. Thanks also to Virginia Young, the scientific referee of our project, for her careful reading of earlier drafts of the manuscript, and her invaluable advice on matters mathematical and stylistic.

We express our gratitude to Paul Embrechts, Christian Genest, Alfred Müller and Moshe Shaked for having read (parts of) the manuscript, and for the numerous remarks they made. We also thank Professors Frees and Valdez for kindly providing the loss–ALAE data set used in Chapter 4, which were collected by the US Insurance Services Office (ISO).

Michel Denuit is grateful for the financial support of the UCL Fonds Spéciaux de Recherche (under projects 'Tarification en assurance: Vers une nouvelle approche intégrée' and 'Nouvelles méthodes de gestion des risques assurantiels et financiers') and the Belgian Fonds National de la Recherche Scientifique (Crédit aux Chercheurs 'Dépendances entre risques actuariels et financiers'). This work was also partly supported by the contract 'Projet d'Actions de Recherche Concertées' nr 98/03–217 from the Communauté Française Wallonie-Bruxelles.

Jan Dhaene and Marc Goovaerts acknowledge the financial support of the Onderzoeksfonds K.U. Leuven (GOA/02 'Actuariële, financiële en statistische aspecten van afhankelijkheden in verzekerings- en financiële portefeuilles').

Finally, we would like to express our gratitude and appreciation to all those with whom we have had the pleasure of working on problems related to this book: Hélène Cossette, Christian Genest, Hans Gerber, Angela van Heerwaarden, Bart Kling, Claude Lefèvre, Etienne Marceau, Alfred Müller, Marco Scarsini, Mhamed Mesfioui, Moshe Shaked, Sergei Utev, Shaun Wang and Virginia Young.

Michel Denuit
Jan Dhaene
Marc Goovaerts
Rob Kaas

Louvain-la-Neuve, Leuven and Amsterdam

Supplementary material for this book can be found at http://www.actu.ucl.ac.be/staff/denuit/mdenuit.html

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The Concept of Risk

Certum est quia impossible est
Tertullian, AD 200