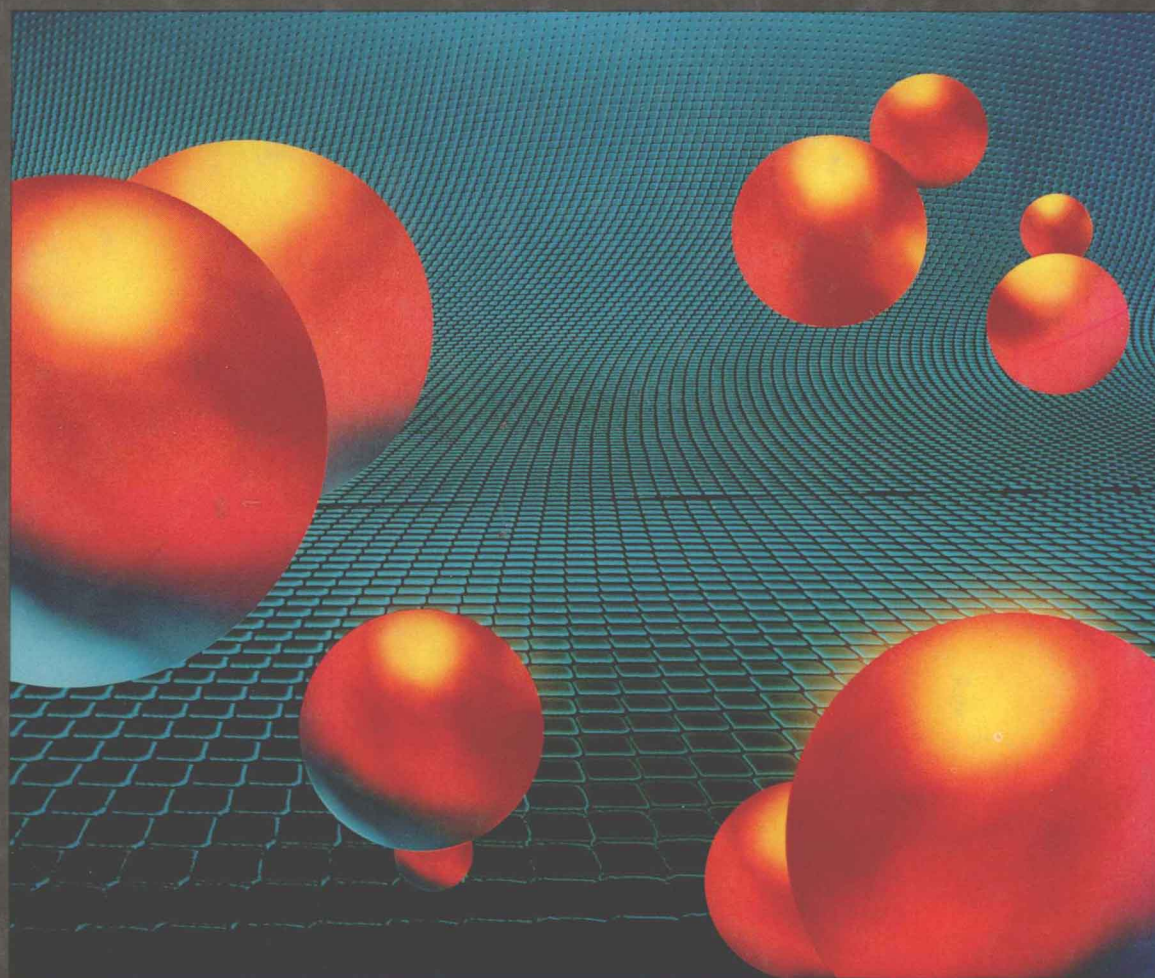


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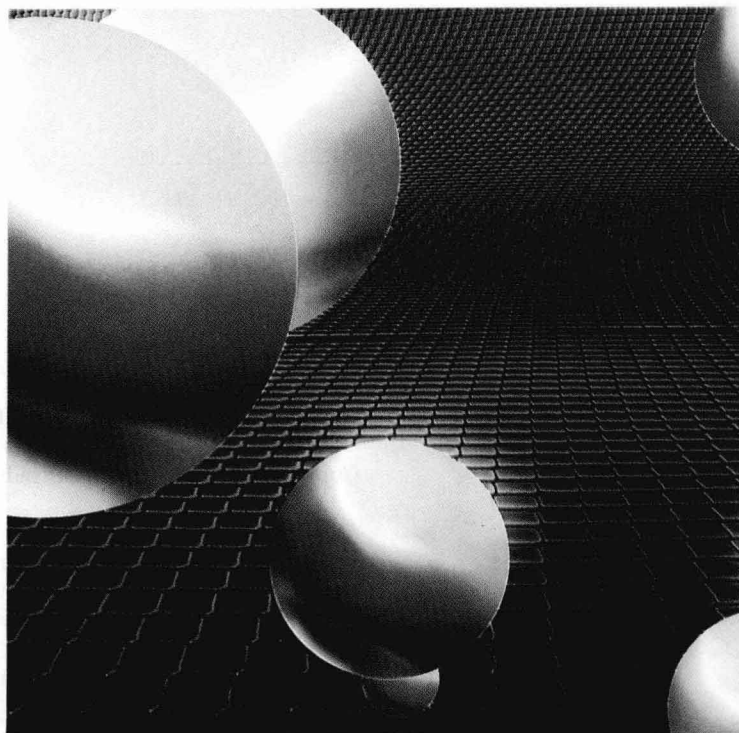
SIXTH EDITION

# ■ PRECALCULUS ■

FUNCTIONS AND GRAPHS



SIXTH EDITION



# PRECALCULUS FUNCTIONS AND GRAPHS

**EARL W. SWOKOWSKI**

Marquette University



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Powers, *Elementary Differential Equations with Linear Algebra*

A Study Guide and Solutions Manual to accompany  
this textbook is available from your college bookstore.  
The guide, by Jeffery A. Cole, contains detailed  
solutions to approximately one-third of the exercises as  
well as strategies for solving other exercises in the text.

## PREFACE

In this sixth edition of *Precalculus: Functions and Graphs*, one of my main goals was to make discussions of concepts easier to understand without sacrificing the mathematical soundness of the previous edition. Solutions to many examples have been rewritten so that important steps stand out clearly. More applied problems have been added to stress the usefulness of the subject matter. Exercise sets have also been improved and expanded. The comments that follow highlight some of the changes and features of this edition.

### CHANGES FOR THE SIXTH EDITION

- ☐ The concept of *interval* and solutions of inequalities are introduced in Section 1.1.
- ☐ Section 1.3 includes methods for solving linear and quadratic equations.
- ☐ In Section 2.3 the maximum or minimum value of a quadratic function is obtained by employing a formula for the vertex of a parabola.
- ☐ Domains of composite functions are covered in Section 2.4.
- ☐ A definition of inverse function is stated without using the formulas  $f^{-1}(f(x)) = x$  and  $f(f^{-1}(y)) = y$ .
- ☐ In Chapter 4 the definition of the number  $e$  is stated using the notation  $\left(1 + \frac{1}{n}\right)^n \rightarrow e$  as  $n \rightarrow \infty$ .
- ☐ The discussion of logarithms has been revised and the change of base formula is given more emphasis.
- ☐ In Section 5.1 the discussion of radian measure has been simplified, and greater emphasis is placed on the length of arc formula  $s = r\theta$ .



- ❑ Trigonometric functions in terms of a unit circle are stressed in Section 5.3, and the graphs of all six trigonometric functions are obtained.
- ❑ In Section 5.5, amplitudes, periods, and phase shifts are emphasized for sketching graphs of equations of the form  $y = a \sin (bx + c)$  and  $y = a \cos (bx + c)$ .
- ❑ Section 5.6 extends the discussion of the preceding section to graphs that involve the tangent, cotangent, secant, and cosecant functions.
- ❑ At the beginning of Section 6.2, methods of obtaining solutions of three simple trigonometric equations are considered before investigating more complicated equations.
- ❑ The graphical significance of cofunctions of complementary angles is illustrated by means of a right triangle in Section 6.3.
- ❑ Section 6.6 on inverse trigonometric functions contains more examples and a brief discussion of the inverse secant function.
- ❑ The *cis* notation for the trigonometric form of a complex number is introduced in Section 7.3.
- ❑ The dot product of vectors is discussed in Section 7.6.
- ❑ Section 8.3 contains an improved notation for row operations on matrices, and the reduced echelon form is emphasized for finding solutions of systems of linear equations.
- ❑ In Section 9.1 the discussion of the axiom of mathematical induction has been deleted so that proofs by induction can be treated earlier.
- ❑ Sections on permutations, combinations, and probability are at the end of Chapter 9.
- ❑ Chapter 10 includes applications involving the eccentricity and reflective properties of conics, orbits of planets, and LORAN navigation.

## FEATURES

**EXAMPLES** Each section contains carefully chosen examples to help students understand and assimilate new concepts. Whenever feasible, applications are included. An innovation in this edition are the labeled *illustrations*, which are brief demonstrations of the use of definitions, laws, and theorems.

**EXERCISES** Approximately half the exercises differ from those in the previous edition. Considerable effort has gone into carefully grading these new problems. Students can obtain practice on all topics in a section by working either the odd-numbered or even-numbered exercises.



**ANSWERS** The answer section includes typical verifications of identities, proofs by mathematical induction, and a consistent scale for graphs.

**APPLIED PROBLEMS** The previous edition contained a great variety of applied problems from many fields. Approximately 125 new exercises and examples involving applications have been added to further strengthen this important aspect of the text.

**CALCULATORS** It is possible to work most of the exercises without a calculator; however, for some problems its use is advisable in order to shorten numerical computations.

**TEXT DESIGN** The text has been redesigned so that concepts and examples stand out. Many figures have been added to help visualize important aspects of problems.

**FLEXIBILITY** Syllabi from schools that have used previous editions attest to the flexibility of the text. Sections and chapters can be rearranged in different ways, depending on the objectives and the length of the course.

## SUPPLEMENTS

Users of this text may obtain the following supplements from the publisher:

*Study Guide and Solutions Manual*, by Jeffery A. Cole of Anoka-Ramsey Community College, contains selected solutions and strategies for solving typical exercises.

*Instructor's Solutions Manual*, by Jeffery A. Cole, provides solutions and answers for all the exercises.

*PWSTest II* is an algorithm-driven test generator for the IBM-PC and compatibles, which can accommodate testing by either chapter or learning objective.

*Printed Test Bank* has three alternate forms of tests for each chapter along with instructor answer keys.

*Transparency Masters* contain figures adapted from the text and statements of key definitions and theorems.

*True BASIC™ Pre-Calculus* software, by John Kemeny and Thomas Kurtz, is ideal for classroom demonstrations, individual study, and problem-solving.

## ACKNOWLEDGMENTS

I wish to thank Jeffery Cole of Anoka-Ramsey Community College for constructing an excellent assortment of well-graded new drill exercises.

Jeffery worked every problem in the text, checking and rechecking answers for accuracy, in addition to meticulously reading the page proofs to help eliminate errata. I also wish to thank Michael Cullen of Loyola Marymount University, whose ideas are the basis of the new applied problems.

This revision has benefited from the comments of many instructors who have used my texts. In particular, I wish to thank the following individuals who reviewed all or portions of the manuscript for this edition of *Precalculus: Functions and Graphs* or for the seventh editions of my precalculus series.

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EARL W. SWOKOWSKI

# PRECALCULUS

## FUNCTIONS AND GRAPHS

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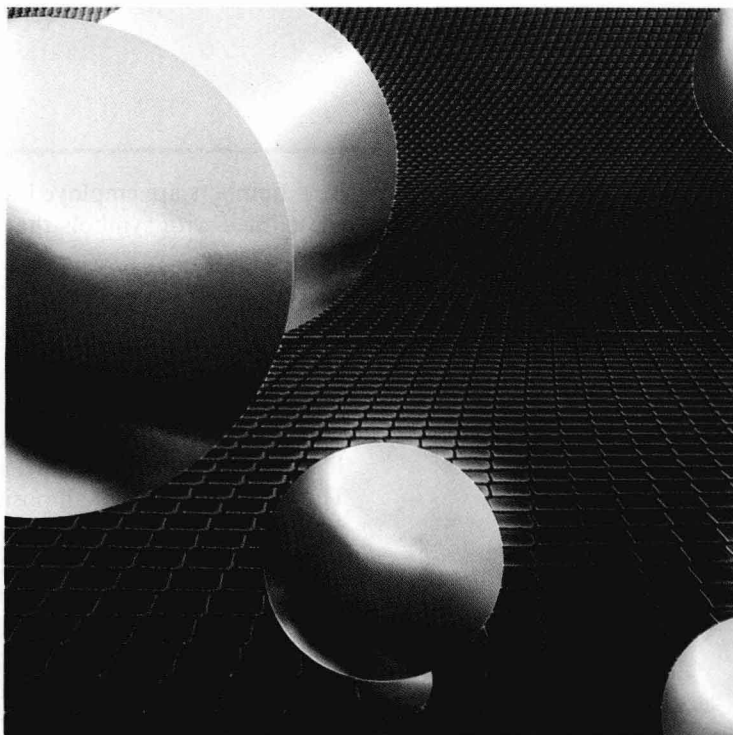
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## CHAPTER 1

**The material in this chapter is basic to the study of precalculus mathematics. After reviewing concepts involving real numbers, we turn our attention to exponents and radicals, and how they may be used to simplify algebraic expressions. Solutions of equations and inequalities are considered next. The chapter concludes with a discussion of rectangular coordinate systems and lines.**



## REAL NUMBERS AND GRAPHS