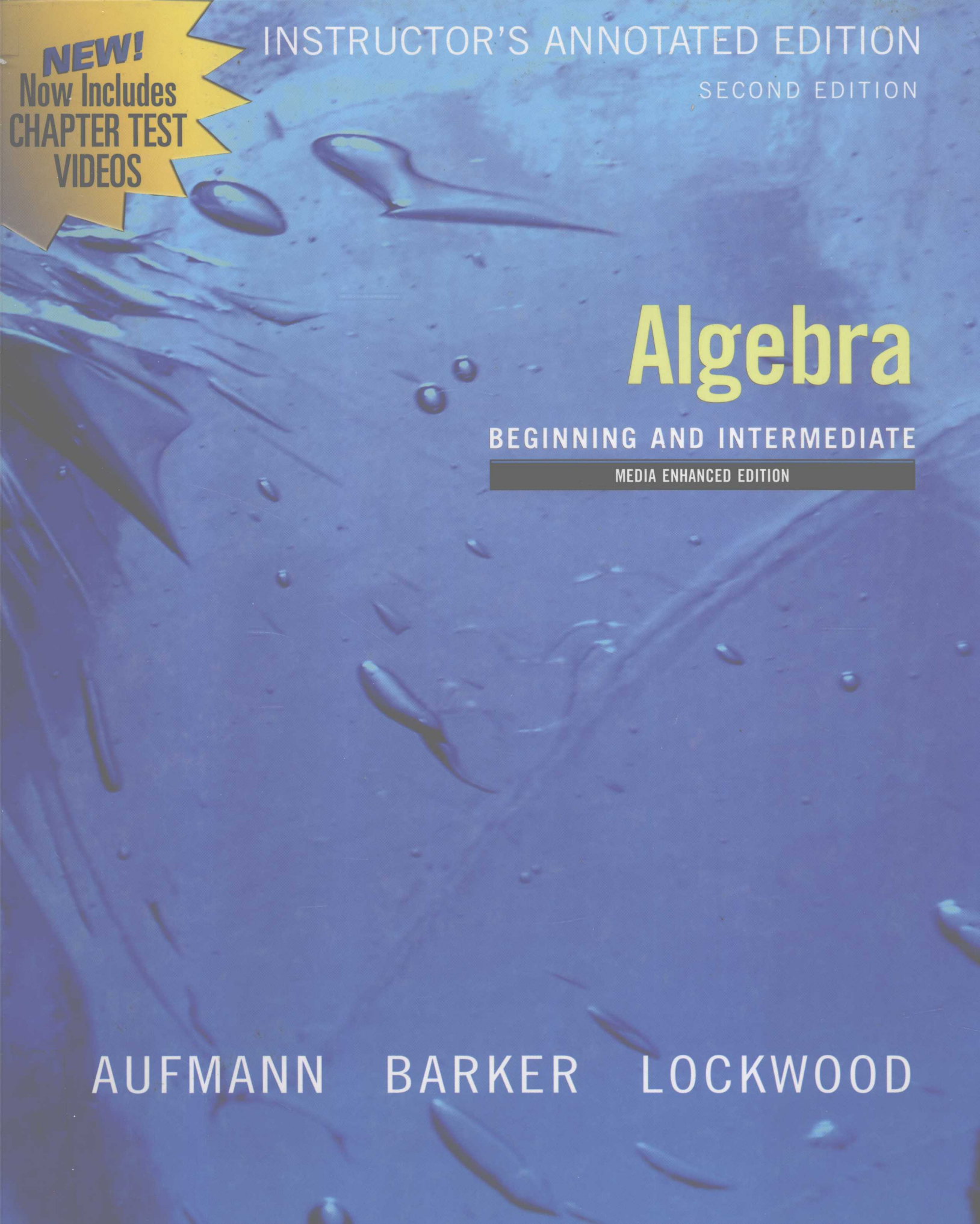

INSTRUCTOR'S ANNOTATED EDITION

SECOND EDITION

Algebra

BEGINNING AND INTERMEDIATE

MEDIA ENHANCED EDITION

AUFMANN BARKER LOCKWOOD

SECOND EDITION

INSTRUCTOR'S ANNOTATED EDITION

MEDIA ENHANCED EDITION

Algebra: Beginning and Intermediate

Richard N. Aufmann
Palomar College, California

Joanne S. Lockwood
New Hampshire Community Technical College

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Richard N. Aufmann and Joanne S. Lockwood

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Preface

Welcome to *Algebra: Beginning and Intermediate*

The second edition of *Algebra: Beginning and Intermediate* examines the fundamental ideas of algebra. The text has been designed not only to meet the needs of the traditional college student but also to serve the needs of returning students whose mathematical proficiency may have declined during years away from formal education.

In this new edition of *Algebra: Beginning and Intermediate*, we have continued to integrate some of the approaches suggested by AMATYC. Each chapter opens by illustrating and referencing a mathematical application within the chapter. At the end of each section there are "Applying Concepts" exercises that include writing, synthesis, critical thinking, and challenge problems. At the end of each chapter there is a "Focus on Problem Solving" section that introduces students to various problem-solving strategies. This is followed by "Projects and Group Activities," which can be used for cooperative learning activities.

Changes to the Second Edition

✔ **NEW! Interactive Exercises** These exercises test students' understanding of the basic concepts presented in a lesson. Included when appropriate, these exercises:

- Generally appear at the beginning of an objective's exercise set.
- Provide students with guided practice on some of the objective's underlying principles.
- Test students' knowledge of the terms associated with a topic OR provide fill-in-the-blank exercises in which students are given part of a solution to a problem and are asked to complete the missing portions.
- Act as stepping stones to the remaining exercises for the objective.

✔ **NEW! Think About It Exercises** These exercises are conceptual in nature. They generally appear near the end of an objective's exercise set and ask the students to think about the objective's concepts, make generalizations, and apply them to more abstract problems. The focus is on mental mathematics, not calculation or computation, and they are designed to help students synthesize concepts.

✔ **REVISED! Chapter 1** In response to user requests, the presentation of operations on fractions in Section 1.3 has been changed from presenting addition and subtraction of rational numbers first, followed by multiplication and division, to presenting multiplication and division of rational numbers first, followed by addition and subtraction.

✔ **REVISED! Chapter 3** This chapter has been revised, in response to user requests:

- Section 3.2 has been expanded to include not only evaluating functions but also graphing functions and the vertical line test.

- Section 3.3 now includes an explanation that the x-coordinate of an x-intercept is a zero of a function; students are asked to find zeros of linear functions. This earlier introduction of zeros lays the groundwork for finding zeros of other types of functions later in the text.

✔ **REVISED! Chapter 5** The first objective in Section 5.1, titled "Evaluate polynomial functions," includes graphs of polynomial functions. Here we revisit the vertical line test by noting that these graphs pass the vertical line test.

✔ **REVISED! Chapter 6** In Objective 6.5.1, in which we are solving equations by factoring, we revisit the concept of zeros of a function by asking students to find zeros of quadratic functions.

✔ **REVISED! Chapter 10** Because the topics of graphing polynomial functions and the vertical line test, which were presented in Chapter 10 of the previous edition, are now presented earlier in the text, Chapter 10 of this edition now begins with graphing absolute value functions as an introduction to translations of graphs in Section 10.1. The earlier presentation and emphasis on functions will provide students with a stronger background for this chapter, the topic of which is "Functions and Relations."

✔ **REVISED! Chapter 11** The introductions to both exponential functions in Section 11.1 and logarithmic functions in Section 11.2 have been rewritten. Both topics are presented using applications. This development should lead to greater student understanding of and interest in these concepts.

Changes to the Media Enhanced Edition

✔ **Media Enhancements** This updated second edition has been enhanced with a variety of multimedia tools to help students succeed in algebra:

- **NEW! Chapter Test Videos** Every chapter test question in the text has an acompanying chapter test video, featuring step-by-step solutions that follow the problem-solving methods used in this textbook. These solutions are presented by real instructors and also feature interactive questions to keep students engaged. Students can access these videos through the Online Study Center.
- **NEW! WebAssign®** Developed by teachers, for teachers, WebAssign allows instructors to focus on what really matters—teaching rather than grading. Instructors can create assignments from a ready-to-use database of algorithmic questions based on end-of-section exercises, or write and customize their own exercises. With WebAssign, instructors can create, post, and review assignments; deliver, collect, grade, and record assignments instantly; offer practice exercises, quizzes, and homework; and assess student performance to keep abreast of individual progress.
- **NEW! Online Multimedia eBook** This electronic version of the textbook includes links to chapter test videos, video lessons, tutorials, and glossary flashcards to expand upon and reinforce concepts as they appear in the text.

✔ **Student Support Resources** This textbook features new tools to support student learning, including:

- **NEW! Pass the Class Media Guide** This student-focused guide provides an overview of the many varied media tools available to students, including Chapter Test Videos, WebAssign, Smarthinking® online tutoring, the Online Multimedia eBook, and more.
- **REVISED! AIM for Success: Getting Started** This updated student preface encourages students to interact with the textbook. Students are asked to fill in the blanks, answer questions, prepare a weekly time schedule, find information within the text and on the class syllabus, write explanations, and provide solutions. This approach will ensure their understanding of how the textbook works and what they need to do to succeed in this course.
- **AIM for Success PowerPoint Slide Show** This PowerPoint Presentation is available on the Online Teaching Center and offers a lesson plan for *AIM for Success: Getting Started* in the text. Visit www.cengage.com/math/aufmann to access this content and more.

Acknowledgments

We would especially like to thank users of the previous edition for their helpful suggestions on improving the text. Also, we sincerely appreciate the suggestions and the time and effort of the following people who reviewed this manuscript and/or its previous edition in preparation for the second edition.

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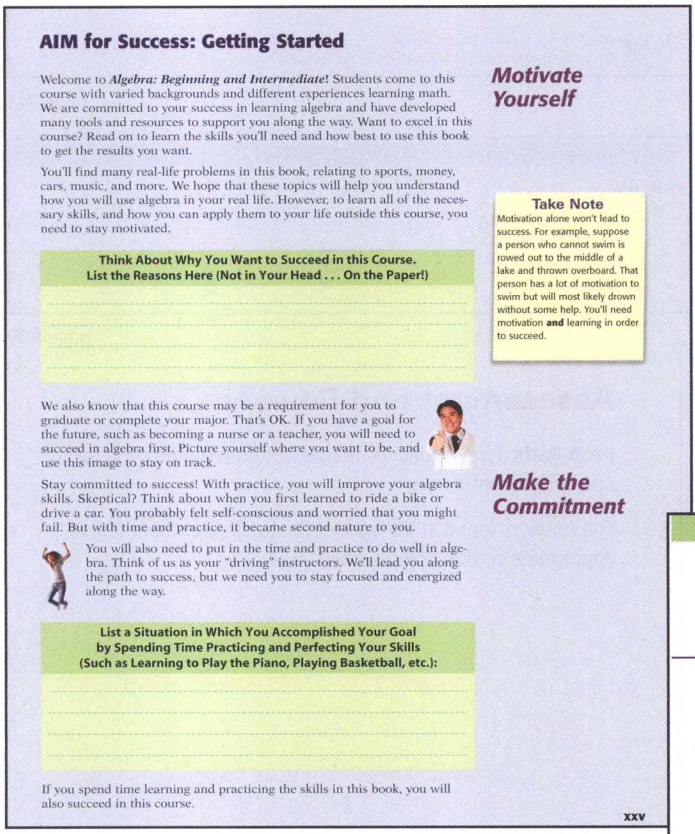
STUDENT SUCCESS

Algebra: Beginning and Intermediate is designed to foster student success through an integrated text and media program.



AIM for Success: Getting Started

Welcome to ***Algebra: Beginning and Intermediate***! Students come to this course with varied backgrounds and different experiences learning math. We are committed to your success in learning algebra and have developed many tools and resources to support you along the way. Want to excel in this course? Read on to learn the skills you'll need and how best to use this book to get the results you want.

You'll find many real-life problems in this book, relating to sports, money, cars, music, and more. We hope that these topics will help you understand how you will use algebra in your real life. However, to learn all of the necessary skills, and how you can apply them to your life outside this course, you need to stay motivated.

Motivate Yourself

Take Note
Motivation alone won't lead to success. For example, suppose a person who cannot swim is rowed out to the middle of a lake and thrown overboard. That person has a lot of motivation to swim but will most likely drown without some help. You'll need motivation **and** learning in order to succeed.

Think About Why You Want to Succeed in this Course. List the Reasons Here (Not in Your Head . . . On the Paper!)

We also know that this course may be a requirement for you to graduate or complete your major. That's OK. If you have a goal for the future, such as becoming a nurse or a teacher, you will need to succeed in algebra first. Picture yourself where you want to be, and use this image to stay on track.

Make the Commitment

Stay committed to success! With practice, you will improve your algebra skills. Skeptical? Think about when you first learned to ride a bike or drive a car. You probably felt self-conscious and worried that you might fail. But with time and practice, it became second nature to you.

You will also need to put in the time and practice to do well in algebra. Think of us as your "driving" instructors. We'll lead you along the path to success, but we need you to stay focused and energized along the way.

List a Situation in Which You Accomplished Your Goal by Spending Time Practicing and Perfecting Your Skills (Such as Learning to Play the Piano, Playing Basketball, etc.):

If you spend time learning and practicing the skills in this book, you will also succeed in this course.

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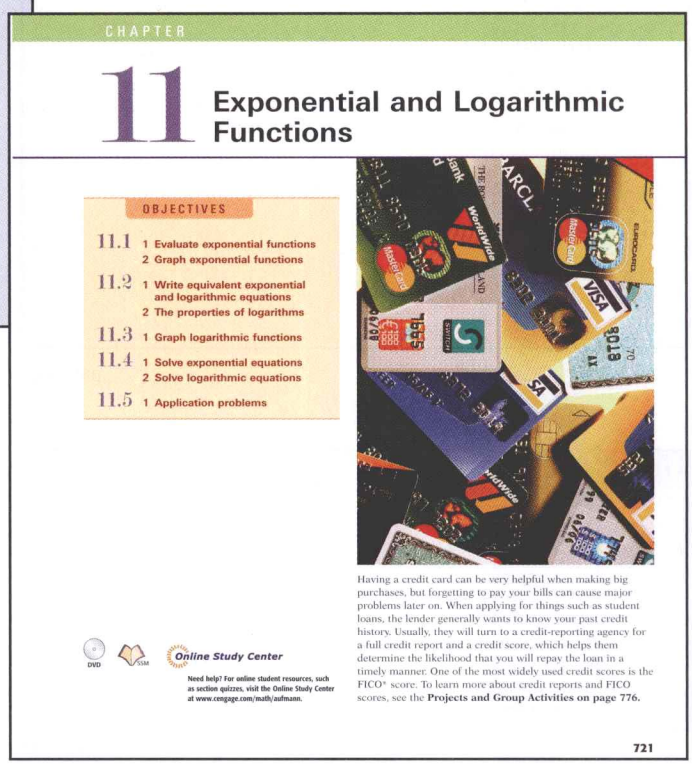
AIM for Success: Getting Started

This preface helps students develop the study skills necessary to achieve success in college mathematics.

It also provides students with an explanation of how to effectively use the features of the text.

AIM for Success: Getting Started can be used as a lesson on the first day of class or as a student project. A lesson plan, with accompanying PowerPoint slides, is available for instructors on the Online Teaching Center.



CHAPTER 11 Exponential and Logarithmic Functions

OBJECTIVES

11.1 1 Evaluate exponential functions
2 Graph exponential functions
11.2 1 Write equivalent exponential and logarithmic equations
2 The properties of logarithms
11.3 1 Graph logarithmic functions
11.4 1 Solve exponential equations
2 Solve logarithmic equations
11.5 1 Application problems

Online Study Center
Need help? For online student resources, such as section quizzes, visit the Online Study Center at www.cengage.com/math/aufmann.

Having a credit card can be very helpful when making big purchases, but forgetting to pay your bills can cause major problems later on. When applying for things such as student loans, the lender generally wants to know your past credit history. Usually, they will turn to a credit-reporting agency for a full credit report and a credit score, which helps them determine the likelihood that you will repay the loan in a timely manner. One of the most widely used credit scores is the FICO® score. To learn more about credit reports and FICO scores, see the **Projects and Group Activities on page 776.**

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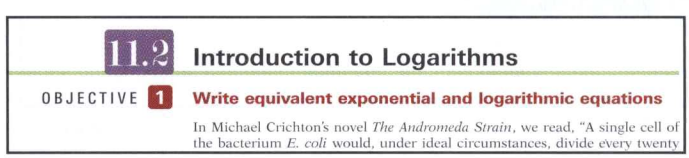
page 721

Objective-Based Approach

All lessons, exercise sets, tests, and supplements are organized around a carefully constructed hierarchy of objectives. This organization serves as a framework for a complete learning system, allowing students to remediate their learning independently at any point in the process.

Each chapter's objectives are listed on the chapter opener page, which serves as a road-map to student learning.

A numbered objective describes the topic of each lesson.



11.2 Introduction to Logarithms

OBJECTIVE 1 Write equivalent exponential and logarithmic equations

In Michael Crichton's novel *The Andromeda Strain*, we read, "A single cell of the bacterium *E. coli* would, under ideal circumstances, divide every twenty

page 732

All exercise sets correspond directly to objectives.

11.2 Exercises

1 Write equivalent exponential and logarithmic equations

1. a. What is a common logarithm?
   b. How is the common logarithm of $4z$ written?

2. a. What is a natural logarithm?
   b. How is the natural logarithm of $3x$ written?

page 743

Answers to the Prep Tests, Chapter Review Exercises, Chapter Tests, and Cumulative Review Exercises refer students back to the original objectives for further study.

Answers to Chapter 11 Selected Exercises

PREP TEST

1. $\frac{1}{9}$ [5.1.2] 2. 16 [5.1.2] 3. −3 [5.1.2] 4. 0; 108 [3.2.1] 5. −6 [2.1.1]
6. −2, 8 [8.1.1] 7. 6326.60 [1.3.2] 8.  [3.2.2]

page A49

Prep Test

1. Simplify: 3^{-2}
2. Simplify: $\left(\frac{1}{2}\right)^{-4}$
3. Complete: $\frac{1}{8} = 2^{?}$
4. Evaluate $f(x) = x^4 + x^3$ for $x = -1$ and $x = 3$.
5. Solve: $3x + 7 = x - 5$
6. Solve: $16 = x^2 - 6x$
7. Evaluate $A(1 + i)^n$ for $A = 5000$, $i = 0.04$, and $n = 6$. Round to the nearest hundredth.
8. Graph: $f(x) = x^2 - 1$

page 722

Assessment and Review

Prep Tests assess students' mastery of prerequisite skills for the upcoming chapter.

The Answers found at the back of the book refer them to the appropriate section objective for review.

Chapter 11 Summary

Key Words	*Examples*
A function of the form $f(x) = b^x$, where b is a positive real number not equal to 1, is an **exponential function**. The number b is the **base** of the exponential function. [11.1.1, p. 723]	$f(x) = 3^x$; 3 is the base of the function.
The function $f(x) = e^x$ is called the **natural exponential function**. [11.1.1, p. 724]	$f(x) = e^x$; e is an irrational number approximately equal to 2.71828183
For $x > 0$, $b > 0$, $b \neq 1$, $y = \log_b x$ is equivalent to $x = b^y$. Read $\log_b x$ as "the **logarithm** of x, base b." [11.2.1, p. 733]	$\log_2 8 = 3$ is equivalent to $8 = 2^3$.

page 778

The **end-of-chapter material** includes:

- Chapter Summary
- Chapter Review Exercises
- Chapter Test

These resources help students prepare for tests.

Cumulative Review Exercises

1. Solve: $4 - 2[x - 3(2 - 3x) - 4x] = 2x$ $\frac{8}{7}$
2. Solve $S = 2WH + 2WL + 2LH$ for L. $L = \frac{S - 2WH}{2W + 2H}$
3. Solve: $|2x - 5| \leq 3$ $\{x \mid 1 \leq x \leq 4\}$
4. Factor: $4x^{2n} + 7x^n + 3$ $(4x^n + 3)(x^n + 1)$

page 782

Cumulative Review Exercises help students maintain skills learned in previous chapters.

AUFMANN INTERACTIVE METHOD (AIM)

Algebra: Beginning and Intermediate uses an interactive style that engages students in trying new skills and reinforcing learning through structured exercises.

An Interactive Approach

EXAMPLE 2 Find the hydrogen ion concentration, H^+, of orange juice that has a pH of 3.6.

Strategy To find the hydrogen ion concentration, replace pH by 3.6 in the equation $pH = -\log(H^+)$ and solve for H^+.

Solution

$$\begin{aligned} pH &= -\log(H^+) \\ 3.6 &= -\log(H^+) \\ -3.6 &= \log(H^+) \\ 10^{-3.6} &= H^+ \\ 0.00025 &\approx H^+ \end{aligned}$$

The hydrogen ion concentration is approximately 0.00025.

PROBLEM 2 On September 3, 2000, an earthquake measuring 5.2 on the Richter scale struck the Napa Valley, 50 mi north of San Francisco. Find the intensity of the quake in terms of I_0.

Solution See page S36.

page 768

- Each section of the text is divided into objectives, and every objective contains one or more sets of matched-pair examples. The first example in each set is worked out.
- The second example, called "Problem," is for the student to work. By solving this problem, the student actively practices concepts as they are presented in the text.

Problem 2

Strategy ▶ To find the intensity, use the equation for the Richter scale magnitude of an earthquake, $M = \log \frac{I}{I_0}$. Replace M by 5.2 and solve for I.

Solution

$$M = \log \frac{I}{I_0}$$

$$5.2 = \log \frac{I}{I_0}$$ • **Replace *M* by 5.2.**

$$10^{5.2} = \frac{I}{I_0}$$ • **Write in exponential form.**

$$10^{5.2} I_0 = I$$

$$158{,}489 I_0 \approx I$$

The earthquake had an intensity that was approximately 158,489 times the intensity of a zero-level earthquake.

page S36

- *Complete worked-out* solutions to these Problems appear at the back of the book for students to check their work.

NEW! Interactive Exercises

Placed at the beginning of an objective's exercise set (when appropriate), these exercises provide guided practice and test students' understanding of the underlying concepts in a lesson. They also act as stepping-stones to the remaining exercises for the objective.

11.4 Exercises

1 Solve exponential equations

1. What is an exponential equation?

2. **a.** What does the Equality of Exponents Property state?
   b. Provide an example of when you would use this property.

3. Solve: $3^{3x-1} = 9^{2x}$.

$3^{3x-1} = 9^{2x}$ • **Both bases are powers of** ___?___.

$3^{3x-1} = (3^{\,?})^{2x}$ • **Write 9 as a power of 3.**

$3^{3x-1} = 3^{\,?}$ • **Simplify the exponent.**

___?___ = ___?___ • **Use the Equality of Exponents Property to equate the exponents.**

$x =$ ___?___ • **Solve for *x*.**

4. Solve: $5^x = 18$

$5^x = 18$ • **The bases are not powers of the same number.**

$\log 5^x = \log 18$ • **Take the** ___?___ **of each side of the equation.**

(___?___) $\log 5 = \log 18$ • **Use the Power Property of Logarithms.**

$x = \frac{?}{?}$ • **Solve for *x* by dividing each side of the equation by** ___?___.

$x \approx$ ___?___ • **Use a calculator to evaluate *x* to the nearest ten-thousandth.**

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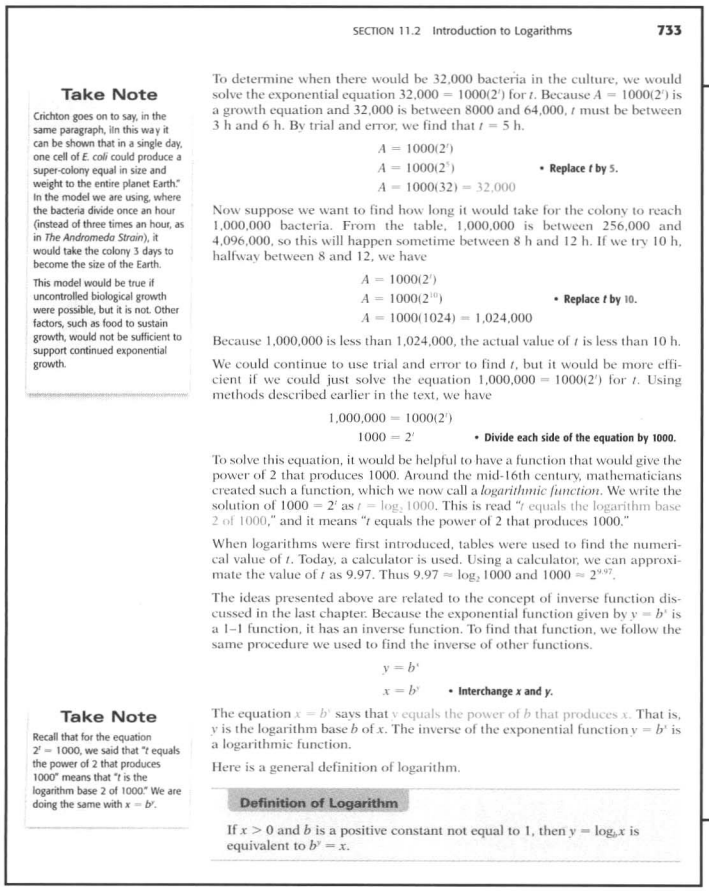
CONCEPTUAL UNDERSTANDING

Algebra: Beginning and Intermediate helps students understand the course concepts through the textbook exposition and feature set.



SECTION 11.2 Introduction to Logarithms 733

Take Note

Crichton goes on to say, in the same paragraph, iIn this way it can be shown that in a single day, one cell of *E. coli* could produce a super-colony equal in size and weight to the entire planet Earth." In the model we are using, where the bacteria divide once an hour (instead of three times an hour, as in *The Andromeda Strain*), it would take the colony 3 days to become the size of the Earth.

This model would be true if uncontrolled biological growth were possible, but it is not. Other factors, such as food to sustain growth, would not be sufficient to support continued exponential growth.

To determine when there would be 32,000 bacteria in the culture, we would solve the exponential equation $32{,}000 = 1000(2^t)$ for t. Because $A = 1000(2^t)$ is a growth equation and 32,000 is between 8000 and 64,000, t must be between 3 h and 6 h. By trial and error, we find that $t = 5$ h.

$A = 1000(2^t)$
$A = 1000(2^5)$ • Replace t by 5.
$A = 1000(32) = 32{,}000$

Now suppose we want to find how long it would take for the colony to reach 1,000,000 bacteria. From the table, 1,000,000 is between 256,000 and 4,096,000, so this will happen sometime between 8 h and 12 h. If we try 10 h, halfway between 8 and 12, we have

$A = 1000(2^t)$
$A = 1000(2^{10})$ • Replace t by 10.
$A = 1000(1024) = 1{,}024{,}000$

Because 1,000,000 is less than 1,024,000, the actual value of t is less than 10 h.

We could continue to use trial and error to find t, but it would be more efficient if we could just solve the equation $1{,}000{,}000 = 1000(2^t)$ for t. Using methods described earlier in the text, we have

$1{,}000{,}000 = 1000(2^t)$
$1000 = 2^t$ • Divide each side of the equation by 1000.

To solve this equation, it would be helpful to have a function that would give the power of 2 that produces 1000. Around the mid-16th century, mathematicians created such a function, which we now call a *logarithmic function*. We write the solution of $1000 = 2^t$ as $t = \log_2 1000$. This is read "t equals the logarithm base 2 of 1000," and it means "t equals the power of 2 that produces 1000."

When logarithms were first introduced, tables were used to find the numerical value of t. Today, a calculator is used. Using a calculator, we can approximate the value of t as 9.97. Thus $9.97 \approx \log_2 1000$ and $1000 \approx 2^{9.97}$.

The ideas presented above are related to the concept of inverse function discussed in the last chapter. Because the exponential function given by $y = b^x$ is a 1–1 function, it has an inverse function. To find that function, we follow the same procedure we used to find the inverse of other functions.

$y = b^x$
$x = b^y$ • Interchange x and y.

Take Note

Recall that for the equation $2^t = 1000$, we said that "t equals the power of 2 that produces 1000" means that "t is the logarithm base 2 of 1000." We are doing the same with $x = b^y$.

The equation $x = b^y$ says that y equals the power of b that produces x. That is, y is the logarithm base b of x. The inverse of the exponential function $y = b^x$ is a logarithmic function.

Here is a general definition of logarithm.

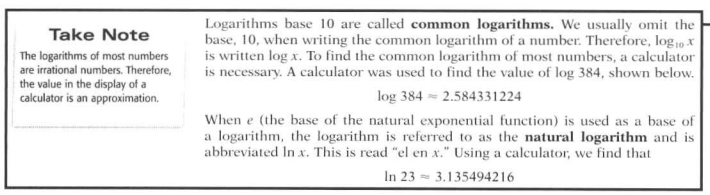
Definition of Logarithm

If $x > 0$ and b is a positive constant not equal to 1, then $y = \log_b x$ is equivalent to $b^y = x$.

page 733

Features in the Exposition

- Margin notes titled Take Note amplify the concept under discussion or alert students to points requiring special attention.
- Key concepts are presented in orange boxes in order to highlight them and provide for easy reference.



Take Note

The logarithms of most numbers are irrational numbers. Therefore, the value in the display of a calculator is an approximation.

Logarithms base 10 are called **common logarithms.** We usually omit the base, 10, when writing the common logarithm of a number. Therefore, $\log_{10} x$ is written $\log x$. To find the common logarithm of most numbers, a calculator is necessary. A calculator was used to find the value of log 384, shown below.

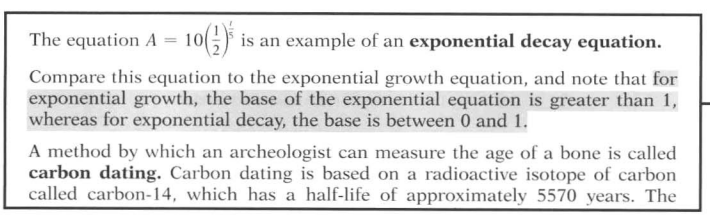
$\log 384 \approx 2.584331224$

When e (the base of the natural exponential function) is used as a base of a logarithm, the logarithm is referred to as the **natural logarithm** and is abbreviated $\ln x$. This is read "el en x." Using a calculator, we find that

$\ln 23 \approx 3.135494216$

page 736

- Key terms, in bold, emphasize important vocabulary.



The equation $A = 10\left(\frac{1}{2}\right)^{\frac{t}{5}}$ is an example of an **exponential decay equation.**

Compare this equation to the exponential growth equation, and note that for exponential growth, the base of the exponential equation is greater than 1, whereas for exponential decay, the base is between 0 and 1.

A method by which an archeologist can measure the age of a bone is called **carbon dating.** Carbon dating is based on a radioactive isotope of carbon called carbon-14, which has a half-life of approximately 5570 years. The

page 765

- Important points are now highlighted, to help students recognize what is most important and to study more effectively.

CONCEPTUAL UNDERSTANDING (*continued*)

Algebra: Beginning and Intermediate helps students understand the course concepts through the textbook exposition and feature set.

Features in the Exercise Sets

11.1 Concept Review

Determine whether the following statements are always true, sometimes true, or never true.

1. The domain of an exponential function $f(x) = b^x$, $b > 0$, $b \neq 1$, is the set of positive numbers.
2. An exponential function $f(x) = b^x$, $b > 0$, $b \neq 1$, is a 1–1 function.
3. The graph of the exponential function $f(x) = b^x$, $b > 0$, $b \neq 1$, passes through the point (0, 0).
4. For the function $f(x) = b^x$, $b > 0$, $b \neq 1$, the base b is a positive integer.
5. An exponential function $f(x) = b^x$, $b > 0$, $b \neq 1$, has two x-intercepts.

page 728

- **Concept Review** exercises appear at the beginning of each exercise set and are designed to test a student's understanding of the topic.

51. $b > 1$ and m and n are positive numbers such that $m > n$. Is $\log_b m$ less than, equal to, or greater than $\log_b n$?

52. $0 < b < 1$ and $m > 1$. Is $\log_b m$ less than, equal to, or greater than zero?

page 744

- **NEW! Think About It Exercises** are conceptual in nature. They ask the students to think about the objective's concepts, make generalizations, and apply them to more abstract problems. The focus is on mental mathematics, and they are designed to help students synthesize concepts.

11.4 Exercises

1 Solve exponential equations

1. What is an exponential equation?
2. **a.** What does the Equality of Exponents Property state?
   b. Provide an example of when you would use this property.

page 760

- **Writing Exercises** require students to verbalize concepts, which is a crucial step in learning them.

11.3 Applying Concepts

31. Employment The proficiency of a typist decreases (without practice) over time. An equation that approximates this decrease is given by $S = 60 - 7\ln(t + 1)$, where S is the typing speed in words per minute and t is the number of months without typing.
a. Graph the equation.
b. The point whose approximate coordinates are (4, 49) is on the graph. Write a sentence that describes the meaning of this ordered pair.

32. Astronomy Astronomers use the *distance modulus* of a star as a method of determining the star's distance from Earth. The formula is $M = 5 \log s - 5$, where M is the distance modulus and s is the star's distance from Earth in parsecs. (One parsec $\approx 2.1 \times 10^{13}$ miles.)
a. Graph the equation.

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- **Applying Concepts** are more challenging problems involving multiple concepts that require interpretation, analysis, and critical thinking. They are designed to help students to apply and internalize concepts.

PROBLEM SOLVING

Algebra: Beginning and Intermediate emphasizes applications, problem solving, and critical thinking.

16. Demography The number of centenarians (people aged 100 or older) in the United States has been increasing dramatically. Based on data from the U.S. Census Bureau, and using $x = 0$ to correspond to the year 2000, the equation $y = 66.77(1.0532^x)$ approximates the projected U.S. centenarian population through 2050, where y is in thousands.

a. According to the model, what is the predicted population of centenarians in 2015? Round to the nearest thousand. **145,000 centenarians**

b. In what year does the model predict that the U.S. centenarian population will first exceed 250,000? **2025**

Chemistry For Exercises 17 and 18, use the equation $pH = -\log(H^+)$, where H^+ is the hydrogen ion concentration of a solution. Round to the nearest tenth.

17. Find the pH of milk, for which the hydrogen ion concentration is 3.97×10^{-7}. **6.4**

18. Find the pH of a baking soda solution, for which the hydrogen ion concentration is 3.98×10^{-9}. **8.4**

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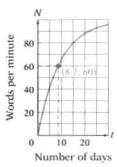
Integrated Real-Life Applications

Wherever appropriate, the last objective of a section presents applications that require students to use problem-solving strategies, along with the skills covered in that section, to solve practical problems. This carefully integrated applied approach generates student awareness of the value of algebra as a real-life tool. Applications are taken from many disciplines and are emphasized through sourced data.



EXAMPLE 1 The number of words per minute a student can type will increase with practice and can be approximated by the equation $N = 100[1 - (0.9)^t]$, where N is the number of words typed per minute after t days of instruction. In how many days will the student be able to type 60 words per minute?

Strategy To find the number of days, replace N by 60 and solve for t.

Solution

$$N = 100[1 - (0.9)^t]$$
$$60 = 100[1 - (0.9)^t]$$
$$0.6 = 1 - (0.9)^t$$ • Divide each side of the equation by 100.

$$-0.4 = -(0.9)^t$$
$$0.4 = (0.9)^t$$ • Subtract 1 from each side of the equation.

$$\log 0.4 = \log(0.9)^t$$
$$\log 0.4 = t \log 0.9$$
$$t = \frac{\log 0.4}{\log 0.9} \approx 8.6967184$$

After approximately 9 days the student will type 60 words per minute.

The graph at the left is the graph of $N = 100[1 - (0.9)^t]$. Note that t is approximately 8.7 when $N = 60$.

PROBLEM 1 An investment of $3000 is placed into an account that earns 12% annual interest compounded monthly. In approximately how many years will the investment be worth twice the original amount? Round to the nearest whole number.

Solution See page S36. 6 years

Point of Interest

Søren Sørensen

The pH scale was created by the Danish biochemist Søren Sørensen in 1909 to measure the acidity of water used in the brewing of beer. pH is an abbreviation for pondus hydrogenii, which translates as "potential hydrogen."

The first applications of logarithms (and the main reason why they were developed) were to reduce computational drudgery. Today, with the widespread use of calculators and computers, the computational uses of logarithms have diminished. However, a number of other applications of logarithms have emerged.

A chemist measures the acidity or alkalinity of a solution by the formula **$pH = -\log(H^+)$**, where H^+ is the concentration of hydrogen ions in the solution. A neutral solution such as distilled water has a pH of 7, acids have a pH less than 7, and alkaline solutions (also called basic solutions) have a pH greater than 7.

Find the pH of vinegar for which $H^+ = 1.26 \times 10^{-3}$. Round to the nearest tenth.

Use the pH equation. $H^+ = 1.26 \times 10^{-3}$.

$$pH = -\log(H^+)$$
$$= -\log(1.26 \times 10^{-3})$$
$$= -(\log 1.26 + \log 10^{-3})$$
$$\approx -[0.1004 + (-3)] = 2.8996$$

The pH of vinegar is approximately 2.9.

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Problem-Solving Strategies

A carefully developed approach to problem solving emphasizes the importance of *strategy* when solving problems.

- Students are encouraged to develop their own strategies as part of their solutions to problems.
- Model strategies are always presented as guides for students to follow as they attempt the parallel Problems that accompany each numbered Example.

SECTION 11.5

Problem 1

Strategy ▶ To find the time, solve the compound interest formula for n. Use $P = 6000$, $A = 3000$, and $i = \frac{12\%}{12} = \frac{0.12}{12} = 0.01$.

Solution

$$P = A(1 + i)^n$$
$$6000 = 3000(1 + 0.01)^n$$
$$6000 = 3000(1.01)^n$$
$$2 = (1.01)^n$$
$$\log 2 = \log(1.01)^n$$
$$\log 2 = n \log 1.01$$
$$\frac{\log 2}{\log 1.01} = n$$
$$70 \approx n$$

70 months ÷ 12 ≈ 5.8 years

In approximately 6 years, the investment will be worth $6000.

page S36

Focus on Problem Solving

Find a Pattern Polya's four recommended problem-solving steps are stated below.

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review your solution

Point of Interest

Carl Friedrich Gauss (1777–1855) has been called the "Prince of Mathematicians" by some historians. He applied his genius to many areas of mathematics and science. A unit of magnetism, the gauss, *is named in his honor. Some electronic appliances (televisions, for instance) contain a* degausser *that controls magnetic fields.*

One of the several ways of devising a plan is first to try to find a pattern. Karl Friedrich Gauss supposedly used this method to solve a problem that was given to his math class when he was in elementary school. As the story goes, his teacher wanted to grade some papers while the class worked on a math problem. The problem the teacher gave to the class was to find the sum

$$1 + 2 + 3 + 4 + \cdots + 100$$

Gauss quickly solved the problem by seeing a pattern. Here is what he saw.

$$\begin{aligned} &101 \\ &101 \\ &101 \\ &101 \\ 1 + 2 + 3 + 4 + \cdots + 97 &+ 98 + 99 + 100 \end{aligned}$$

Note that

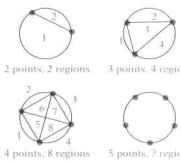
$1 + 100 = 101$
$2 + 99 = 101$
$3 + 98 = 101$
$4 + 97 = 101$

Gauss noted that there were 50 sums of 101. Therefore, the sum of the first 100 natural numbers is

$$1 + 2 + 3 + 4 + \cdots + 97 + 98 + 99 + 100 = 50(101) = 5050$$

Try to solve the following problems by finding a pattern.

1. Find the sum $2 + 4 + 6 + \cdots + 96 + 98 + 100$.
2. Find the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$.
3. Find another method of finding the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$ given in the previous exercise.
4. Find the sum $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{49 \cdot 50}$.
   Hint: $\frac{1}{1 \cdot 2} = \frac{1}{2}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} = \frac{2}{3}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{3}{4}$



5. The following problem shows that checking a few cases does not always result in a conjecture that is true for *all* cases. Select any two points on a circle and draw a *chord*, the line connecting the points (see the drawing in the left margin). The chord divides the circle into two regions. Now select three different points and draw chords connecting each of the three points with every other point. The chords divide the circle into four regions. Now select four points and connect each of the points with every other point. Make a conjecture about the relationship between the number

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Focus on Problem Solving

Students are introduced to various successful problem-solving strategies within the end-of-chapter material. Strategies include:

- drawing a diagram,
- applying solutions to other problems,
- working backwards,
- inductive reasoning,
- and trial and error.

NEW! Think About It Exercises

These exercises focus on concepts, not on calculation or computation, and will help develop students' critical thinking skills.

29. Which function(s) will have the same graph as the graph of the function $f(x) = \log_3 3x$?

a. $g(x) = 3 \log_3 x$ **b.** $h(x) = 1 + \log_3 x$

c. $F(x) = \log_3 x^3$ **d.** $G(x) = \dfrac{(\log 3x)}{(\log 3)}$

30. Which function(s) will have the same graph as the graph of the function $f(x) = -\log_b x$?

a. $g(x) = \log_b(-x)$ **b.** $h(x) = \dfrac{1}{\log_b x}$

c. $F(x) = \log_b \dfrac{1}{x}$ **d.** $G(x) = \dfrac{(\log b)}{(\log x)}$

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Additional Resources—Get More Value from Your Textbook!

INSTRUCTOR RESOURCES

Instructor's Annotated Edition (IAE)

This edition contains a replica of the student text and additional items just for the instructor. Answers to all exercises are provided.

Online Teaching Center

This website offers instructors a variety of resources, including instructor's solutions, digital art and figures, course sequences, drill sheets, printed test bank, and more.

Computerized Testing

Computerized Testing provides instructors with a wide array of new algorithmic exercises along with improved functionality and ease of use. Instructors can create, author/edit algorithmic questions, customize, and deliver multiple types of tests.

STUDENT RESOURCES

Student Solutions Manual

Contains complete solutions to all odd-numbered exercises and all of the solutions to the end-of-chapter material.

Math Study Skills Workbook, Third Edition, by Paul Nolting

Helps students identify their strengths, weaknesses, and personal learning styles in math. Nolting offers study tips, proven test-taking strategies, a homework system, and recommendations for reducing anxiety and improving grades.

Online Study Center

This free website gives students access to Chapter Test Videos, the Online Multimedia eBook, ACE practice tests, glossary flashcards, and other study resources.

DVD Instructional DVDs

Hosted by Dana Mosely, these text-specific DVDs cover all sections of the text and provide explanations of key concepts, examples, exercises, and applications in a lecture-based format. DVDs are now close-captioned for the hearing-impaired.

Online Study and Teaching Centers

Instructors and students have powerful course management tools and text-specific content to support all of their online teaching and learning needs. The Online Study and Teaching Centers now include:

- **NEW! WebAssign®** Developed by teachers, for teachers, WebAssign allows instructors to create assignments from an abundant ready-to-use database of algorithmic questions, or write and customize their own exercises. With WebAssign, instructors can create, post, and review assignments 24 hours a day, 7 days a week; deliver, collect, grade, and record assignments instantly; offer more practice exercises, quizzes, and homework; assess student performance to keep abreast of individual progress; and capture the attention of online or distance learning students.
- **SMARTHINKING® Live, Online Tutoring** SMARTHINKING provides an easy-to-use and effective online, text-specific tutoring service. A dynamic **Whiteboard** and a **Graphing Calculator** function enable students and e-structors to collaborate easily.

Online Course Management Content for Blackboard®, WebCT®, and eCollege®

Deliver program- or text-specific content online using your institution's local course management system. Cengage Learning offers homework, tutorials, videos, and other resources formatted for Blackboard, WebCT, eCollege, and other course management systems. Add to an existing online course or create a new one by selecting from a wide range of powerful learning and instructional materials.

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Reduce Class Preparation Time with the Instructor's Annotated Edition

The Instructor's Annotated Edition is ideal for seasoned instructors, teaching assistants, or adjuncts. Reduced student pages leave room in the margin of the IAE for extensive instructor support materials at point-of-use.

- **Answers to Exercises** Answers to all the exercises are given in the Instructor's Annotated Edition. Only answers to the odd exercises are given in the Answer Section in the student edition. Answers to writing exercises are not included in the student edition.
- **Answers to Writing Exercises, Focus on Problem Solving, and Projects & Group Activities** Suggested answers to all the writing exercises are given in the Instructor's Annotated Edition. Also included are answers to Focus on Problem Solving exercises and exercises in the Projects & Group Activities feature.
- **Digital Art and Tables** Next to many of the graphs and tables in the text, there is a  indicating that a Microsoft PowerPoint® slide of the figure is available. These slides (along with PowerPoint Viewer) may be downloaded from our web site. Alternatively, the slides may also be printed as transparency masters.
- **Suggested Assignment** At the beginning of every exercise set, there is a suggested homework assignment that covers the essential topics of the section.
- **Quick Quiz** For every objective, there is a short quiz that can be given to students. These quizzes are designed to check basic concepts. These quizzes can also be downloaded from our web site.
- **In-Class Examples** Extra examples, in addition to those found in the student text, accompany every objective to provide material to be used as reinforcement during the in-class presentation.
- **Instructor Notes** Instructor notes include teaching ideas, warnings about common student errors, and historical notes.
- **New Vocabulary** A list of new vocabulary introduced within a lesson is provided for appropriate objectives. There are similar lists for New Symbols, Formulas, Rules, Properties, Definitions, and Equations.
- **Vocabulary to Review** A list of vocabulary introduced in a previous objective that students will need to recall in order to understand the material in the present lesson is provided for appropriate objectives. There are similar lists for Symbols, Formulas, Rules, Properties, Definitions, and Equations.
- **Discuss the Concepts** These questions, or requests for an explanation, can be used for class discussion or for writing exercises. They require students to verbalize the basic concepts presented in the lesson.
- **Concept Check** These questions or exercises can be used after the presentation of a lesson to test student understanding of the concepts developed.
- **Optional Student Activity** These exercises can be assigned at the conclusion of the lesson. They can serve as a class activity, individual work, or as cooperative learning projects. Most of the activities are designed to take most students approximately five minutes to complete.

AIM for Success: Getting Started

Motivate Yourself

Welcome to ***Algebra: Beginning and Intermediate***! Students come to this course with varied backgrounds and different experiences learning math. We are committed to your success in learning algebra and have developed many tools and resources to support you along the way. Want to excel in this course? Read on to learn the skills you'll need and how best to use this book to get the results you want.

You'll find many real-life problems in this book, relating to sports, money, cars, music, and more. We hope that these topics will help you understand how you will use algebra in your real life. However, to learn all of the necessary skills, and how you can apply them to your life outside this course, you need to stay motivated.

Take Note

Motivation alone won't lead to success. For example, suppose a person who cannot swim is rowed out to the middle of a lake and thrown overboard. That person has a lot of motivation to swim but will most likely drown without some help. You'll need motivation **and** learning in order to succeed.

Think About Why You Want to Succeed in this Course. List the Reasons Here (Not in Your Head . . . On the Paper!)

We also know that this course may be a requirement for you to graduate or complete your major. That's OK. If you have a goal for the future, such as becoming a nurse or a teacher, you will need to succeed in algebra first. Picture yourself where you want to be, and use this image to stay on track.

Make the Commitment

Stay committed to success! With practice, you will improve your algebra skills. Skeptical? Think about when you first learned to ride a bike or drive a car. You probably felt self-conscious and worried that you might fail. But with time and practice, it became second nature to you.

You will also need to put in the time and practice to do well in algebra. Think of us as your "driving" instructors. We'll lead you along the path to success, but we need you to stay focused and energized along the way.

List a Situation in Which You Accomplished Your Goal by Spending Time Practicing and Perfecting Your Skills (Such as Learning to Play the Piano, Playing Basketball, etc.):

If you spend time learning and practicing the skills in this book, you will also succeed in this course.

Think You Can't Do Math? Think Again! Skills for Success

You can do math! When you first learned the skills you just listed, you may have not done them well. With practice, you got better. With practice, you will be better at math. Stay focused, motivated, and committed to success.

We can't emphasize enough how important it is to overcome the "I Can't Do Math Syndrome." If you listen to interviews of very successful athletes after a particularly bad performance, you will note that they focus on the positive aspect of what they did, not the negative. Sports psychologists encourage athletes to always be positive—to have a "Can Do" attitude. Develop this attitude toward math and you will succeed.

Get the Big Picture If this were an English class, we wouldn't encourage you to look ahead in the book. But this is algebra—go right ahead! Take a few minutes to read the table of contents. Then, look through the entire book. Move quickly: scan titles, look at pictures, notice diagrams.

Getting this big picture view will help you see where this course is going. To reach your goal, it's important to get an idea of the steps you will need to take along the way.

As you look through the book, find topics that interest you. What's your preference? Horse racing? Sailing? TV? Amusement parks? Find the Index of Applications in the back and pull out three subjects that interest you. Then, flip to the pages in the book where the topics are featured, and read the exercises or problems where they appear. Write these topics here:

Write the Topic Here	Write the Corresponding Exercise/Problem Here

You'll find it's easier to work at learning the material if you are interested in how it can be used in your everyday life.

Use the following activities to think about more ways you might use algebra in your daily life. Flip open your book to the exercises below to answer the questions.

✔ (see p. 70, #112) I am moving across the country and want to maintain my current cost of living, but I'm unsure of what my salary will be. I need algebra to . . .

✔ (see p. 244, #112) I've accumulated a credit card debt of $1000. I need algebra to . . .