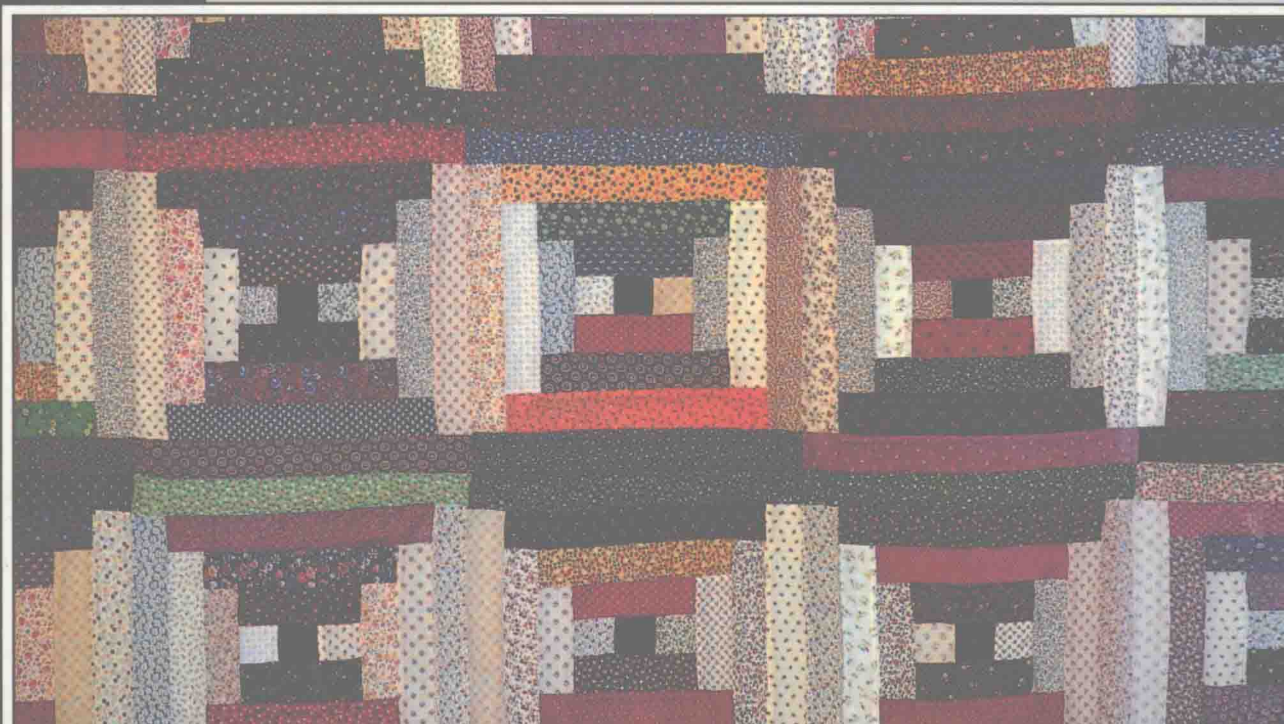
INTRODUCTORY ALGEBRA FOR COLLEGE STUDENTS



ROBERT BLITZER

Introductory Algebra for College Students

Robert Blitzer

Miami-Dade Community College

PRENTICE HALL, Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging-in-Publication Data

Blitzer, Robert.
Introductory algebra for college students / Robert F. Blitzer.
p. cm.
Includes index.
ISBN 0-02-310851-7
1. Algebra. I. Title.
QA152.2.B586 1995 94-37465
512.9—dc20 CIP

Acquisitions Editor: Melissa S. Acuña
Director of Production and Manufacturing: David W. Riccardi
Production Editor: Phyllis Niklas
Marketing Manager: Karie Jabe
Interior Designer: HRS Electronic Text Management
Cover Designer: Bruce Kenselaar
Cover Photo: STG International
Creative Director: Paula Maylahn
Art Director: Amy Rosen
Manufacturing Manager: Trudy Pisciotti
Photo Researcher: Clare Maxwell
Supplements Editor: Audra J. Walsh
Editorial Assistant: April Thrower

Printed in the United States of America

10 9 8 7 6 5 4 3

ISBN 0-02-310851-7

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

Dedicated to the memory of my father,

Richard Blitzer,

a master teacher, whose intelligence and humor in the classroom enriched the lives of his students and shaped the professional direction of my life.

Preface

Introductory Algebra for College Students provides comprehensive, in-depth coverage of the topics required in a one-term course in beginning or introductory algebra. The book is written for college students who have had no previous experience in algebra and for those students who need a review of basic algebraic concepts.

The book has three fundamental goals: First, to help students acquire a solid foundation in the basic skills of algebra; second, to enable students to develop problem-solving skills, fostering critical thinking, within a varied and interesting setting; and third, to show students how algebra can be used to solve real-life problems in numerous disciplines.

Unlike other books at this level, algebra is integrated into the whole spectrum of learning through a vast collection of historical references, multidisciplinary applications, enrichment essays, critical thinking exercises, and unique word problems. Using an interdisciplinary approach that draws from the history of mathematics, algebra is viewed as both interesting and relevant.

Key Pedagogical Features

The features of this book are designed to support its three fundamental goals.

Detailed Step-by-Step Explanations Students learn a great deal of algebra by studying examples and working problems. This book makes algebraic skills accessible by providing numerous illustrative examples that are presented one step at a time. No steps are omitted, and each step is clearly explained. Detailed explanations appear to the right of each mathematical step, enabling students to study the examples and follow the mathematics.

Example Titles All illustrative examples have titles so that students immediately see the purpose of each example.

Unique Chapter and Section Introductions Each chapter begins with an interdisciplinary introduction to stimulate student interest. Chapter introductions cover such diverse topics as Fibonacci numbers in nature's designs, uncertainty in mathematics, TOE (the theory of everything), and Nelson Cole's show-stopping performance at the 1903 meeting of the American Mathematical Society. Furthermore, each section opens with an introduction leading to a statement of specific objectives that prepare students for the material covered in the section.

Enrichment Essays Interspersed throughout the book are enrichment essays—some pictorial—that germinate from ideas appearing in expository sections. Providing historical information and interdisciplinary connections, topics of some of the enrichment essays include fractals, relativistic time, friendly numbers, paradoxes, palindromes, Fermat's Last Theorem, the aging male voice, inequalities and aerobic exercise, and the use of art to convey ideas expressed by algebraic formulas. Full-color photographs, including works by Seurat, Mondrian, Dali, Magritte, Escher, Melvin Prueitt, and the Chudnovsky brothers, add visual impact to many of the essays.

Relevant Word Problems Thousands of applied problems from every conceivable discipline show students the most up-to-date real-life applications of algebra. An index of applications is included inside the front cover. This is intended to enable students to locate the vast array of realistic, relevant, and unique applications of algebra discussed throughout the book.

Emphasis on Problem Solving The emphasis of the book is on learning to use the language of algebra as a tool for solving relevant, real-world problems. Translating from English phrases to algebraic expressions is taught throughout Chapter 1 and is utilized in Chapter 2, where a substantial introduction to problem solving is presented. Chapter 3, devoted entirely to problem solving, has been structured with flexibility in mind. Other than covering the section on problem solving in geometry, instructors can select whatever other sections of the chapter they find appropriate. (Omitting all of Chapter 3 other than the discussion of Euclidean geometry will not affect the continuity of topics in subsequent chapters.) Word problems and strategies for solving them are explained and developed throughout the remainder of the book.

Extensive Application to Geometric Problem Solving Many students enter a beginning algebra course with little or no knowledge of Euclidean geometry. Consequently, this book teaches (rather than reviews) geometric concepts that are important to a student's understanding of algebra. Section 3.3 covers measuring angles, parallel lines, alternate interior angles, sum of the measures of the interior angles of a triangle, perimeter, area, and volume. Literacy in geometry is developed throughout the remainder of the book by the use of extensive applications involving geometric problem solving.

Writing in Mathematics Each problem set contains writing exercises intended to help students communicate their mathematical knowledge by thinking and writing about algebraic topics.

Critical Thinking Analytic skills that go beyond the routine application of basic algebraic concepts are developed in a separate section (Section 3.1) devoted to critical thinking. Critical thinking exercises requiring students to recognize patterns, use logical reasoning, think visually, engage in organized counting, and approach problems creatively appear in many of the problem sets.

Early Graphing and Functions Graphing is introduced in Chapter 4 and applied throughout the remainder of the book to illustrate the solutions to problems. Chapter 4 also contains an introduction to some basic ideas about functions. Material on functions is developed and expanded in subsequent chapters.

Vast Collection of Problems A problem set is included at the end of each section. This book contains nearly 7000 problems that are intended to provide drill, provoke critical thinking, develop and enhance problem-solving skills, motivate, amuse, and illustrate how algebra is used in numerous disciplines. The *Instructor's Manual with Test Bank* contains a list of basic suggested assignments for each problem set. The flexibility of the problem sets permits instructors to add to the suggested core assignments those problems that they find useful and appropriate.

Review Problems Every exercise set concludes with three review problems. A comprehensive collection of chapter review problems is included at the end of each chapter. In addition, Chapters 3–9 conclude with cumulative review problems, helping students to continually review topics covered in previous chapters. Cumulative review problems do not appear at the end of the final chapter, but are included in a separate Appendix B, which contains a complete set of problems covering all important concepts discussed in the entire book.

Calculator Applications Calculator applications for both scientific and graphing calculators are provided at appropriate places throughout the book. Calculator problems appear in most of the exercise sets. Although the book recognizes and encourages the use of calculators, coverage of this material is optional.

Screened Boxes Extensive use is made of screened boxes to highlight important definitions, formulas, rules, and procedures. Procedure boxes clearly state step-by-step summaries of the processes to be used for solving problems.

Chapter Summaries Inclusive summaries appear at the conclusion of each chapter, helping students to bring together what they have learned after reading the chapter.

Four-Color Format A four-color system is used to highlight the pedagogical features of this book. Full-color photographs in the enrichment essays provide students with visually interesting material not found in comparable algebra books.

Supplements

The supplement package includes an instructor's manual, testing materials, solutions manuals, videotapes, and tutorial software.

Instructor's Manual This manual contains lecture notes for each section in the book, and a list of suggested minimum assignments for each problem set.

TestPro (IBM and Macintosh) This versatile testing system allows the instructor to easily create up to 99 versions of a customized test. Users may add their own test items and edit existing items in WYSIWYG format. Each objective in the text has at least one multiple-choice and free-response algorithm. Free upon adoption.

Instructor's Solutions Manual This manual contains worked-out solutions to all the problems in the book.

Student's Solutions Manual This manual provides students with detailed solutions to selected odd-numbered problems, all review problems in each exercise set, all review and cumulative review problems at the end of each chapter, and all cumulative review problems in Appendix B.

Videotapes A set of videotapes gives students a chance to review key topics from each chapter in the book.

Computer-Assisted Tutorials The tutorials offer self-paced, interactive review in IBM and Macintosh formats.

Acknowledgments

I wish to express my appreciation to all the reviewers for their helpful criticisms and suggestions. In particular, I want to thank: Howard Anderson, Skagit Valley College; John Anderson, Illinois Valley Community College; Michael H. Andreoli, Miami-Dade Community College—North Campus; Warren Jay Burch, Brevard Community College; Alice Burstein, Middlesex Community College; Sandra Pryor Clarkson, Hunter College; Robert A. Davies, Cuyahoga Community College; Ben Divers, Jr., Ferrum College; Irene Doo, Austin Community College; Charles C. Edgar, Onondaga Community College; Susan Forman, Bronx Community College; Jay Graening, University of Arkansas; Robert B. Hafer, Brevard Community College; Donald Herrick, Northern Illinois University; Beth Hooper, Golden West College; Tracy Hoy, College of Lake County; Gary Knippenberg, Lansing Community College; Mary Koehler, Cuyahoga Community College; Hank Martel, Broward Community College; John Robert Martin, Tarrant County Junior College; Irwin Metviner, State University of New York at Old Westbury; Allen R. Newhart, Parkersburg Community College; Gayle Smith, Lane Community College; Dick Spangler, Tacoma Community College; Janette Summers, University of Arkansas; Robert Thornton, Loyola University; Lucy C. Thrower, Francis Marion College; and Andrew Walker, North Seattle Community College.

Additional acknowledgments are extended to Richard Orellana and Sarina Abromowitz, for deciphering my handwriting while typing the manuscript; Gloria Langer, for preparing the solutions manuals; Bob Martin, for double-checking the answers to the problems; Donna Gerken, for working with me in proofreading the galleys; Clare Maxwell, for playing detective and pursuing the photographs across the globe; Chris Migdol, for making sure that Clare's work culminated in the stunning photographs that grace the book; Carl Brown, whose superb illustrations provide visual support to the verbal portions of the text; Betty Berenson and Kathy Lee, whose meticulous work as copy editor and proofreader, respectively, put me at my syntactical best; and, especially, Phyllis Niklas, whose talents as supervisor of production resulted in the book's wonderful look.

Most of all I wish to thank my editors, Melissa Acuña and Robert Pirtle. They guided this project from its initial inception to its finished form. The quality of the book is a testimony to their diligence, expertise, and talent.

Few books ever spring directly from their author's head into a typesetter's machine without requiring a great deal of help from a variety of experts. It is doubtful that any book involving hundreds of interdisciplinary applications such as this could be so presumptuous. My research has enabled me to synthesize a wealth of interesting and unique ideas. I extend my heartfelt thanks to the artists, authors, mathematicians, poets, and museum personnel who gave me permission to share their ideas, their art, and, ultimately, their humanity within the pages of this book.

To the Student

The process of learning mathematics requires that you do at least three things—read the book, work problems, and get your questions answered if you are stuck. This book has been written so that you can learn directly from its pages. All concepts are carefully explained, important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear throughout. A great deal of attention has been given to show you the vast and unusual applications of algebra in order to make your learning experience both interesting and relevant. As you begin your studies, I would like to offer some specific suggestions for using this book and for being successful in algebra.

- *Read the book.*
  - **a.** Begin with the chapter introduction. This will present a particular perspective from which you can view the entire chapter, making your work in the chapter more meaningful and enjoyable.
  - **b.** Move on to the introduction to a particular section. This will tell you what will be covered, why this information is significant, and exactly what you should be able to do once you have completed the section.
  - **c.** At a slower and more deliberate pace, read the section. Move through the illustrative examples with great care. These worked-out examples provide a model for doing the problems in the exercise sets.
  - **d.** Read the enrichment essays that appear throughout the section. Although omission of these essays will not interfere with learning the subject matter, they should help make the course more interesting, provide a temporary break from the more formal mathematics, and enhance your appreciation of algebra.

  As you proceed through the reading, do not give up if you do not understand every single word. Things will become clearer as you read on and see how various procedures are applied to specific illustrative examples.

- *Work problems every day, and check your answers.* The way to learn mathematics is by *doing* mathematics, which means by *solving problems.* The more problems you work, the better you will become at solving problems which, in turn, will make you a better algebra student.

  - **a.** Work the assigned problems in each problem set. The problems in the exercise sets move from fairly routine to more difficult questions that are intended to stimulate your ability to think and reason. It is often better to think about a particular question, even if confused and somewhat frustrated, than to immediately have someone else show you how to work the problem. The answers to most of the odd-numbered problems are given in the back of the book. Once you have completed a problem, be sure to check your answer. If you made an error, find out what it was. Getting help is a good idea once you have attempted the work on your own.

b. Work the review problems at the end of each problem set. By continuously reviewing, you will remember the material you learned for a much longer period of time.

- *After completing a chapter, study the chapter summary and work all the chapter review problems.* Unlike the problems that appear in the exercise sets throughout the chapter, you should be able to do *all* the chapter review problems (answers are given in the back of the book). This is a good way to test yourself on whether you truly understand the objectives of the chapter.

- *After completing the book, work the review problems that appear in the cumulative review in Appendix B.* This is an effective way to bring together the procedures and problem-solving skills learned throughout the course. By doing this, you should be ready for a cumulative final examination and to pursue more advanced mathematics courses. (Your instructor will let you know which problems you should omit if your course did not include every section in the book.)

- *Attend all lectures.* No book is intended to be a substitute for the valuable insights and interactions that occur in the classroom. In addition to arriving for a lecture on time and prepared, you might find it helpful to read the section that will be covered in class beforehand so that you have a clear idea of the new material that will be discussed.

Algebra is often viewed as the foundation for more advanced mathematics. It is my hope that this book will make algebra accessible, relevant, and an interesting body of knowledge in and of itself.

Introductory Algebra for College Students

Contents

1 The Real Number System

The English language contains all the abstraction and complexity of the compact symbolic language used in algebra. Consider, for example, the following sentence: "A woman once gave her husband something so offensive that he tossed it into a trash can and refused to talk about it again, although she occasionally asks him where it is." If we use letters to represent the unknowns, the sentence becomes: "x gave x's husband y a z, and y found z so offensive that y tossed z into a trash can and refused to talk about z to x, although x occasionally asks y where z is." This second form of the sentence replaces the variable pronouns with letters, but also contains English words.

Until the 13th century, all algebraic formulas and equations were written out in verbal sentences. Later, abbreviations replaced some of the words. Algebra's current notation, originating near the beginning of the 17th century with the work of the French mathematician René Descartes, utilizes compact symbols without the use of words. In this notation, variables such as x represent any number, with every number condensed into one all-embracing symbol. Using modern notation, mathematicians write x^n, where n is an exponent, a symbol attached to a symbol that condenses all repeated multiplications for all numbers into one compact space.

The symbolic notation used in algebra is the result of a slow and tedious evolution of ideas and symbols. It wasn't until the late 16th century that the French mathematician François Viète came upon the idea of using letters such as x, y, and z to represent numbers in much the same way that English pronouns represent nouns.

The problem that many people experience with algebra is not that it is abstract, but rather that its xs, ys, and zs seem too often ungrounded and meaningless. The real issue is to discover what lies beyond the symbolic notation and how the notation reflects a meaningful description of the world in which we live.

The use of algebra to solve problems and describe our world in a significant way is the primary focus of this book. In our first chapter, we look at the real numbers, establishing procedures for adding, subtracting, multiplying, and dividing positive and negative numbers. This will lead to the use of compact symbolic formulas that describe phenomena as diverse as perceptions of being underpaid at work, bowlers' handicaps, life expectancy, blood pressure, learning and memory, and the cost of cleaning up chemical contamination. An understanding of the basic ideas underlying the real numbers will enable us to use algebra's unique symbolic language not only to describe the world, but also to solve some of its problems.

SECTION 1.1 THE REAL NUMBERS

Numbers and their properties have intrigued humankind since the beginning of civilization. Pythagoras discovered that harmony in music was the result of ratios of whole numbers, and he developed a philosophy of the universe based on these ratios. There is a harmony in nature, he said, and it has a language—numbers are the language of nature.

The transition from counting specific things to the abstract concept of a number represents a long, slow cultural evolution. As mathematician and philosopher Bertrand Russell wrote, "It must have required many ages to discover that a brace of pheasants and a couple of days were both instances of the number 2."

We begin our study of algebra by looking at numbers, in particular the *real numbers.* After completing this section, you should be able to:

1. Use the roster method to write sets of numbers expressed in set-builder notation.
2. Use the symbols $\in$ and $\notin$.
3. Define the sets that make up the real numbers.
4. Classify given numbers as belonging to one or more of the sets that make up the real numbers.
5. Understand and use inequality symbols.
6. Find the opposite (additive inverse) and the absolute value of a given real number.

Set Notation

In this section, we consider the sets that make up the real numbers. The term *set* appears extensively in mathematics.

> A *set* is a collection of objects. The objects in a set are the *elements* or *members* of the set. A set is *well defined* if it is possible to determine whether or not a given element belongs to it.

The *roster method* of writing a set encloses the elements of the set in braces, { }. For example, the set of numbers used for counting that are less than 6 is written {1, 2, 3, 4, 5}. This set has a limited number of elements and is an example of a *finite* set.

To express the fact that 4 is an element of the set {1, 2, 3, 4, 5}, we use the symbol $\in$.

> The symbol $\in$ means *is an element of.*

EXAMPLE 1 *Set Notation and the Symbol* $\in$

Given the set {1, 2, 3, 4, 5}, each statement listed below is true.

$2 \in \{1, 2, 3, 4, 5\}$	2 is an element of the set.
$4 \in \{1, 2, 3, 4, 5\}$	4 is an element of the set.
$7 \notin \{1, 2, 3, 4, 5\}$	7 is not an element of the set.

In algebra, letters, called *variables,* are used to represent numbers. Variables are used to express sets in *set-builder notation.* The set {1, 2, 3, 4, 5} can be written using this notation as

$\{x|x$ is a counting number between 1 and 5 inclusively$\}$

which is read "the set of all elements x such that x is a counting number between 1 and 5 inclusively." (The word *inclusively* includes both 1 and 5 as elements of the set.)

EXAMPLE 2 *Representing Sets Using Two Notations*

Table 1.1 represents sets in both set-builder and roster notations. The sets in each row are *equal* because they contain the *same elements.*

TABLE 1.1 SETS IN SET-BUILDER AND ROSTER NOTATIONS

Set-Builder Notation	Roster Method
$\{x\|x$ is an even number between 2, inclusively, and 10, exclusively$\}$	{2, 4, 6, 8}
$\{x\|x$ is a counting number less than 8$\}$	{1, 2, 3, 4, 5, 6, 7}
$\{x\|x$ is a counting number greater than 8$\}$	{9, 10, 11, 12, 13, . . .}

Observe that the last set in Table 1.1 contains an unlimited number of elements and is an example of an *infinite set.* The three dots indicate that the pattern continues; the dots are read "and so on." There are infinitely many counting numbers that are greater than 8, so the set {9, 10, 11, 12, 13, . . .} also contains the numbers 14, 15, 16, and so on. ■

The Set of Real Numbers

We are now in a position to define the various kinds of sets that make up the set of real numbers.

When a child learns to talk, the names of the first few counting numbers are almost as essential to an emerging vocabulary as *mommy, dog,* and *bird.* Counting is followed by words for numbers, which, in turn, are followed by symbolic notation for numbers. It should come as no surprise that the first kinds of numbers children are introduced to are the *natural numbers.* They can find "models" in external reality for these abstractions—one *thing,* two *things,* three *things,* and so on. Our primeval ancestors must have had a parallel experience.

The Natural Numbers

The set of *natural numbers* or *counting numbers* is represented by the infinite set {1, 2, 3, 4, 5, . . . }.

Our early mathematical experience with subtraction imposed a very strange idea on us. Many of us can remember our first-grade teacher showing us three things, then removing them and saying, "Now what do you see? Nothing." But can we,