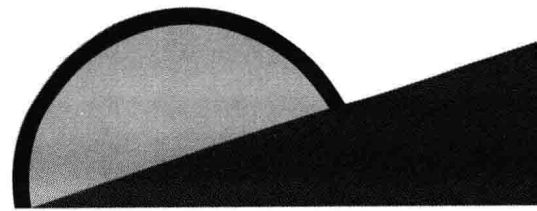


LIAL • HORNSBY • SCHNEIDER

SIXTH EDITION

# TRIGONOMETRY



**Sixth Edition**

# Trigonometry

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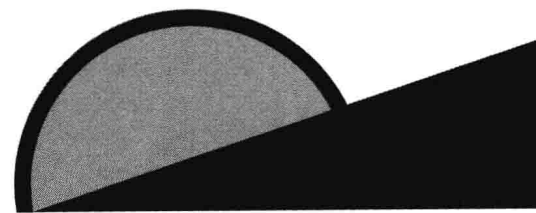
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# Preface

*Trigonometry, Sixth Edition*, is written for students in a traditional college trigonometry course. We assume students have had at least one course in algebra. Geometry is a desirable prerequisite, but many students reach trigonometry with little or no background in geometry. Because of this, we explain the necessary ideas from geometry as needed. Although this book is intended for a traditional course, we have acknowledged the growing interest in using graphing calculators to augment and deepen the concepts typically presented in trigonometry.

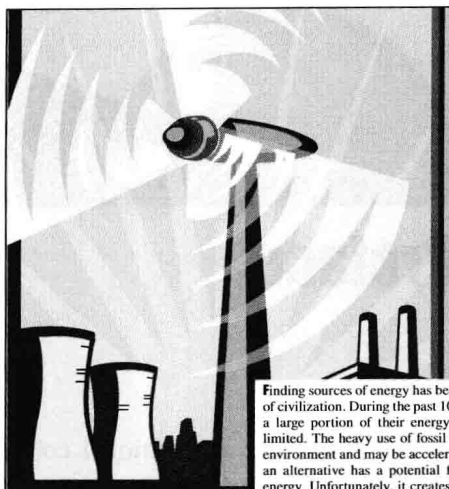
## Changes in Content ▼▼▼

We have given less emphasis to showing specific calculator keystrokes, to allow for more variation in calculators. We assume that all students will be using at least a scientific calculator, and that many will use graphing calculators.

## New Features ▼▼▼

Several new features have been incorporated in this edition. The design has been developed to enhance the pedagogical features and increase their accessibility.

- Each chapter opens with a genuine application of the material to be presented. Corresponding examples and exercises, identified with a special icon, are located throughout the chapter.
- We have made an effort to point out the many connections between mathematical topics in this course and those studied earlier, as well as connections between mathematics and the “real world.” Optional Connections boxes presenting such topics are included in many sections throughout the book. Most of them include thought-provoking questions for writing or class discussion. A few topics, such as the sum and product identities, are now included in a Connections box, rather than in a complete section. In addition, we have included a feature in many exercise sets called Discovering Connections. These groups of exercises tie together different topics and highlight the relationships among various concepts and skills.



3

## Radian Measure and the Circular Functions

Finding sources of energy has been an important concern since the beginning of civilization. During the past 100 years, people have relied on fossil fuels for a large portion of their energy requirements. Fossil fuels are finite and limited. The heavy use of fossil fuels has caused irreversible damage to our environment and may be accelerating a greenhouse effect. Nuclear energy as an alternative has a potential for providing almost unlimited amounts of energy. Unfortunately, it creates health risks and dangerous nuclear wastes. Currently there is no completely safe disposal method for nuclear wastes. As a result, no new nuclear power plants have been ordered in the United States since 1978.

Over the past twenty-five years the production of solar energy has evolved from a mere kilowatt of electricity to hundreds of megawatts. Solar energy has many advantages over traditional energy sources in that it does not pollute and has the potential of being an unlimited, cheap source of energy. Its use and production is not limited to a small number of countries but is readily available throughout the United States and the world. The North American Southwest has some of the brightest sunlight in the world with a potential to provide up to 2500 kilowatt-hours per square meter.

In the design of solar power plants, engineers need to position solar panels perpendicular to the sun's rays so that maximum energy can be collected. Understanding the movement and position of the sun at any time and

Source: Winter, C., R. Sizmann, and Vant-Hunt (Editors), *Solar Power Plants*, Springer-Verlag, 1991.

- 3.1 Radian Measure
- 3.2 Applications of Radian Measure
- 3.3 Circular Functions of Real Numbers
- 3.4 Linear and Angular Velocity

**Chapter Openers** present a genuine application of the material to be presented.

### 3.3 CIRCULAR FUNCTIONS OF REAL NUMBERS 127

Find the value of  $s$  in the interval  $[0, \pi/2]$  that has  $\cos s = .96854556$ .

The value of  $s$  can be found with a calculator set for radian mode. Recall from Section 2.3 how we found an angle measure given a trigonometric function value of the angle. The same procedure is repeated here with the calculator set to radian mode to find that

$$\cos .25147856 = .96854556,$$

and  $0 < .25147856 < \pi/2$ , so  $s = .25147856$ .

Find the exact value of  $s$  in the interval  $[0, \pi/2]$  for which  $\sin s = \sqrt{2}/2$ .

Sketch a triangle in quadrant I and use the definition of  $\sin s$  to label the sides as shown in Figure 11. To relate it to the definition of the trigonometric function  $\sin \theta$ , multiply the lengths of each side by 2. We recognize this as a right triangle with the two acute angles of  $45^\circ$ . To find  $s$ , convert  $45^\circ$  to radians, to get  $s = \pi/4$ .

**Titled Examples** include detailed, step-by-step solutions and descriptive side comments. Examples relating to the Chapter Openers are marked with a special symbol.

#### EXAMPLE 4 Finding the angle of elevation of the sun



Knowing the position of the sun in the sky is essential for solar-power plants. Solar panels need to be positioned perpendicular to the sun's rays for maximum efficiency. The angle of elevation  $\theta$  of the sun in the sky at any latitude  $L$  can be calculated using the formula

$$\sin \theta = \cos D \cos L \cos \omega + \sin D \sin L$$

where  $\theta = 0$  corresponds to sunrise and  $\theta = \pi/2$  occurs if the sun is directly overhead.  $\omega$  is the number of radians that the Earth has rotated through since noon when  $\omega = 0$ .  $D$  is the declination of the sun which varies because the Earth is tilted on its axis. (Source: Winter, C., R. Sizmann, and Vant-Hunt (Editors), *Solar Power Plants*, Springer-Verlag, 1991.)

Sacramento, California, has a latitude of  $L = 38.5^\circ$  or .6720 radians. Find the angle of elevation  $\theta$  of the sun at 3 P.M. on February 29, 2000, where at that time,  $D = -.1425$  and  $\omega = .7854$ .

Use the formula for  $\sin \theta$ .

$$\begin{aligned} \sin \theta &= \cos D \cos L \cos \omega + \sin D \sin L \\ &= \cos(-.1425) \cos(.6720) \cos(.7854) + \sin(-.1425) \sin(.6720) \\ &\approx .4593 \end{aligned}$$

Thus,  $\theta \approx .4773$  radians or  $27.3^\circ$ .

**Boxes** highlight words, definitions, rules, and procedures.

**Connections Boxes** point out the many connections between mathematics and the “real world” or other mathematical concepts.

### Graphing General Sine and Cosine Functions

To graph the general function  $y = c + a \sin b(x - d)$  or  $y = c + a \cos b(x - d)$ , where  $b > 0$ , follow these steps.

1. Find an interval whose length is one period ( $2\pi/b$ ) by solving the compound inequality

$$0 \leq b(x - d) \leq 2\pi.$$

2. Divide the interval into four equal parts.
3. Evaluate the function for each of the five  $x$ -values resulting from Step 2. The points will be maximum points, minimum points, and points that intersect the line  $y = c$  (“middle” points of the wave).
4. Plot the points found in Step 3, and join them with a sinusoidal curve.
5. Draw the graph over additional periods, to the right and to the left, as needed.

The amplitude of the function is  $|a|$ . The vertical translation is  $c$  units up if  $c > 0$ ,  $|c|$  units down if  $c < 0$ . The horizontal translation (phase shift) is  $d$  units to the right if  $d > 0$ , and  $|d|$  units to the left if  $d < 0$ .

**CONNECTIONS** You have probably noticed that the graphs of  $\sin x$  and  $\cos x$  are the same shape and each is a horizontal translation (or phase shift) of the other. Because of this we can rewrite any cosine function as a sine function or any sine function as a cosine function. The table of values below was produced by a graphing calculator. It shows that the corresponding  $y$ -values for  $Y_1 = 2 \sin 2x$  and  $Y_2 = 2 \cos 2(x - \pi/4)$  are the same in the intervals shown.

X	Y <sub>1</sub>	Y <sub>2</sub>
0	0	0
.5	1.6829	1.6829
1	1.8186	1.8186
1.5	2.0224	2.0224
2	1.914	1.914
2.5	1.518	1.518
3	-.5588	-.5588

Y<sub>1</sub>: 2sin 2X

X	Y <sub>1</sub>	Y <sub>2</sub>
2.5	1.314	1.314
3	1.9787	1.9787
3.5	.82424	.82424
4	-1.088	-1.088
4.5	2	2
5	-1.073	-1.073
5.5	.84033	.84033

Y<sub>2</sub>: 2cos 2(X-π/4)

### DISCUSSION OR WRITING

Graph these two functions and compare their graphs. What do you expect to find?

Find a sine function that is equivalent to  $y = .5 \cos(x - \pi)$ .

(c)  $\csc \frac{10\pi}{3}$

Because  $10\pi/3$  is not between 0 and  $2\pi$ , we must first find a number between 0 and  $2\pi$  with which it is coterminal. To do this subtract  $2\pi$  as many times as needed. Here,  $2\pi$  must be subtracted only once.

$$\frac{10\pi}{3} - 2\pi = \frac{10\pi}{3} - \frac{6\pi}{3} = \frac{4\pi}{3}$$

$$\csc \frac{10\pi}{3} = \csc \frac{4\pi}{3} = \csc 240^\circ = -\frac{2\sqrt{3}}{3}$$

**NOTE** The values found in Example 1 can also be determined without converting to degrees by using reference angles measured in radians.

In order to use a calculator to find an approximation of a circular function of a real number, we must first set the calculator to *radian* mode. The next example shows how to find such approximations.

### EXAMPLE 2 Finding approximate circular function values

Use a calculator to find an approximation for each of the following circular function values.

(a)  $\cos .5149$

$$\cos .5149 \approx .87034197$$

(b)  $\cot 1.3209$

Because calculators do not have keys for cotangent, secant, and cosecant, to find these values we must use the appropriate reciprocal function. To find  $\cot 1.3209$ , we first find  $\tan 1.3209$  and then find the reciprocal.


$$\cot 1.3209 = \frac{1}{\tan 1.3209} \approx .25523149$$

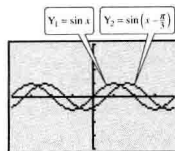
(c)  $\sec(-2.9234)$

$$\sec(-2.9234) = \frac{1}{\cos(-2.9234)} \approx -1.0242855$$

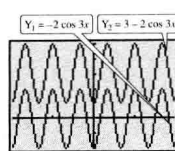
**CAUTION** One of the most common errors in trigonometry involves using calculators in degree mode when radian mode should be used. Remember that if you are finding a circular function value of a real number, the calculator *must* be in radian mode.

**Notes and Cautions** highlight common student errors and address concepts that students often find difficult or confusing.

 The graphing calculator also helps to reinforce the concepts of horizontal and vertical translations as seen in Figures 14 and 18 earlier. Again, let  $Y_1$  remain as  $\sin x$ , and enter  $Y_2$  as  $\sin(x - \pi/3)$ , taking care to insert parentheses as necessary. The graph of  $Y_2$  should be the same as the graph of  $Y_1$  shifted  $\pi/3$  units to the right. See the figure on the left.



Trig Window

[-2 $\pi$ , 2 $\pi$ ] by [-2, 5]  
Xscl =  $\frac{\pi}{3}$  Yscl = 1

To see the effect of a vertical translation, enter  $Y_1 = -2 \cos 3x$  and  $Y_2 = 3 - 2 \cos 3x$ . The constant  $c = 3$  in  $Y_2$  will have the effect of shifting the graph of  $Y_1$  three units upward. Adjust the range so that the minimum and maximum values of  $y$  are  $-2$  and  $5$ , respectively. Now graph these two functions and observe the results, shown in the figure on the right.

In the next example we graph a function that involves all the types of stretching, compressing, and shifting studied in the previous section and this one.

**EXAMPLE 5**  
Graphing  $y = c + a \sin b(x - d)$

Graph  $y = -1 + 2 \sin 4\left(x + \frac{\pi}{4}\right)$ .

Here, the amplitude is 2, the period is  $2\pi$  down 1 unit and  $\pi/4$  units to the left as cos. Since the graph is translated  $\pi/4$  units to the left, the first period will end at  $-\pi/4 + \pi$  will be  $2 - 1 = 1$  and the minimum  $y$ -values graph using the typical sine curve. See Figure 19.

Alternatively, start by finding an interval one cycle. To do this, use the argument

Many examples include optional graphing calculator coverage.

**EXAMPLE 6**  
Modeling temperature with a sine function



The maximum average monthly temperature in New Orleans is  $82^\circ\text{F}$  and the minimum is  $54^\circ\text{F}$ . The table shows the average monthly temperatures.

Month	Temperature
Jan	54
Feb	55
Mar	61
Apr	69
May	73
June	79
July	82
Aug	81
Sept	77
Oct	71
Nov	59
Dec	55

- (a) Using only the maximum and minimum temperatures, determine a function of the form  $f(x) = a \sin b(x - d) + c$ , where  $a$ ,  $b$ ,  $c$ , and  $d$  are constants, that models the average monthly temperature in New Orleans. Let  $x$  represent the month, with January corresponding to  $x = 1$ .

We can use the maximum and minimum average monthly temperatures to find the amplitude  $a$ .

$$a = \frac{82 - 54}{2} = 14$$

The average of the maximum and minimum temperatures is a good choice for  $c$ . The average is

$$\frac{82^\circ + 54^\circ}{2} = 68^\circ\text{F}.$$

Since the coldest month is January, when  $x = 1$ , and the hottest month is July, when  $x = 7$ , we should choose  $d$  to be about 4. The table shows that temperatures are actually a little warmer after July than before, so we try  $d = 4.2$ . Since temperatures repeat every 12 months,  $b$  is  $2\pi/12 = \pi/6$ . Thus,

$$f(x) = a \sin b(x - d) + c = 14 \sin \left[ \frac{\pi}{6}(x - 4.2) \right] + 68.$$

- (b) On the same coordinate axes, graph  $f$  for a two-year period together with the actual data values found in the table.

We show a graphing calculator graph of  $f$  together with the data points in Figure 20. The function models the data quite accurately.  $\blacksquare$

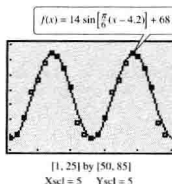


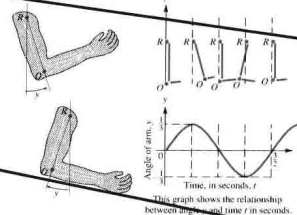
FIGURE 20

**Exercises** corresponding to the Chapter Openers are marked with a special symbol.

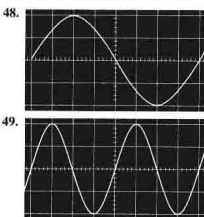
**Writing and Conceptual Exercises** are included to aid students in applying the concepts presented.

47. The figure shows schematic diagrams of a rhythmically moving arm. The upper arm  $RO$  rotates back and forth about the point  $R$ ; the position of the arm is measured by the angle  $y$  between the actual position and the downward vertical position. (Source: De Sapia, Rodolfo, *Calculus for the Life Sciences*. Copyright © 1978 by W. H. Freeman and Company. Reprinted by permission.)

- (a) Find an equation of the form  $y = a \sin kt$  for the graph shown.  
(b) How long does it take for a complete movement of the arm?



Pure sounds produce single sine waves on an oscilloscope. Find the amplitude and period of each sine wave in the following photographs. On the vertical scale, each square represents 5, and on the horizontal scale each square represents  $30^\circ$  or  $\pi/6$ .



50. The voltage  $E$  in an electrical circuit is given by  $E = 5 \cos 120\pi t$ , where  $t$  is time measured in seconds.

- (a) Find the amplitude and the period.  
(b) How many cycles are completed in one second? (The number of cycles (periods) completed in one second is the **frequency** of the function.)  
(c) Find  $E$  when  $t = 0, .03, .06, .09, .12$ .  
(d) Graph  $E$  for  $0 \leq t \leq 1/30$ .

51. For another electrical circuit, the voltage  $E$  is given by  $E = 3.8 \cos 40\pi t$ ,

- where  $t$  is time measured in seconds.  
(a) Find the amplitude and the period.  
(b) Find the frequency. See Exercise 50(b).  
(c) Find  $E$  when  $t = .02, .04, .08, .12, .14$ .  
(d) Graph one period of  $E$ .



52. At Mauna Loa, Hawaii, atmospheric carbon dioxide levels in parts per million (ppm) have been measured regularly since 1958. The function defined by

$$L(x) = .022x^2 + .55x + 316 + 3.5 \sin(2\pi x)$$

can be used to model these levels, where  $x$  is in years and  $x = 0$  corresponds to 1960. (Source: Nilsen, A., *Greenhouse Earth*, John Wiley & Sons, New York, 1992.)

- (a) Graph  $L$  for  $15 \leq x \leq 35$ . (Hint: Use  $325 \leq y \leq 365$ .)  
(b) When do the seasonal maximum and minimum carbon dioxide levels occur?  
(c)  $L$  is the sum of a quadratic function and a sine function. What is the significance of each of these functions? Discuss what physical phenomena may be responsible for each function.



53. Refer to the previous exercise. The carbon dioxide content in the atmosphere at Barrow, Alaska, in parts per million (ppm) can be modeled using the function defined by

$$C(x) = .04x^2 + .6x + 330 + 7.5 \sin(2\pi x),$$

where  $x = 0$  corresponds to 1970. (Source: Zeilik, M., S. Gregory, and E. Smith, *Introductory Astronomy and Astrophysics*, Saunders College Publishing, 1992.)

- (a) Graph  $C$  for  $5 \leq x \leq 25$ . (Hint: Use  $320 \leq y \leq 380$ .)  
(b) Discuss possible reasons why the amplitude of the oscillations in the graph of  $C$  are larger than the amplitude of the oscillations in the graph of  $L$  in Exercise 52, which models Hawaii.  
(c) Define a new  $C$  function that is valid if  $x$  represents the actual year where  $1970 \leq x \leq 1995$ .

## 4.2 TRANSLATING GRAPHS OF

55. The average temperature (in  $^\circ\text{F}$ ) in Austin, Texas, can be modeled using the trigonometric function

$$f(x) = 17.5 \sin \left[ \frac{\pi}{6}(x - 4) \right] + 67.5$$

where  $x$  is the month and  $x = 1$  corresponds to January. (Source: Miller, A., and J. Thompson, *Elements of Meteorology*, Charles E. Merrill Publishing Company, Columbus, Ohio, 1975.)

- (a) Graph  $f$  over the interval  $1 \leq x \leq 25$ . Determine the amplitude, period, phase shift, and vertical translation of  $f$ .  
(b) What is the average monthly temperature for the month of December?  
(c) Determine the maximum and minimum average monthly temperatures and the months when they occur.  
(d) What would be an approximation for the average yearly temperature in Austin? How is this related to the vertical translation of the sine function in the formula of  $f$ ?



56. The average monthly temperature (in  $^\circ\text{F}$ ) in Vancouver, Canada, is shown in the table. (Source: Miller, A., and J. Thompson, *Elements of Meteorology*, Charles E. Merrill Publishing Company, Columbus, Ohio, 1975.)

Month	Temperature
Jan	36
Feb	39
Mar	43
Apr	48
May	55
June	59
July	64
Aug	63
Sept	57
Oct	50
Nov	43
Dec	39

- (a) Plot the average monthly temperature over a two-year period by letting  $x = 1$  correspond to the month of January during the first year. Do the data seem to indicate a translated sine graph?

- (b) The big in July is Graph. What d peratur  
(c) Approx shift of data.  
(d) Determin a sin b constan  
(e) Graph dinat e given d



57. The average monthly temperature (in  $^\circ\text{F}$ ) in Phoenix, Arizona, is shown in the table. (Source: Miller, A., and J. Thompson, *Elements of Meteorology*, Charles E. Merrill Publishing Company, Columbus, Ohio, 1975.)

Month	Temperature
Jan	51
Feb	55
Mar	63
Apr	67
May	77
June	86
July	90
Aug	90
Sept	84
Oct	71
Nov	59
Dec	52

- (a) Predict the average yearly temperature and compare it to the actual value of  $70^\circ\text{F}$ .  
(b) Plot the average monthly temperature over a two-year period by letting  $x = 1$  correspond to January of the first year.  
(c) Determine a function of the form  $f(x) = a \cos b(x - d) + c$ , where  $a, b, c$ , and  $d$  are constants, that models the data.  
(d) Graph  $f$  together with the data on the same coordinate axes.


Many exercises and examples are based on **Real Data**, and many require reading graphs and charts.



- Graphing calculator comments and screens are given throughout the book as appropriate. These are identified with an icon, so that an instructor may choose whether or not to use them. We know that many students own graphing calculators and may need guidance for using them, even if they are not a required part of the course.
- Many examples and exercises are based on real data, and many require reading charts and graphs.
- The exercise sets have been completely rewritten and contain many new exercises, including more conceptual and writing exercises (which are marked by symbols in the instructor's edition), as well as optional graphing calculator exercises. Those exercises that require applying the topics in a section to ideas beyond the examples are marked as challenging in the instructor's edition.
- Cautions and notes are included to highlight common student errors and misconceptions. Some of these address concepts that students often find difficult or confusing.

## Supplements ▼▼▼

### For the Instructor

**Annotated Instructor's Edition** With this volume, instructors have immediate access to the answers to every exercise in the text, excluding proofs and writing exercises. In a special section at the end of the book, each answer is printed next to or below the corresponding text exercise. In addition, challenging exercises, which will require most students to stretch beyond the concepts discussed in the text, are marked with the symbol ▲. The conceptual (⊙) and writing (✍) exercises are also marked in this edition so instructors may assign these problems at their discretion. (Graphing calculator exercises will be marked by  in both the student's and instructor's editions.)

**Instructor's Resource Manual** Included here are four versions of a chapter test for each chapter; additional test items for each chapter; and two forms of a final examination. Answers to all tests and additional exercises also are provided. Answers to most of the textbook exercises are included as well.

**Instructor's Solution Manual** This manual includes complete, worked-out solutions to every even exercise in the textbook (excluding most writing exercises).

***Test Generator/Editor for Mathematics with QuizMaster*** is a computerized test generator that lets instructors select test questions by objective or section or use a ready-made test for each chapter. The software is algorithm driven so that regenerated number values maintain problem types and provide a large number of test items in both multiple-choice and open-response formats for one or more test forms. The **Editor** lets instructors modify existing questions or create their own including graphics and accurate math symbols. Tests created with the **Test Generator** can be used with **QuizMaster**, which records student scores as they take tests on a single computer or network, and prints reports for students, classes, or courses. CLAST and TASP versions of this package are also available. (IBM, DOS/Windows, and Macintosh)

### For the Student

***Student's Solution Manual*** Complete, worked-out solutions are given for odd-numbered exercises and chapter review exercises in a volume available for purchase by students. In addition, a practice chapter test, with answers, is provided for each chapter. All-new cumulative review exercises with worked-out solutions are also included.

***Videotapes*** A new videotape series has been developed to accompany *Trigonometry*, Sixth Edition. In a separate lesson for each section of the book, the series covers all objectives, topics, and problem-solving techniques within the text.

***Interactive Mathematics Tutorial Software with Management System*** is an innovative software package that is objective-based, self-paced, and algorithm driven to provide unlimited opportunity for review and practice. Tutorial lessons provide examples, progress-check questions, and access to an on-line glossary. Practice problems include hints for the first incorrect responses, solutions, textbook page references, and on-line tools to aid in computation and understanding. Quick Reviews for each section focus on major concepts. The optional **Management System** records student scores on disk and lets instructors print diagnostic reports for individual students or classes. Student versions, which include record-keeping and practice tests, may be purchased by students for home use.

## Acknowledgments ▼▼▼

We are grateful to the many users of the fifth edition and to our reviewers for their insightful comments and suggestions. It is because they take the time to write thoughtful reviews that our textbooks continue to meet the needs of students and their instructors.

### Reviewers

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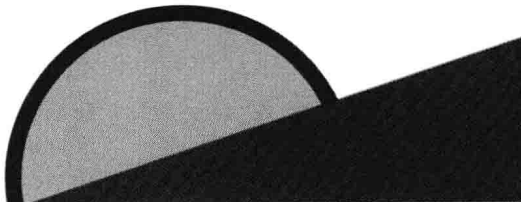
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# An Introduction to Scientific and Graphing Calculators

In the past, some of the most brilliant minds in mathematics and science spent long, laborious hours calculating values for logarithmic and trigonometric tables. These tables were essential to solve equations in real applications. Because they could not predict what values would be needed, the tables were sometimes incomplete. During the second half of the twentieth century, computers and sophisticated calculators appeared. These computing devices are able to evaluate mathematical expressions and generate tables in a fraction of a second. As a result, the study of mathematics is changing dramatically.

Although computers and calculators have made a profound difference, they have *not* replaced mathematical thought. Calculators cannot decide whether to add or subtract two numbers in order to solve a problem—only you can do that. Once you have made this decision, calculators can efficiently determine the solution to the problem. In addition, graphing calculators also provide important graphical and numerical support to the validity of a mathematical solution. They are capable of exposing errors in logic and pointing to patterns. These patterns can lead to conjectures and theorems about mathematics. The human mind is capable of mathematical insight and decision making, but is not particularly proficient at performing long arithmetic calculations. On the other hand, calculators are incapable of possessing mathematical insight, but are excellent at performing arithmetic and other routine computations. In this way, calculators complement the human mind.

If this is your first experience with a scientific or graphing calculator, the numerous keys and strange symbols that appear on the keyboard may be intimidating. Like any learning experience, take it a step at a time. You do not have to understand every key before you begin using your calculator. Some keys may not even be needed in this course. The following explanations and suggestions are intended to give you a brief overview of scientific and graphing calculators.

It is not intended to be complete or specific toward any particular type of calculator. You may find that some things are different on your calculator. *Remember, a calculator comes with an owner's manual.* This manual is essential in learning how to use your calculator.

## Scientific Calculators ▼▼▼

Two basic parts of any calculator are the keyboard and the display. The keyboard is used to input data—the display is used to output data. Without correct input, the displayed output is meaningless. Most scientific calculators do not display the entire arithmetic expression that is entered, but only display the most recent number inputted or outputted. If an arithmetic expression is entered incorrectly, it is not possible to edit it. The entire expression must be entered again.

### Order of Operation

The order in which expressions are entered into a calculator is essential to obtaining correct answers. Operations on a calculator can usually be divided into two basic types: unary and binary. Unary operations require that only one number be entered. Examples of unary operations are  $\sqrt{x}$ ,  $x^2$ ,  $x!$ ,  $\sqrt[3]{x}$ , and  $\log x$ . When entering a unary operation on a scientific calculator, the number is usually entered first, followed by the unary operation. For example, to find the square root of 4, press the key  $\boxed{4}$ , followed by the square root key. Binary operations require that two numbers are entered. Examples of binary operations are  $+$ ,  $-$ ,  $\times$ ,  $\div$ , and  $x^y$ . When evaluating a binary operation on a scientific calculator, the operation symbol is usually entered between the numbers. Thus, to add the two numbers 4 and 5, enter  $\boxed{4} \boxed{+} \boxed{5} \boxed{=}$ . However, on some calculators, such as those made by Hewlett Packard, it is necessary to use *Reverse Polish Notation* (RPN). In RPN the operation is entered last, after the operands. One advantage of RPN is that parentheses are usually not necessary.

Every calculator has a set of built-in precedence rules that can be found in the owner's manual. For example, suppose that the expression  $3 + 4 \times 2 =$  is entered, from left to right, into a scientific calculator. The output will usually be 11, and not 14. This is because multiplication is performed before addition in the absence of parentheses. Parentheses can always be used to override existing precedence rules. *When in doubt, use parentheses.* Try evaluating  $\frac{24}{4 - 2}$ . It should be entered as  $24 \div (4 - 2) =$  in order to obtain the correct answer of 12. This is because division has precedence over subtraction.

### Scientific Notation

Numbers that are either large or small in absolute value are often displayed using scientific notation. The numeric expression 2.46 E12 refers to the large number  $2.46 \times 10^{12}$ , while the expression 2.46 E-12 refers to the small positive number  $2.46 \times 10^{-12}$ . Try multiplying one billion times ten million. Observe the output on your calculator.

### Precision and Accuracy

Precision refers to the number of digits a calculator will display. When  $\frac{1}{3}$  is evaluated, a calculator may display 0.333333333. This answer is approximate. The displayed precision of most calculators is between 8 and 12 digits. Accuracy is different from precision. It refers to the number of correct digits that an answer contains, compared to the true value. If a scale is misread as 129.6 pounds, when the actual answer is 145.8 pounds, then the number 129.6 has four digits of precision, but only one digit of accuracy. Many times when using a calculator to solve a real application, it will display ten digits of precision, but only a few digits will be accurate or meaningful. For example, suppose you drive 100 miles on 3 gallons of gas. A calculator would say that your mileage is  $100 \div 3 \approx 33.33333333$ . The precision of this answer is ten digits. The accuracy is probably not ten digits unless both the mileage and amount of gasoline were measured in an exceedingly accurate manner. It would be more reasonable or accurate to say that the mileage is about 33 miles per gallon, rather than 33.33333333 miles per gallon.

### Second and Inverse Keys

Because the size of the keyboard is limited, there is often a 2nd or INV key. This key can be used to access additional features. These additional features are usually labeled above the key in a different color.

## Graphing Calculators ▼▼▼

Graphing calculators provide several features beyond those found on scientific calculators. The bottom rows of keys on a graphing calculator are often similar to those found on scientific calculators. Graphing calculators have additional keys that can be used to create graphs, make tables, analyze data, and change settings. One of the major differences between graphing and scientific calculators is that a graphing calculator has a larger viewing screen with graphing capabilities.

### Editing Input

The screen of a graphing calculator can display several lines of text at a time. This feature allows the user to view both previous and current expressions. If an incorrect expression is entered, a brief error message is displayed. It can be viewed and corrected by using various editing keys—much like a word-processing program. You do not need to enter the entire expression again. Many graphing calculators can also recall past expressions for editing or updating.

### Order of Operation

Arithmetic expressions on graphing calculators are usually entered as they are written in mathematical equations. As a result, unary operations like  $\sqrt{x}$ ,  $\sqrt[3]{x}$ , and  $\log x$  are entered first, followed by the number. Unary operations like  $x^2$  and  $x!$  are entered after the number. Binary operations are entered in a manner similar to most scientific calculators. The order of operation on graphing calculators is also important. For example, try evaluating the expression  $\sqrt{2} \times 8$ . If this expression is entered as it is written, without any parentheses, a graphing calculator may display 11.3137085 and not 4. This is because a square root is performed before multiplication. To prevent this error, use parentheses around  $2 \times 8$ .

### Calculator Screen

If you look closely at the screen of a graphing calculator, you will notice that the screen is composed of many tiny rectangles or points called pixels. The calculator can darken these rectangles so that output can be displayed. Many graphing calculator screens are approximately 96 pixels across and 64 pixels high. Computer screens are usually 640 by 480 pixels or more. For this reason, you will notice that the resolution on a graphing calculator screen is not as clear as on most computer terminals. With a graphing calculator, a straight line will not always appear to be exactly straight and a circle will not be precisely circular. Because of the screen's low resolution, graphs generated by graphing calculators may require mathematical understanding to interpret them correctly.

### Viewing Window

The viewing window for a graphing calculator is similar to the viewfinder in a camera. A camera cannot take a picture of an entire view in a single picture. The camera must be centered on some object and can only photograph a subset of the available scenery. A person may want to photograph a close-up of a face or a person standing in front of a mountain. A camera with a zoom lens can capture different views of the same scene by zooming in and out. Graphing calculators have similar capabilities. The  $xy$ -coordinate plane is infinite. The calculator screen can show only a finite, rectangular region in the  $xy$ -coordinate plane. This rectangular region must be specified before a graph can be drawn. This is done by setting minimum and maximum values for both the  $x$ - and  $y$ -axes. Determining an appropriate viewing window is often one of the most difficult things to do. Many times it will take a few attempts before a satisfactory window size is found. Like many cameras, the graphing calculator can also zoom in and out. Zooming in shows more detail in a small region of a graph, whereas zooming out gives a better overall picture of the graph.



### Graphing and the Free-moving Cursor

Once a viewing window has been determined, an equation in the form of  $y = f(x)$  can be graphed. A simple example of this form is  $y = 3x$ . Four or more equations of this type can be graphed at once in the same viewing window. A graphing calculator has a free-moving cursor. By using the arrow keys, a small cross-hair can be made to move about on the screen. Its  $x$ - and  $y$ -coordinates are usually displayed on the screen. The cursor can be used to approximate the locations of features on the graph, such as  $x$ -intercepts and points of intersection. Using the trace key, the free-moving cursor can also be made to trace over the graph, displaying the corresponding  $x$ - and  $y$ -coordinates located on the graph.

### Tables

Some graphing calculators have the ability to display tables. For example, if  $y = x^2$ , then a vertical table like the following can be generated automatically. This is an efficient way to evaluate an equation at selected values of  $x$ .

X	Y
0	0
1	1
2	4
3	9
4	16
5	25
6	36

### Programming

Graphing calculators can be programmed, much like computers. Complex problems can be solved with the aid of programs. In this course, it will not be necessary for you to program your calculator. However, the capability is there, if you choose to use it.

### Additional Features

Graphing calculators have additional features too numerous to list completely. They may be able to generate sequences, find maximums and minimums on graphs, do arithmetic with complex numbers, solve systems of equations using matrices, and analyze data with statistics. The most advanced calculators are capable of performing *symbolic manipulation*. Using these calculators, one can factor  $x^2 - 1$  into  $(x - 1)(x + 1)$  and simplify  $\frac{x^2y^3}{x^{-2}y}$  to  $x^4y^2$  automatically. If the solution to a problem is  $\pi$ , symbolic manipulation routines will display  $\pi$  rather than 3.141592654.