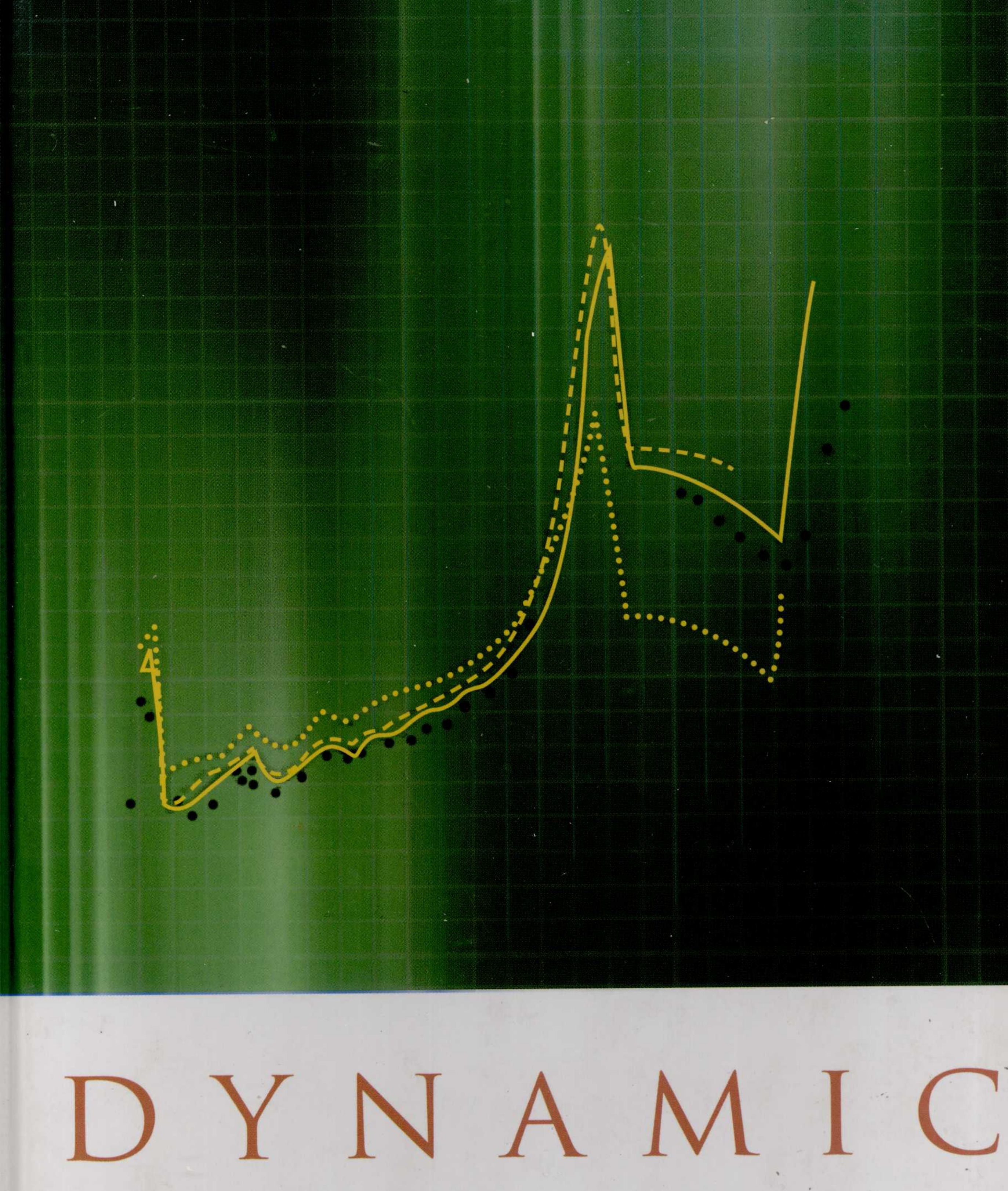
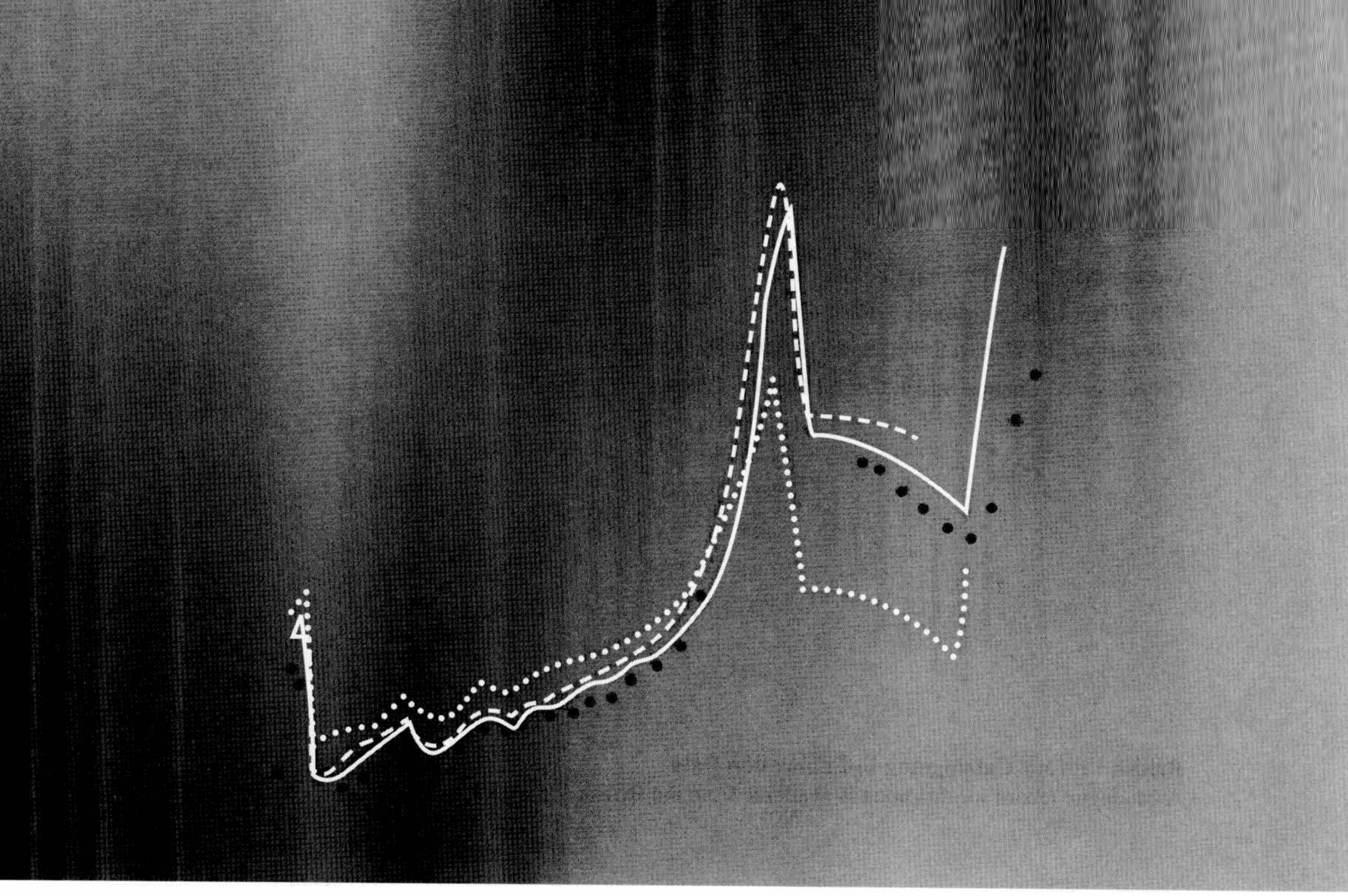


DYNAMIC PLASTICITY

N D Cristescu



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IN MEMORIAM

I would like to mention, with all the sadness possible,
the memory of my wife Cornelia, which disappeared meantime,
and which helped me always so much.

Preface

The present book is not simply a new addition of the book Dynamic Plasticity, initially published in 1967, a long time ago. Certainly this edition is not only a new version, containing the essential of the old book and what has been done meantime. Why again Dynamic Plasticity? Well because very many books published meantime on the subject are not mentioning the waves which are to be considered in Dynamic Plasticity. Also, generally, the plastic waves are slower than the elastic one. Thus, when considering a simple problem of propagation of waves in thin bars, for any loading at the end, the plastic waves are reached by the elastic ones, and will not propagate any more. Only a part of the bar is deforming plastically. Examples of this kind are very few.

I thought that this new version is too restrictive for the today students which know little of static plasticity, differential equations, dynamic elastic–plastic properties, etc. Therefore, I thought to write a simpler book, containing the main concepts of dynamic plasticity, but also something else. Thus I thought that this new version would contain the elementary concepts of static plasticity, etc., which would be useful to give. Also it would be good to give other problems, not directly related to dynamic plasticity. Thus I started with some classical problems on static plasticity, but only the simplest things, so that the readers would afterwards understand also the dynamic problems. Also, since in dynamic problems the soils and rocks played a fundamental role, I thought to write a chapter on rocks and soils. Then were expressed several chapters about dynamic plasticity, as propagation of elastic–plastic waves in thin bars, the rate influence and the propagation of waves in flexible strings. It is good to remember here that all problems related to dynamic problems, are to be considered using the mechanics of the wave propagation; without the wave propagation mechanics all results concerning constitutive equations, rate effect, etc. are only informative. Such problems are mentioned however in the book. We have presented mainly the different aspects on constitutive equations of materials, as resulting from dynamic problems. Rate effects are considered in this way. They have been used by a variety of authors. The same with the mechanics of flexible strings, presented afterwards. Not very many authors have considered till now the mechanics of deformable cables.

Therefore I thought to write a very simple book, which can be read by the students themselves, without any additional help. They can understand what "plasticity" is after all. Then several other problems have been presented. Not trying to remove the fundamentals, I have thought also to add some additional problems, which are in fact dynamic, though the inertia effect is disregarded. They are the stationary problems, quite often met in many applications. It is question obviously, about problems involving Bingham bodies, as wire drawing, floating with working plug, extrusion, stability of natural inclined plane, etc.

Further I have considered various problems of plastic waves, using various theories. Also the perforation problems, was presented, using various symmetry assumptions, or any other assumption made.

The last chapter is on hypervelocity impact. To keep it simple, I have given only very few information about. Thus I wished to show what hypervelocity is and how is it considered now.

Though the book is a very simple one, I wished to ask any author to disregard possible missing of some papers. All literature is certainly incomplete. One has done today much more than given here. It was impossible for me to mention "all" authors in this field.

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Contents

Introduction

Theory of Plasticity studies the distribution of stresses and particle velocities (or displacements) in a plastically (irreversible) deformed body, when are known the external factors which have acted upon him and the history of variation of these factors. The theory was applied to metals to describe working processes both at cold (drawing, rolling, etc.) and warm (extrusion, forging, etc.), to describe term behavior (high and law) involving also temperatures, to short term behavior, to describe impact, shocks, perforation, etc. It was applied to geomaterials, as soils, rocks, sands, clays, etc., with the description of civil engineering applications as tunnels, wells, excavations of all sorts, etc. It was applied to other materials as concrete, asphalt, ceramics, ice, powder-like materials, various pastes, slurries, etc.

In the classical sense the Plasticity Theory is time independent. However a time dependent theory was also developed and called Viscoplasticity. Besides Rheology deals with any flow or deformation in which time is the main parameter.

From the point of view of formulation of problems, in plasticity one considers in some of the problems, as in elasticity, that the strains are small; whoever in some other problems the consideration of the problems are as in nonlinear fluid mechanics when the strain are finite.

1 Diagnostic Tests

These are the slow tests in compression or in tension ($\dot{\varepsilon} \leq 10^{-2}\ \mathrm{s}^{-1}$, say) so as the strain is uniform along the specimen. We denote by

$$\sigma_{\mathrm{PK}} = \frac{F}{A_0} \quad \text{and by} \quad \sigma_C = \frac{F}{A}$$

the Piola–Kirchhoff and the Cauchy's stresses. Here F is the total force applied axially to the specimen, and A the current area, and by A_0 the initial area of the cross section of the specimen.

We also denote by

$$\varepsilon_H = \ln \frac{l}{l_0} \quad \text{and by} \quad \varepsilon_c = \frac{l - l_0}{l_0}$$

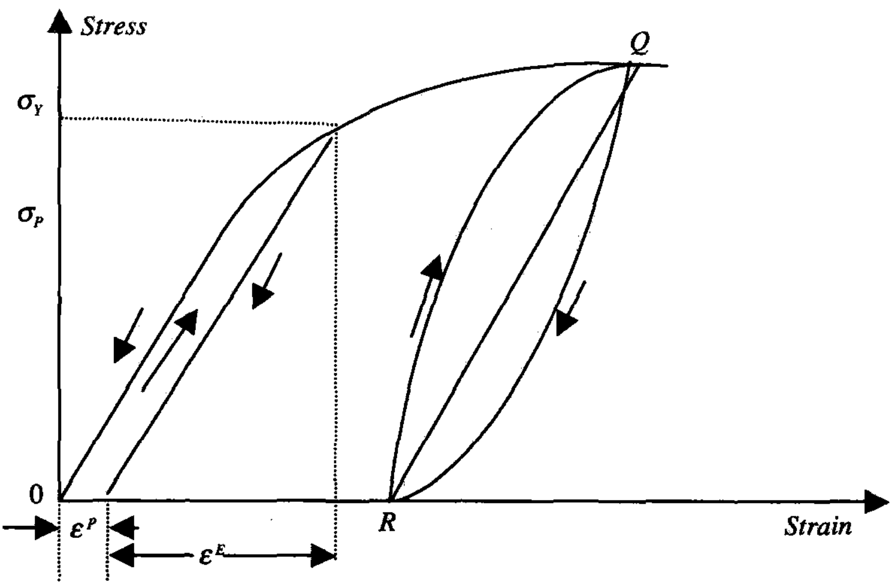

Fig. 1 Typical diagram of a diagnostic test.

the Henchy's strain or the Cauchy's strain. Here again l is the length of the working area of the specimen, and l_0 is the initial length of the same area. As a sign convention, $\sigma > 0$ in tension for metals, but it is a reverse convention for rocks and soils (see Fig. 1). σ_P is the proportionally limit of the specimen where we apply the Hooke's law $\sigma = E\varepsilon$ with E the Young's modulus which is constant, and independent on the loading rate and on the loading history. Up to σ_P we apply the Hooke's law in both loading und unloading. σ_Y is a conventional or offset yield limit defined by the permanent ε_Y, generally $0.1\% \doteq 0.5\%$ of the total strain. Essentially is that ε_Y is defined by a convention. Thus for $\varepsilon < \varepsilon_Y$ the unloading is perfectly elastic without hysteresis loop, as

$$\sigma = \sigma_Q + E(\varepsilon - \varepsilon_Q) .$$

Thus we assume small strains and

$$\varepsilon = \varepsilon^E + \varepsilon^P .$$

In elasticity we apply the Hooke's law written as

$$\boldsymbol{\sigma} = \mathbf{C}[\boldsymbol{\varepsilon}] \quad \text{or} \quad \sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

where $\mathbf{C}$ is a fourth order tensor. We apply this law during loading and unloading and the natural reference configuration is the stress-free strain-free state. If we introduce the two deviators by:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \frac{\operatorname{tr}\boldsymbol{\sigma}}{3}\mathbf{1} \quad \text{and} \quad \boldsymbol{\varepsilon}' = \boldsymbol{\varepsilon} - \frac{\operatorname{tr}\boldsymbol{\varepsilon}}{3}\mathbf{1} ,$$

the Hooke's law can be written

$$\boldsymbol{\sigma}' = 2G\boldsymbol{\varepsilon}' \quad \text{and} \quad \operatorname{tr}\boldsymbol{\sigma} = 3K\operatorname{tr}\boldsymbol{\varepsilon} ,$$

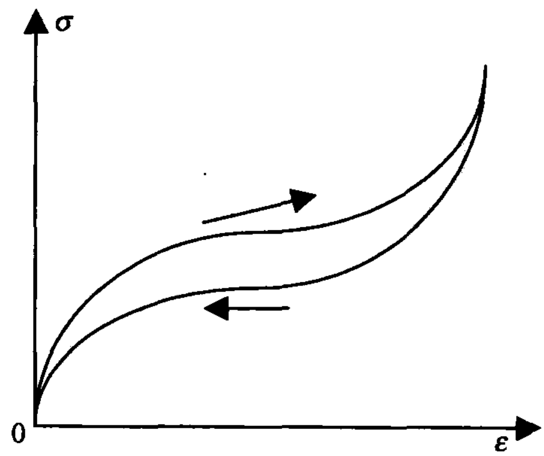

Fig. 2 Nonlinear elastic curve.

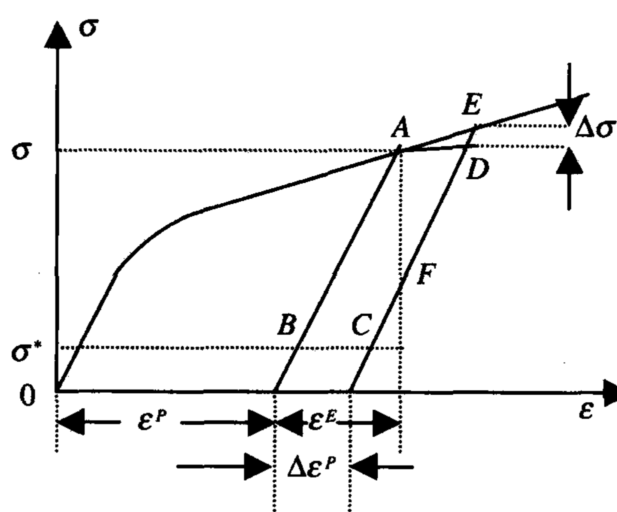

Fig. 3 Stress work per unit volume.

where G and K are the two elastic constants. The above relations are applied for any elastic isotropic body.

There are nonlinear elastic bodies as for instance rubber (see Fig. 2). The stress–strain curve is nonlinear but reversible. The unloading is according to some other law, exhibiting a significant hysteresis loop. We cannot describe this behavior by the Hooke's law but with a nonlinear law, giving a one-to-one correspondence. Since the material remembers his initial configuration, the reference configuration is the initial one.

For dissipative materials as the plastic ones, we can define an irreversible stress work per unit volume by:

$$W(T) = \int_0^T \sigma(t)\dot{\varepsilon}^P(t)\,dt\,.$$

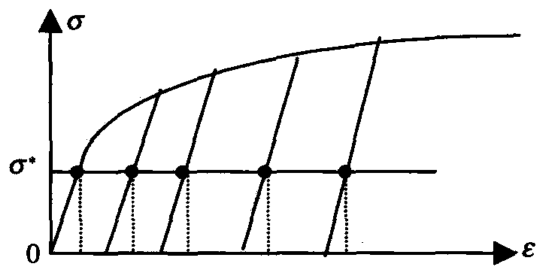

Fig. 4 Lack of one-to-one correspondence.

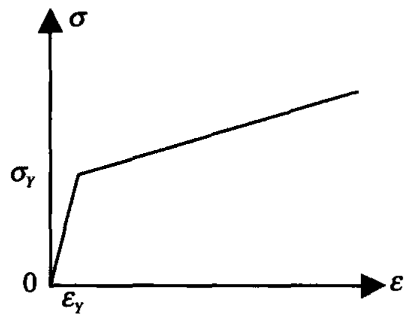

Fig. 5 Linear work-hardening.

That is shown in Fig. 3; it is the total irreversible area under the curve. The remaining area is the potential energy of deformation reversible for the reversible elastic materials (conservative).

In order to define work-hardening, we start with the Fig. 3. In a loading loop BAEDC producing the irreversible strain $\Delta\varepsilon^P$ and returning to the same stress σ^*, we define by:

$$(\sigma - \sigma^*)\Delta\varepsilon^P > 0 \quad \text{irreversibility},$$

$$\Delta\sigma\Delta\varepsilon^P > 0 \quad \text{stability}.$$

These two conditions are known as the Drucker's postulate and are used to define the plastic work-hardening.

Another postulate is due to Iliushin's; it says that the loading–unloading FAEDF must be positive.

In plasticity there is no one-to-one stress–strain correspondence. That is very clear in Fig. 4. The loading history must be known; to a single stress correspond several strains. Plasticity starts with unloading, as compared with nonlinear elastic behavior; and with the plastic strains which can develop only if $\sigma > \sigma_Y$.

The linear work-hardening is defined by two straight lines (Fig. 5):

$$\sigma = E\varepsilon \qquad \text{if } \sigma \le \sigma_Y\,,$$

$$\sigma = \sigma_Y + E_1(\varepsilon - \varepsilon_Y) \quad \text{if } \sigma \ge \sigma_Y\,.$$

Here E_1 is the constant work-hardening parameter, and $E_1 \ll E$.

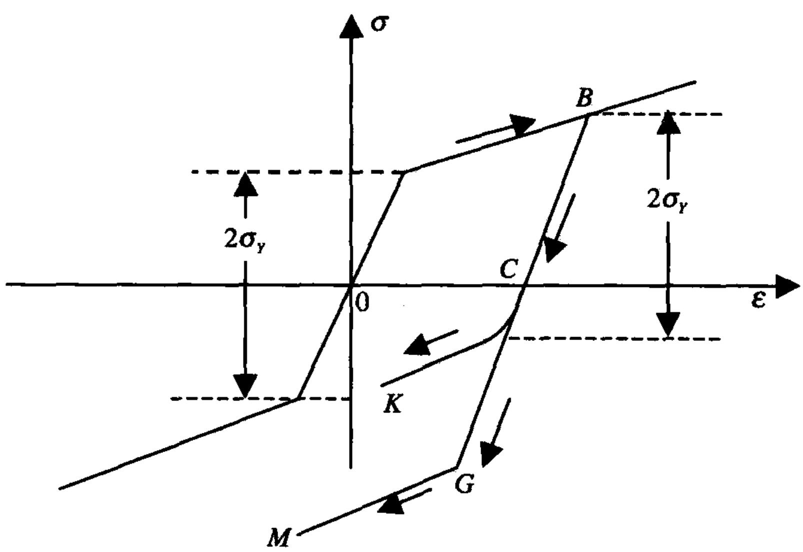

Fig. 6 Various hardening laws.

If the stress is increasing very slowly, in the so-called "soft" machines, one is observing some steps on the stress–strain curve. It is question of the so-called Savart–Masson effect, later rediscovered by Portevin–Le Chatelier effect. It was shown that this effect can by described by a rate-type constitutive equation (Suliciu [1981]). The viscosity coefficient has strong variation in some regions of the ε, σ plane that lie above the equilibrium curve.

For most materials, if $\boldsymbol{\sigma}$ is on a plastic state, then $-\boldsymbol{\sigma}$ is also on the plastic state. As it is well known, there are a lot of materials which do not satisfy this condition. For rocks for instance, if σ_{Yt} is the yield stress in tension, then σ_{Yc} is in compression, and $|\sigma_{Yc}| \gg |\sigma_{Yt}|$. That is also for concrete, cast iron, soils, glass, powders, etc. That is called Bauschinger effect, discovered in 1886. In Fig. 6 it is along BCK, that is the segment $2\sigma_Y$ stays more or less constant during loading.

For metals the elastic domain has a constant size $2\sigma_Y$ during loading. If during unloading we follow $BCGM$ then the hardening is said to be isotropic. If during unloading we follow the path BCK we say that the hardening is kinematic. Generally, if the yield stress in one direction is diminished by a previous plastic deformation in the opposite direction we have a Bauschingr effect. It introduces anisotropy, sough it can be removed by annealing at high temperature. If we do a loading in a single direction, we cannot distinguish between the two hardening.

2 Tests Performed at Long and Short Term Intervals

If dx is a material element in current configuration, and dX in the initial configuration, we call $\lambda = dx/dX$ elongation. The rate of elongation is $D = \dot{\lambda}/\lambda$ $(= \dot{\varepsilon}$ sometimes).

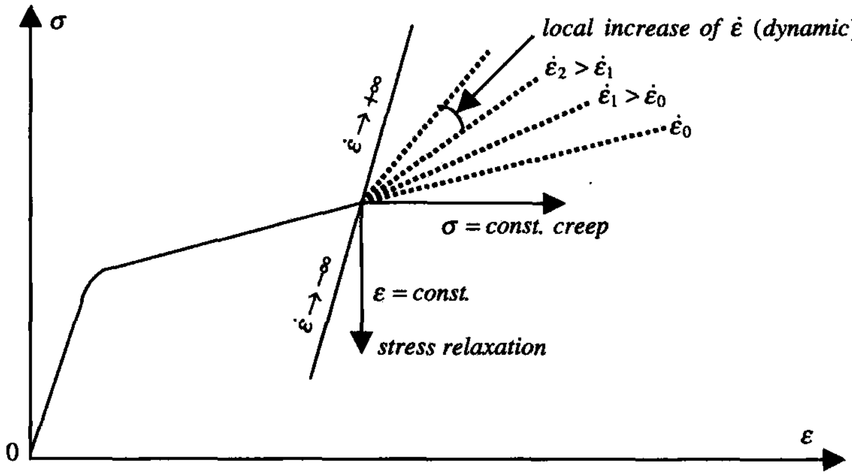

Fig. 7 Effect of change of rate of elongation.

Table 1 Variation of $\dot{\varepsilon}$.

$\dot{\varepsilon} < 10^{-20}\ \mathrm{s}^{-1}$	slow tectonic motion,
$10^{-8}\ \mathrm{s}^{-1} \geq \dot{\varepsilon} \geq 10^{-12}\ \mathrm{s}^{-1}$	creep tests,
$10^{-4}\ \mathrm{s}^{-1} \geq \dot{\varepsilon} \geq 10^{-1}\ \mathrm{s}^{-1}$	testing machines,
$\dot{\varepsilon} \simeq 1\ \mathrm{s}^{-1}$	hammer drop,
$\dot{\varepsilon} \simeq 10\ \mathrm{s}^{-1}$	the strain is not uniform, wave propagation is needed,
$\dot{\varepsilon} \simeq 10^2\ \mathrm{s}^{-1}$	metal drawing, air gun bullet,
$\dot{\varepsilon} \simeq 10^4\ \mathrm{s}^{-1}$	high speed impact, ballistics,
$\dot{\varepsilon} \simeq 10^6\ \mathrm{s}^{-1} - 10^7\ \mathrm{s}^{-1}$	high speed drawing of very fine wires, or very fast tests.

The change of rate of deformation is shown in Fig. 7. An increase of $\dot{\varepsilon}$ is raising the curves. But this raise in technically limited by the machine we have. For an additional increase, we need dynamic curves, with an local increase of $\dot{\varepsilon}$. This increase is done by elastic waves propagating with the velocity $c_0 = \sqrt{E/\rho}$. A table of approximate increase of $\dot{\varepsilon}$ is given in Table 1. This is a very approximate table of variation of $\dot{\varepsilon}$. For constant stress we have *creep*, but for constant strain we have *stress relaxation*. Any other intermediate variation of the strain rate is possible. For $|\dot{\varepsilon}| \to \infty$ we have very fast variation of the strain rate, impossible to realize practically.

In order to have a representation we take into account that mainly the plastic properties are influenced by the change of the rate of strain (see Fig. 8). A relationship was proposed by Ludwik from 1909, and is of the kind shown on Fig. 8. Thus we have for a fixed strain:

$$\sigma = \sigma_Y + \sigma_0 \ln \frac{\dot{\varepsilon}^P}{\dot{\varepsilon}_0^P} \quad \text{and} \quad \sigma > \sigma_Y > 0$$

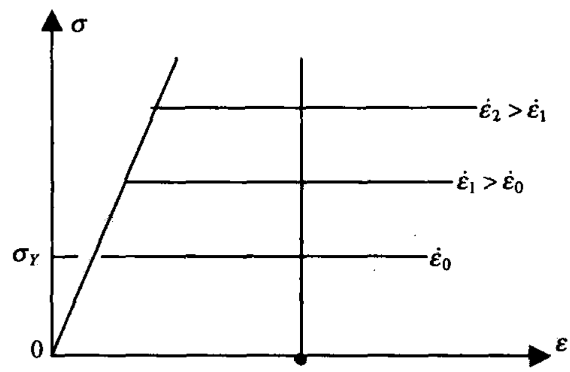

Fig. 8 Influence of the strain rate on the stress–strain curve.

with σ_0 = constant. If the elastic strains are disregarded, the stress–strain curves are

$$\dot{\varepsilon} = \begin{cases} \dfrac{\sigma - \sigma_Y}{3\eta} & \text{if } \sigma > \sigma_Y \,, \\ 0 & \text{if } 0 \leq \sigma \leq \sigma_Y \,, \end{cases}$$

where $\sigma - \sigma_Y$ is the overstress. σ_Y is the yield stress for a conventional small $\dot{\varepsilon}_0$ obtained in very slow performed tests, when flow starts being possible. η is a viscosity coefficient; if two tests are performed with the strain rates $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ we have:

$$3\eta = \frac{\sigma_2 - \sigma_1}{\dot{\varepsilon}_2 - \dot{\varepsilon}_1} \,,$$

to determine η. If η is constant for any strain rates, the relation is linear, otherwise nonlinear. Since in this relation there is no strain, the reference configuration is the actual one.

For work-hardening materials (Malvern):

$$\dot{\varepsilon} = \begin{cases} \dfrac{\sigma - f(\varepsilon)}{3\eta} & \text{if } \sigma > f(\varepsilon) \,, \\ 0 & \text{if } 0 \leq \sigma \leq f(\varepsilon) \,. \end{cases}$$

The reference configuration is the initial one or a relative one, corresponding to the state when the test started (for geomaterials, for instance).

Let as give several examples. In Fig. 9 is given the stress–strain curves for schist. One can see that the influence of the strain rate is felt from the beginning. The hole curve is influenced, not only the plastic part. Also, the last points correspond to failure. Thus with an increase of loading rate the stress at failure is increased, while the strain at failure is decreased. Thus a theory of failure expressing in stresses only, would not work.

In order to see that the influence of the strain rate is not always to be seen always on the stress–strain curves. In Fig. 10 are given the curves for granite, obtained in