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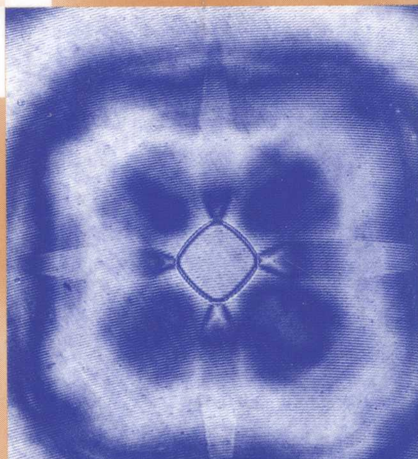

Shock Focussing Effect in Medical Science and Sonoluminescence

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Shock Focussing Effect in Medical Science and Sonoluminescence

With 107 Figures and 12 Tables

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Preface

In 1942 Guderley investigated the behaviour of the typical parameters near the axis when he was studying the focusing of cylindrical shock waves by solving the partial differential equations using the similarity method. He predicted an infinitely large pressure and temperature at the axis of convergence. Since then many theoretical and experimental studies have been performed on shock focusing as an energy source for the generation of very high pressures and temperatures. Many experimental investigations confirmed the idea of Guderley that at the centre of convergence extremely high pressures and temperatures can be obtained.

At the Technical University of Darmstadt we performed a numerical experiment to investigate the shock-focusing phenomenon, and many test problems were solved using the Rusanov scheme. At the centre of convergence we obtained extremely high pressures and temperatures. The results of these investigations were compared with the experimental work done at the Max Planck Institute for Fluid Dynamics Research in Göttingen. The three-dimensional time-dependent shock wave which was created by a centred gas volume under high pressure interacts with the plane walls of the cavity and leads to a focusing effect after the explosion. The symmetry-preserving character of the invariant difference scheme under use was proved numerically by calculation over a long interval of time. A test run of our investigation was also made at the Institute of Computational Fluid Dynamics in Tokyo, Japan. The numerical and experimental work done by the other researchers was taken into account and we found that our formulation in quasi-conservative form simulates the focusing phenomena in a way that is better and faster, and at the same time it gives a pointwise variation of the physical situation near the focus.

Sonoluminescence is a weak emission of light which occurs in an engassed liquid when it is cavitated by a sound field. Although this phenomenon has been known since 1930, extensive research work was started only at the beginning of this decade when Gaitan discovered that sonoluminescence is not only associated with the transient cavitation but can occur during stable oscillation of a single bubble. A clear understanding of the mechanism that produces sonoluminescence has not been achieved so far, but shock theory explains many if not most of the phenomena. The investigations undertaken so far indicate that the behaviour of the gas inside the bubble near the centre is the key element in understanding the sonoluminescence phenomena.

The shock wave got a new dimension when it was applied in medical science to treat kidney stones in 1980. Since then the research and development effort for this treatment has covered a wide range of therapy modalities. New applications have been developed in ureter stone lithotripsy, gallstone lithotripsy, gall duct stone lithotripsy, salivary stone lithotripsy, pseudarthrosis treatment, tendinosis calcarea pain therapy, therapy for motion infringement, and induratio penis plastica. The biological side effects of extracorporeal shock have also been investigated. The development of accurate pressure sensors for focused shock is still ongoing. The new modalities open up further questions. In pain therapy it is of particular interest to find limits between the wanted effects on tissue and the unwanted side effects.

The phenomena of shock focusing in liquid and gases has attracted not only gas dynamicists, scientists who are interested in creating certain conditions using shock waves, but also scientists devoted to the investigation of the collapsing bubble, a phenomenon of sonoluminescence, and medical doctors who use shock waves in treatment.

This book is an attempt to bring the fundamental research into shock focusing and its applications in medical science and sonoluminescence together. It contains articles devoted to the investigation of shock focusing and bubble dynamics and their applications in medical science. The destructive action of cavitation bubbles collapsing near boundaries is also covered by the book, to signify its importance in causing damage to materials, with special reference to kidney stones. The article on extracorporeal shock-wave bioeffects and the mechanisms of action is very interesting and contains new findings, shedding some light on the possible future applications of shock waves which ultrasound might open up one day. We discuss the current status of the development of shock-measurement techniques and clinical findings correlated with measurement results, the types of sensors, their advantages and disadvantages, and the related literature. The shock theory of sonoluminescence plays an important part, as do converging shock and thermal waves emanating from sonoluminescing gas bubbles and upscaling single-bubble sonoluminescence. The articles presented here are of great interest to medical doctors, technical researchers of lithotripsy, gas dynamicists and scientists working on sonoluminescence, and cavitation bubbles, and we have adopted an interdisciplinary approach.

We would like to express our gratitude to all the contributors. Our sincere thanks are also due to Professor W. Beiglböck, Mrs. Brigitte Reichel-Mayer, and their colleagues at Springer-Verlag, Heidelberg, for their friendly cooperation in producing this volume.

Darmstadt, August 2002

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Contents

1 The Shock-Wave Theory of Sonoluminescence

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Abstract

When driven into violent pulsation by a sufficiently strong source of sound, a bubble of air in water emits light, a phenomenon known as 'sonoluminescence'. The reasons for this are not yet fully understood. The most popular explanation at this time is the shock-wave theory of sonoluminescence. This supposes that, because the bubble surface moves inwards supersonically with respect to the air in the bubble during the compressive parts of the acoustic cycle, it launches an imploding spherical shock wave that becomes so strong, as it focuses at the center of the bubble, that it ionizes the air, the observed light being emitted from the resulting plasma ball. We discuss here the structure and stability of spherical shocks in ideal and van der Waals gases, paying particular attention to similarity shocks of the Guderley type and their relevance to sonoluminescence. We discuss the status of the shock-wave theory of sonoluminescence and alternative explanations. We pose a number of theoretical challenges.

1.1 Sonoluminescence

No one who encounters sonoluminescence for the first time, as we did in 1992, can fail to be impressed. One enters a darkened laboratory attracted by a small pinprick of light. On closer inspection he finds that this is emitted by a small bubble of air centered on a spherical flask of water, and is told that this is the result of bombarding the bubble with sound waves generated by transducers at the walls of the flask. Questions flood into his mind. Why does the bubble not rise under its buoyancy to the top of the flask? Why does the bubble not dissolve into the surrounding water? But most of all, why does a small bubble of air bombarded by sound produce light? The deeper one digs into the phenomenon, the more fascinating are the questions that arise. This is indeed a rich subject for experimenter and theoretician alike, rich because it is still not exhausted and because it spans so many interesting areas of physics and chemistry.

Although the phenomenon of sonoluminescence, or "SL" as we shall call it for short, has been known [1] since 1934, and although interest in the phenomenon never subsequently died, it was not until early this decade that the subject really took fire. By now there is a very considerable body of literature devoted to it, a body so large that it would be foolish to try to summarize it here. And indeed our remit is to focus on the generation of shocks in the bubble. The reader wishing to know more is referred to papers [2–4], which review much of the relevant material. Suffice it to say here that the bubble is driven so strongly towards the acoustic pressure antinode at the center of the flask that it is prevented from rising under gravity; see, for example, [5]. Also, to make a bubble sonoluminesce, the source of sound must be strong; the acoustic pressure, $p'_{\rm a}(t) = -P'_{\rm a} \sin \omega t$, at the walls of the container (or "at infinity" in theoretical models) must significantly exceed the atmospheric pressure, P_0. The radius, $R(t)$, of the bubble then varies strongly with time, t; see the example shown in Fig. 1.1. During part of the acoustic cycle, when the net pressure, $p(R,t)$, of the gas at its surface $r = R(t)$ is small, the bubble expands to many times its ambient radius, R_0 but, during the compressive part of the cycle, R decreases dramatically to a fraction of R_0. Although the

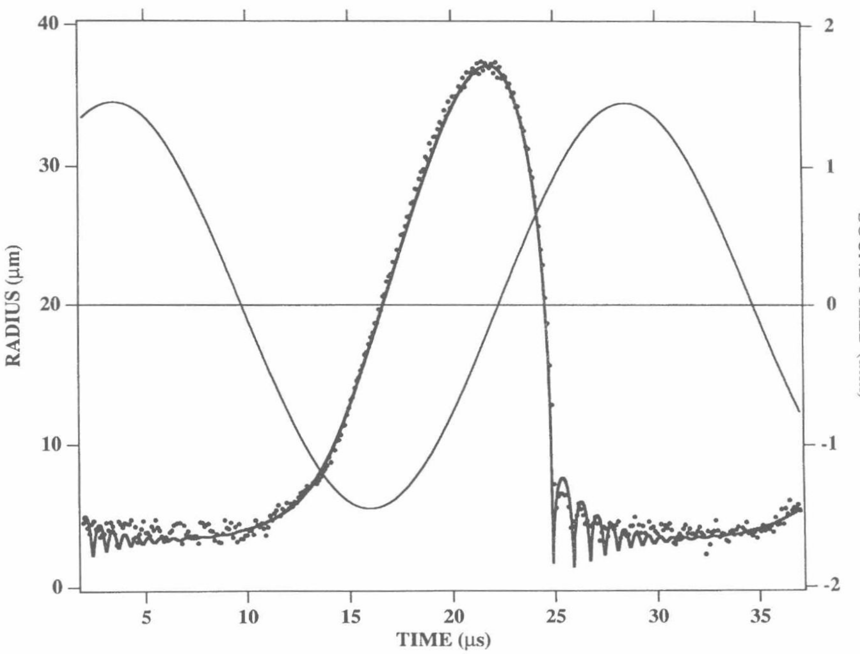

Fig. 1.1. Radius of a Xenon bubble of ambient radius 4.3 μm and acoustic driving pressure $P'_{\rm a}$ of 1.45 atm. The *dots* are from experiment, the *full line* from the appropriate solution of the Rayleigh–Plesset equation. The thin line is the acoustic waveform (after [2])

"giant fall" of R is violent, it is also short in duration. The bubble spends a greater fraction of each period P of the acoustic cycle in expansion than in contraction. During a contraction, gas is pushed out of the bubble into the neighboring fluid and it is returned during an expansion. These processes can balance and, when they do so, the bubble lives, with a finite ambient radius R_0 that is determined by this mechanism, which is called "rectified diffusion"; e.g. see [6].

Many theoretical works assume that R_0 is known, and determine $R(t)$ by solving the so-called Rayleigh–Plesset equation, which follows from the assumption that the fluid is viscous and almost incompressible. By retaining compressibility as a perturbation ($|\dot{R}|/a_L$ being the small parameter, and a_L being the speed of sound in the liquid), the damping of the bubble motion by the sound waves it radiates through its oscillations is included as a perturbation. This damping is usually more effective than that created by the viscosity of the liquid. To close the Rayleigh–Plesset equation, it is necessary to specify not only the pressure, p_∞, of the liquid at infinity (as we have done above; $p_\infty(t) = P_0 + p'_a(t)$) but also the pressure $p_L(R,t)$ at the surface of the bubble, which differs from $p(R,t)$ only by interfacial tension. Frequently, $p(R,t)$ is specified through simple models of the gas. For example, it may be assumed that the gas is ideal and in a *uniform* (though time-varying) state, perhaps isothermal or adiabatic. This is a reasonable approach for a bubble that is weakly-driven acoustically, and it allows us to make a first crude estimate of the temperature T in the bubble. This is significant since, to explain SL, one must understand how the gas is heated to high temperatures. If we assume that the gas is adiabatically compressed during the giant fall, we find that T is at most of order 5000–10 000 K. It is hard to explain SL unless the maximum T is much greater than this.

When the acoustic drive is strong ($P'_a > P_0$), it is unreasonable to suppose that the gas remains in a uniform state. It has been conclusively demonstrated experimentally that the surface of a sonoluminescing bubble moves supersonically inwards during the giant fall; the Mach number M of this "piston", defined as the ratio of its inward velocity $-\dot{R}$ to the speed of sound a_0 in the gas in its ambient state, may be as large as 4 [7]; see Fig. 1.2. Inevitably, the bubble wall launches an inwardly moving spherical shock wave, as indeed was originally proposed by Trilling [8] and Jarman [9]. The idea behind what we shall call "the shock-wave theory" of SL is that temperatures high enough to explain SL arise through shock heating of the gas. We also note that the shock wave in the bubble creates large rapid fluctuations in $p(R,t)$. Strictly speaking, the Rayleigh–Plesset equation and the equations of motion for the gas should be solved simultaneously, as we were the first to do [10, 11]. In this review, however, in which we focus on shocks in the bubble, we shall suppose that $R(t)$ is a known function of t.

Initially, as the bubble wall moves inwards, there is no shock, and the gas in the bubble is everywhere compressed adiabatically. This compression

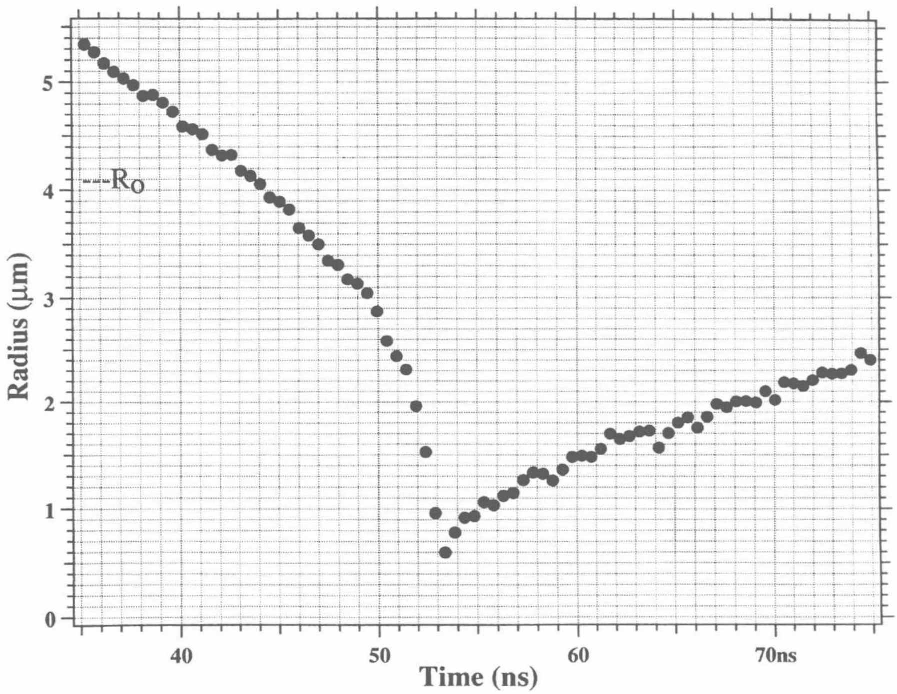

Fig. 1.2. Radius of a sonoluminescing bubble as the moment of collapse is approached (1% xenon in oxygen at 150 mm); $P_{\mathrm{a}}' = 1.45$ atm (after [2])

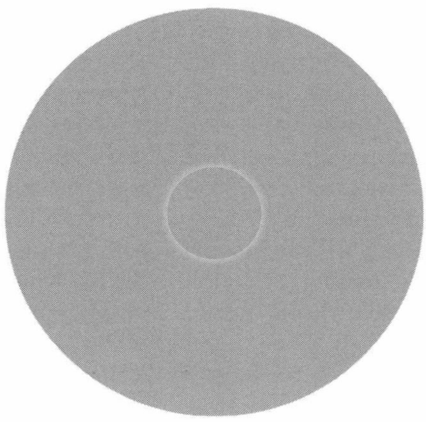

Fig. 1.3. The formation of the shock in a collapsing bubble, illustrated by the specific entropy, S. The interior of the bubble is shown shaded. In the lighter area S is greater than in the darker area. The increase in S signals the formation of the shock; S is the same in the region within the shock and in the region between the shock and the bubble wall. (Based on case III of [11])

continues until the shock forms. Figure 1.3 shows the specific entropy, S, at approximately the time when the shock first appears. From that time onwards, the central region ahead of the shock is cut off from the regions behind it, and the gas in the central region compresses relatively little until shocks pass through it. In SL applications, the gas is already strongly compressed when the shock first forms, and deviations from the ideal gas law are already significant. Indeed, models we describe later suggest that the density ρ of the gas may momentarily become as large as that of water! A popular alternative to the ideal gas is a simplified van der Waals model, and we too adopt this as our model of an imperfect gas. Other possibilities have been studied [12].

Deviations from the ideal gas law are significant for a second reason. SL has been observed [13] from bubbles of ethane, a gas for which the ratio of specific heats γ is close to 1. At first sight, there would seem to be no significant shock heating, and the shock-wave theory has been criticized on those grounds [14]. We have shown, however, that shock heating also arises through deviations from the ideal gas law and is by no means negligible [15, 16]. In fact, no experimental fact has as yet ruled out the shock-wave theory of SL, though none has positively supported it. Other explanations for SL have been advanced (e.g. [14, 17]) that are perhaps even harder to evaluate; see Sect. 1.5. The success of shock-wave theory may ultimately be decided by how successful shocks are at depositing energy near the center O of the bubble. This in turn may depend on how well the shocks preserve their spherical shape during the implosion. For this reason, we believe that shock stability is a significant issue faced by the theory.

These considerations have motivated much of our own theoretical work. We have examined the structure of spherical shocks [10, 11] and their stability [15, 16], both for ideal and nonideal gases; see Sects. 1.2–1.4 below. We speculated [10, 11] that the shock would at first dissociate the air, and then ionize it to create a dense central plasma ball, "a star in a jar" [18]. We argued [10, 11] that the light that gives sonoluminescence its name is principally bremmstrahlung, i.e. the light emitted during free–free transitions of electron motions in the plasma ball. This interpretation has not yet been shown inconsistent with the available data, although much more detailed models have by now been integrated [19–23]. According to our model, the light is emitted as a pulse lasting for a few tens of picoseconds (ps), before the outgoing shock strikes the bubble surface and drives it outward. And the experimental data, though currently inconclusive, does not contradict the idea that $\dot{R}$ reverses sign only after the light pulse has been emitted. It is now known experimentally that diatomic gases do not provide the most striking examples of SL; noble gases such as argon and xenon make brighter bubbles [24]. Though many gases sonoluminesce, few fluids are "friendly" to the phenomenon [2].

Most crucial for the success of the shock-wave theory of SL is what happens near the "moment of implosion", the instant (which we shall usually take to be $t = 0$) when the shock implodes onto O (which we take to be the origin of spherical coordinates r, θ, ϕ). We find that, as the shock focuses, its structure asymptotically approaches a similarity form, of the same general type as that first derived by Guderley [25] for an ideal gas; see also Sect. 107.8 of [26]. It seems to us that similarity solutions are very significant in the understanding of SL, for we find that, although light is not emitted at $t = 0$, where the hottest region fills zero volume, it is emitted very soon after $t = 0$, from a hot shell created as the outward traveling shock encounters the gas that is still moving inwards. At that time the shock is close to having a similarity form.

The Guderley solution has been the object of many studies. It has been shown that the well-known CCW approximation (named after Chisnell, Chester and Whitham [27–29]) gives a very good account of the Guderley solution, and that even better agreement is obtained after further refinements [30]. Somewhat surprisingly perhaps, we found [15, 16] that the CCW approximation does not perform well for our simple van der Waals model, and in fact misses a new branch of solutions in its entirety. It also is inadequate for studying the linear instability of the shock front. This is demonstrated by our new analysis which, though restricted to similarity shocks, is more complete than any that have gone before, such as [31]; see Sect. 1.4.

Spherical implosions have some unphysical characteristics that are not easily removed. In particular, both the temperature T and pressure p become instantaneously infinite at O for $t = 0$. This absurdity arises from the strengthening of the shock as its energy is focused to a point at implosion. Mathematically, strong shocks are regarded as discontinuities, and physically as thin layers a few mean-free paths thick. To investigate the internal structure of such a shock, it is necessary to abandon the continuum description and use transport theory instead. Similarly, the infinities at the moment of implosion could in principle be removed by reverting to a particle description of the gas. So far, this has not been attempted. In the meantime, the infinities provide, in a sense, a measure of the accuracy of numerical integrations: the higher the truncation level, the larger the central T and p at implosion, i.e. the better the numerical representation of infinity! Fortunately, the infinities have no bearing on the SL application since (as we mentioned above) the light is plausibly emitted after the implosion, during times in which T, p and ρ, though large, are well determined by the similarity solution. The fact that ρ and T are large has suggested the possibility of "table-top fusion" [20] and of an accurately periodic source of neutrons [32]. We have pointed out [15, 16] that, these possibilities are improved if, instead of using gases such as D_2 or DT, hydrogenic gases of large molecular weight are used in which H is replaced by D or T.

Although similarity solutions are useful and ubiquitous, they have one serious drawback: they contain a free parameter, the amplitude A_i of the incoming shock, that cannot easily be determined from the conditions at the bubble surface when the shock was launched. To match the two, it is necessary [10, 11, 15, 16] to integrate the shock from its inception until it approaches its similarity form near O, something that is not always done [33].

1.2 Basic Equations; Guderley Solutions

The laws governing the motion of the gas are the Euler equations, which, in conservative form, are

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \boldsymbol{v}) = 0, \tag{1.1}$$

$$\frac{\partial E}{\partial t} + \nabla\cdot[(E+p)\boldsymbol{v}] = 0, \tag{1.2}$$

$$\frac{\partial}{\partial t}(\rho v_i) + \nabla_j\cdot(\rho v_i v_j) = -\nabla_i p, \tag{1.3}$$

where $\boldsymbol{v}$ is the gas velocity and $E = \rho e + \frac{1}{2}\rho v^2$ is the total energy density, e being the internal energy density. In addition tensor notation is used in (1.3). As we pointed out in Sect. 1.1, SL arises in extreme conditions under which the ideal gas is not a sufficiently accurate description. To discover how deviations from the ideal can affect the solutions, we adopt a simple model; we assume that the gas obeys a simplified van der Waals equation of state of the form

$$p = \frac{\mathcal{R}T}{\mathcal{V}-b}, \qquad e = c_{\mathrm{v}}T = \frac{\mathcal{V}-b}{\gamma-1}p, \qquad S = c_{\mathrm{v}}\ln[p(\mathcal{V}-b)^{\gamma}] + \text{constant}, \tag{1.4}$$

where $\mathcal{V} = 1/\rho$ is the specific volume, $\mathcal{R}$ is the gas constant, and $c_{\mathrm{v}} = \mathcal{R}/(\gamma-1)$ is the specific heat at constant volume. The constant b is the 'van der Waals excluded volume'; it places a limit, $\rho_{\max} = 1/b$, on the density of the gas. It follows from (1.1)–(1.4) that (1.3) may also be written as

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\right)[p(\mathcal{V}-b)^{\gamma}] = 0, \qquad \text{or} \qquad \frac{DS}{Dt} = 0, \tag{1.5}$$

where $D/Dt = \partial/\partial t + \mathbf{v}\cdot\boldsymbol{\nabla}$ is the motional derivative. By (1.5), the specific entropy following the motion does not change.

Equations (1.1)–(1.4) must be solved subject to appropriate initial and boundary conditions, and in addition they must be connected across shocks, whenever they occur, by Rankine–Hugoniot jump conditions. Excluded volume effects (associated with b) change these conditions to

$$\frac{\boldsymbol{n}\cdot\boldsymbol{v_2}}{\boldsymbol{n}\cdot\boldsymbol{v_1}} = \frac{\rho_1}{\rho_2} = 1 - \frac{2(M^2-1)}{(\gamma+1)M^2}(1-b\rho_1) \tag{1.6}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma(M^2-1)}{\gamma+1}, \tag{1.7}$$

$$\mathbf{n}\wedge\mathbf{v}_2 = \mathbf{n}\wedge\mathbf{v}_1, \tag{1.8}$$

$$p_2(\mathcal{V}_2-b)^{\gamma} \geq p_1(\mathcal{V}_1-b)^{\gamma}, \qquad \text{or} \qquad S_2 \geq S_1. \tag{1.9}$$

The last of these emphasizes that (1.5) does not apply to material passing through the shock front, where the continuum description breaks down. The suffices $_1$ and $_2$ denote respectively values immediately ahead of, and immediately behind, the shock front, $\mathbf{n}$ is the unit normal to the front, and

$$M = \frac{\mathbf{n}\cdot\mathbf{v}_1}{a_1} \tag{1.10}$$

is the Mach number of the shock relative to the state ahead of it, a being the speed of sound:

$$a = \sqrt{\left[\frac{\gamma p}{\rho(1-b\rho)}\right]}. \tag{1.11}$$

Conditions (1.6)–(1.9) apply in the reference frame co-moving with the shock; in the application below, they are transformed to the laboratory frame.

Guderley [25] discovered similarity solutions for spherically symmetric shocks in an ideal gas, and we have found [15, 16] that the imperfect gas ($b \neq 0$) admits solutions of a similar type. We shall now review the Guderley theory; see also Landau and Lifshitz [26, Sect. 107.8]. This is a necessary preparation for our generalization to an imperfect gas in Sect. 1.3. Guderley took $t = 0$ as the moment of implosion, and sought similarity solutions in which the incoming shock is situated at time $t < 0$ at $r = R_S(t)$, where

$$R_S(t) = A_i(-t)^\alpha, \qquad \text{for} \qquad t < 0 \tag{1.12}$$

and in which the density, the *outward* radial velocity, and the sound speed are of the form

$$\rho = \rho_0 G(\xi), \qquad v = \frac{\alpha\, r}{t} V(\xi), \qquad a^2 = \left(\frac{\alpha\, r}{t}\right)^2 Z(\xi), \tag{1.13}$$

where ρ_0 is defined below and ξ is the similarity variable

$$\xi = \frac{r}{A_i(-t)^\alpha} \qquad \text{for} \qquad t < 0, \tag{1.14}$$

so that

$$\xi = 1 \qquad \text{at the shock front.} \tag{1.15}$$

The value of the constant α can only be determined by numerical computation (see below). In (1.14) and below, the suffix $_i$ is used when it is necessary to distinguish the incoming shock from the outgoing shock; $_o$ is used for the latter. Since $t < 0$, the inward motion of the gas ($v < 0$) implies that $V > 0$ in (1.13) during implosion.