



SALAS / HILLE / ETGEN

EIGHTH EDITION

CALCULUS

ONE AND SEVERAL VARIABLES

**EIGHTH
EDITION**

SALAS

HILLE

ETGEN

CALCULUS

ONE AND SEVERAL VARIABLES

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PREFACE

This text is designed for an introductory single and multivariable calculus sequence. It is a text on mathematics and the underlying emphasis throughout is on the three basic concepts: limit, derivative, and integral.

Our goal in preparing the Eighth Edition was to continue the style and approach that have characterized the previous editions. At the same time, we recognize that the rapid advances in computer technology and changes in mathematics education are impacting the calculus sequence. This text, therefore, continues to evolve with the needs of the students.

FEATURES OF THE EIGHTH EDITION

Precision and Clarity

The emphasis is on mathematical exposition—an accurate, understandable treatment of the topics. The mathematical statements are careful and precise; the basic concepts and important points are not obscured by excess verbiage.

Accessibility

This text is completely accessible to the beginning calculus student without sacrificing appropriate mathematical rigor. The important theorems are proved, and the mathematical methods are justified. These may be covered or omitted according to the theoretical level desired in the course.


Balance of Theory and Applications

Imaginative problems drawn from the physical sciences are used to motivate the basic concepts of calculus; in turn, the concepts and methods of calculus are applied in a variety of fields in the sciences, engineering, business, and the social sciences in the form of text examples and exercises. Because the presentation is flexible, instructors can vary the balance of theory and applications according to the needs of their students.

Visualization

We recognize the importance of visualization in developing students' understanding of mathematical concepts. For that reason over 1200 illustrations accompany the examples and help students work exercises.

Technology

Technology-based examples in the text support numerous exercises requiring the use of a graphing utility (graphing calculators or computer software). We use a generic approach rather than attempting to teach any particular technology. Technology problems are clearly designated with an icon () and may be omitted by instructors who prefer that their students not use calculators or computers.

Projects

New projects with an emphasis on problem solving have been integrated into the text. The projects investigate a variety of special topics that supplement the text material. Typically, they require an approach that involves both theory and applications, including the use of technology, and they are intended to foster collaborative learning.

Early Introduction of Differential Equations

Differential equations are first introduced in Chapter 7 in connection with exponential growth and decay. First-order linear equations and separable equations are covered in optional sections in Chapter 8 to take advantage of their natural association with the techniques of integration. Chapter 15 has an optional section on exact differential equations where it is related to the reconstruction of a function from its gradient.

CONTENT AND ORGANIZATION CHANGES IN THE EIGHTH EDITION

In our effort to create an even more effective text, we consulted users of the Seventh Edition and other calculus instructors. As a consequence, the primary goals in preparing the Eighth Edition of the text were the following:

- 1. Reduce the overall length without sacrificing essential content.** This has been accomplished by eliminating some of the optional topics, reducing the coverage of redundant material, and by combining some sections.
- 2. Streamline the exercise sets.** Every exercise set was examined in terms of the balance between the routine, drill problems and the more challenging applications and conceptual problems. In many instances, the number of routine problems was reduced and where appropriate, new applications or conceptual problems were added.
- 3. Improve the illustrative examples.** Many existing examples were modified and new examples have been added to enhance the students' understanding of the material.
- 4. Improve discussions and explanations.** Every topic was reviewed for possible improvements in the presentation. A number of sections and subsections were completely rewritten and many others were refined.

Specific changes in the organization and content of the chapters have been made to meet the needs of today's students and instructors. They include:

Precalculus Review (Chapter 1)

This material has been reduced in length but not in the coverage of the major topics. The exercise sets have been reduced where appropriate by cutting back on the number of drill problems. The subsections on "Distance Between a Point and a Line" and "Translations" have been moved to Chapter 9 where they are treated with the "Conic Sections."

Limits and Continuity (Chapter 2)

The introductory section on the “Idea of Limit” has been rewritten. There are new examples, a brief discussion of infinite limits, and an application to finding the slope of the tangent line to the graph of a function. The application of the intermediate-value theorem to solving inequalities is now covered; the subsection on the “bisection method” has been converted to a Project.

Differentiation and Applications of the Derivative (Chapters 3 and 4)

The content and organization of the material in these two chapters have not been changed. A number of examples have been modified, and the exercise sets have been revised.

Integration and Applications of the Integral (Chapters 5 and 6)

Some of the material in Chapter 5 has been reorganized and rewritten. The number of sections has been reduced by combining some sections. The approach to the definite integral has been modified to include both Riemann sums and upper and lower sums.

The Transcendental Functions (Chapter 7)

The length of this chapter has been reduced by combining two sections from the previous edition and by eliminating some material. The integration of the trigonometric functions is now treated with the logarithm function. The treatment of separable differential equations has been moved to Chapter 8.

Technique of Integration (Chapter 8)

The review material in Section 8.1 has been expanded to include the use of integral tables. The coverage of powers and products of trigonometric functions has been reduced; this material is covered in one section rather than two. Rationalizing substitutions are now treated in an optional section. Optional sections on first-order linear differential equations and separable equations, together with applications, have been moved from Chapter 18.

Conic Sections: Polar Coordinates; Parametric Equations (Chapter 9)

The coverage of the conic sections has been reorganized; the material is now covered in two sections rather than the one long section in the previous edition. The treatment of polar coordinates has been reduced by combining the sections on graphing and intersections of polar curves. The optional sections on the conics in polar coordinates and the cycloid have been converted to Projects.

Sequences and Series (Chapters 10 and 11)

There has been some rewriting to reduce the overall length; the material on sigma notation has been integrated into the introduction of infinite series. The examples illustrating approximation by Taylor series have been revised.

Vectors and Vector Calculus (Chapters 12 and 13)

Vectors are introduced through a brief discussion of displacements, forces and velocities in a separate sections without exercises.

Functions of Several Variables, Gradients, Extreme Values (Chapters 14 and 15)

The introduction to the gradient and differentiability has been rewritten, the calculation of the gradient from the formal definition has been reduced. The coverage of the mean-value theorem and chain rules has been reduced; this material is now covered in one section. Local and absolute extrema and the second partials test are covered in one section rather than two separate sections as in previous editions. An optional section on exact differential equations has been moved from Chapter 18.

Double and Triple Integrals; Line and Surface Integrals (Chapter 16 and 17)

The sections on the double integral over a rectangle and the double integral over a region have been combined into one section. Some examples have been modified.

Differential Equations (Chapter 18)

First-order linear equations and separable equations have been moved to Chapter 8; exact equations are covered in Chapter 15. New material on numerical methods for first-order equations has been added. The material on second-order, non-homogeneous equations has been rewritten.

SUPPLEMENTS

Student Aids

Answers to Odd-Numbered Exercises Answers to all the odd-numbered exercises are included at the back of the text.

Student Solutions Manual, Prepared by Bradley E. Garner, University of Houston, and Carrie J. Garner This manual contains worked-out solutions to all the odd-numbered exercises and is available through your bookstore.

Instructor Aids

Instructor's Solutions Manual, by Bradley E. Garner, University of Houston, and Carrie J. Garner This manual contains solutions to all the problems in the text.

Test Bank, by Deborah Bettthausen Briff A wide range of problems and their solutions are keyed to the text and exercises sets.

Computerized Test Bank Available in both IBM and Macintosh formats, the Computerized Test Bank allows instructors to create, customize, and print a test containing any combination of questions from the test bank. Instructors can also edit the questions or add their own.

Instructor's Resource CD-ROM This CD-ROM, accessible from both IBM and Macintosh platforms, will include the Instructor's Solutions Manual and the Test Bank.

Getting Started Technology Manual Series

Two introductory tutorials offer students quick and easy access to information they need to know for their calculus courses. The authors provide multiple examples, tips, and special trouble-shooting Q&A sections to answer student questions. Maple covers releases 3, 4, and 5. Mathematica covers versions 2.0 and 3.0.

- *Getting Started with Mathematica*, by C-K. Cheung, G. E. Keough, Charles Landraitis, and Robert H. Gross of Boston College
- *Getting Started with Maple* by C-K. Cheung, G. E. Keough both of Boston College, and Michael May of St. Louis University

These introductory graphing calculator tutorials instruct students on how to fully utilize their calculators as a tool in their calculus courses. Contains multiple examples, tips, and special trouble-shooting Q&A sections.

- *Getting Started with the TI-83/82 Graphing Calculator* by Carl Swenson of Seattle University
- *Getting Started with the TI-86/85 Graphing Calculator* by Carl Swenson of Seattle University
- *Getting Started with the TI-92/92 Plus Graphing Calculator* by Carl Swenson, Brian Hopkins both of Seattle University

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Garret J. Etgen

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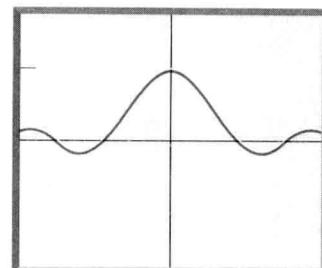
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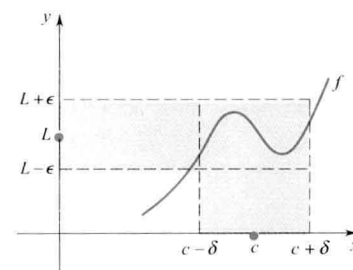
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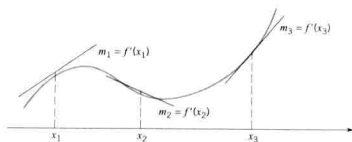


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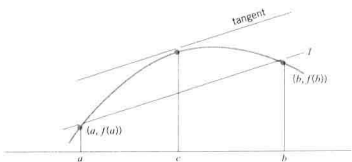




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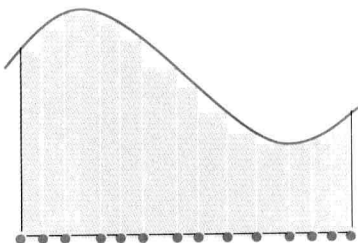
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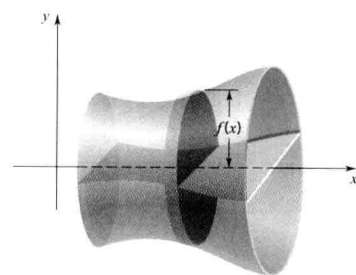
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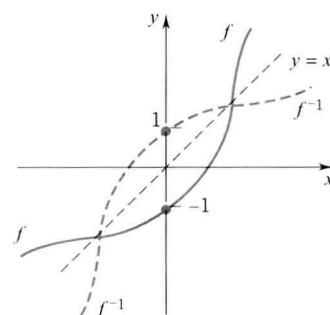
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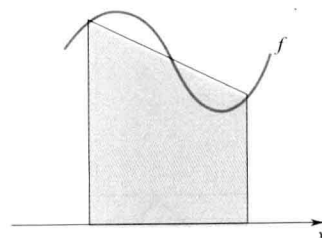
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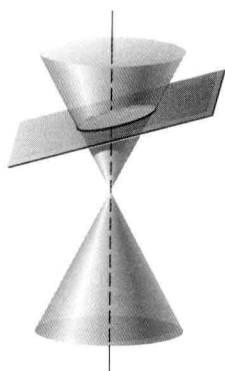
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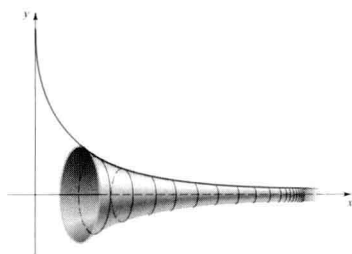
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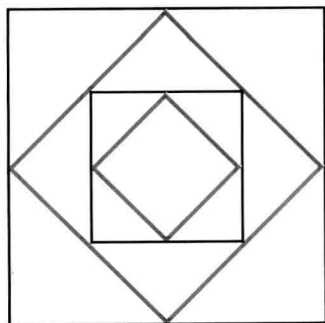
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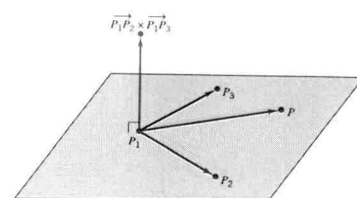
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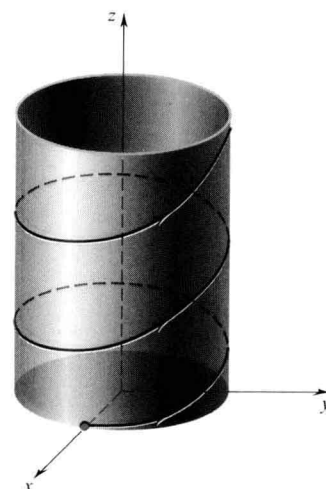
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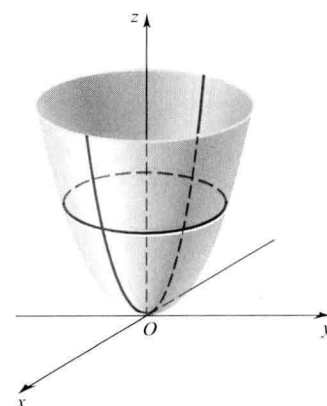
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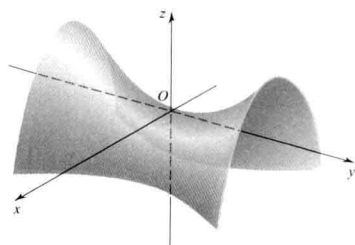
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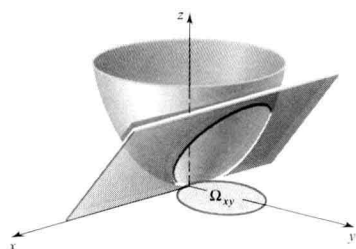
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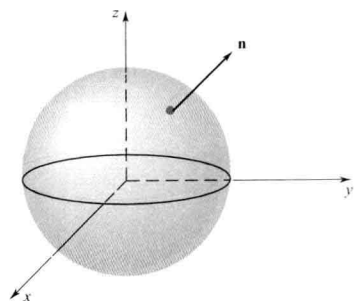
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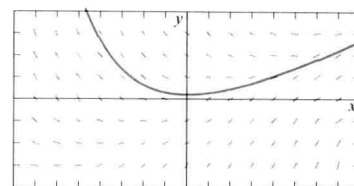
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