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# Cement-Based Composites: Strain Rate Effects on Fracture

EDITORS

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Surendra P. Shah

MRS

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# Preface

There is increasing interest in the effect of high strain rates on the mechanical properties of cementitious materials. However, the response of concrete structures, or even single elements, to impact or impulsive loading is still not understood. If we are to be able to design structures for very high loading rates, much more work will have to be done in this area; this symposium is a step in that direction.

The papers in this symposium deal with both the fundamental concepts of high strain rate loading, and with experimental studies of strain rate effects on cementitious materials. The papers in this volume, with the exception of two, were all presented orally at the Symposium. Unfortunately, two papers which were on the Symposium program were never put into manuscript form. The papers are not ordered as in the original program; rather, they have been arranged according to topic areas.

We would like to thank the sponsors of the Symposium for their financial support; the anonymous reviewers; and the Material Research Society officers and staff. We would also like to thank Mrs. Kelly Lamb for her efficient clerical support.

Sidney Mindess
Surendra P. Shah
March 1986

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*Invited Paper

*Invited Paper

# STRAIN RATE EFFECTS ON THE TENSILE STRENGTH OF CONCRETE AS PREDICTED BY THERMODYNAMIC AND FRACTURE MECHANICS MODELS

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## ABSTRACT

Models are presented and discussed which predict the influence of stress rate or strain rate on the tensile strength of concrete. Three of them are based on linear fracture mechanics and two on thermodynamics. It is shown that four models are appropriate and useful in the low to medium stress rate whereas two can predict realistic values at high stress rates. Finally a proposal is made how to apply the knowledge to engineering practice.

## 1. INTRODUCTION

The last two decades are characterized by an increasing interest in the tensile strength of concrete under high rates of loading. This is rather peculiar since the tensile strength is usually neglected in the analyses of concrete structures. But there are a few developments which justify this new interest: first, the importance of the tensile strength has been recognized by studying the bending shear behaviour of beams, the punching shear behaviour of slabs and shells, the anchorage and bond of reinforcing steel in concrete, the thermal and shrinkage cracking of concrete with continuous deterioration of structures; second, the numerical methods, such as the finite element method, require comprehensive material models including the tensile behaviour of concrete; third, fracture mechanics has penetrated the concrete field with an increasing treatment of cracks in tensile stress fields; fourth, extreme loading conditions, such as impulse and impact loads which have mostly been treated qualitatively, are now subject of quantitative assessment. The combination of these four aspects has led to considerable research efforts in the experimental field as well as in the theoretical field.

This paper deals mainly with a few theoretical approaches to the explanation of the stress or strain rate influence on the tensile strength of concrete. These theories are based on thermodynamics or fracture mechanics or both. The predictions of these theories are compared with experimental results as far as possible. It is tried to show the influence of concrete composition and temperature on the stress rate dependency.

Before going into the subject a remark should be made on the correctness of the tensile strength as a material property. In engineering terms tensile strength means the maximum attained force in a uniaxial tensile experiment divided by the initial total cross-section. That means that tensile strength is defined as an average stress which does not account for flaws and cracks and stress peaks within the material. If concrete were a really brittle material tensile strength would make no sense because specimen dimensions and flaw size would highly determine the maximum force in an experiment. However, since concrete is a strain softening material stress peaks are flatted and specimen size becomes of minor importance (in a uniaxial test; this is not true for a bending test).

Most theories which will be discussed, treat concrete on a micro scale considering flaws and particles, but come out with a prediction of the tensile strength, i.e. an average property. Some theories are concerned with a discrete crack in a homogeneous and isotropic material thus leading to a dynamic stress intensity factor or crack opening resistance.

## 2. FRACTURE MECHANICS APPROACHES

### 2.1 Linear Fracture Mechanics

Linear fracture mechanics can be applied to concrete on two ways: first, on a macro scale taking concrete as an elastic homogeneous isotropic material, and, second, on a micro scale considering concrete as a regularly flawed material. In the first case the size of the concrete element and the crack should exceed a certain minimum value which depends on the concrete grade. The order of the dimensions is meter. The second model can be a fictitious material with equidistant flaws which are physically due to pores in the hydrated cement paste and the shrinkage and thermal cracks around the large aggregate grains. It can also be a numerical model of concrete with aggregate, matrix and bond between the two constituents (Wittmann [1] calls it numerical concrete).

Macro level.

As linear fracture mechanics with one discrete crack is concerned there are analytical solutions for the stress intensity factor under dynamic loading [2]. One loading configuration shall be recalled. This is the example with a semi-infinite crack in a finite strip. At time t = 0 the faces of the strip are moved by an instantaneous displacement of magnitude $\delta_0$ which is kept constant. The crack propagation velocity v is also assumed constant. Fig. 1 gives the theoretical solution [3] in terms of normalized dynamic stress intensity factor versus crack velocity $v/c_2$ with $c_2$ the propagation velocity of a shear wave. It shows unity at v = 0 and an almost linear decay with the velocity until v = $c_R$, i.e. the Rayleigh wave speed.

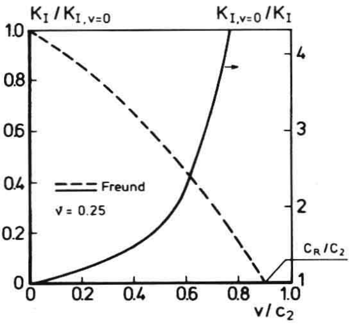



Fig. 1 Semi-infinite crack propagating in a finite strip [3]

This theoretical result may be discussed by regarding the stress waves which cause a stress intensity at the crack tip. At low crack velocities the information from the stressed body and the wake region of the crack is transferred to the crack tip more rapidly than the crack velocity. At higher crack velocities the crack faces may not move fast enough to provide the strains at the crack tip necessary for a high stress intensity factor. If the limiting value of the energy transport along the crack faces is reached, i.e. Rayleigh wave speed, the stress intensity drops to zero.

Since $K_I \sim \delta_0$ the result means that a running crack causes less stress intensity at the same displacement than a static crack. If fracture is initiated at $K_I = K_{IC}$ and $K_{IC}$ is a material property this would mean that the loading capacity is larger the greater the crack velocity is. The dashed line in fig. 1 gives an idea of such an interpretation which cannot be more than a rough indication.

Kipp et al. [4] extended the theory of constant stress to arbitrary stress loading by an appropriate use of the stress-time relation. From the special loading case of a constant strain rate $\dot{\varepsilon}_0$ and thus a constant stress rate $\dot{\sigma}_0$ in an elastic material the following relation for the stress intensity factor for a penny-shaped crack is derived

$$K_I\ (t) = \frac{4\alpha}{3\sqrt{\pi}}\ \dot{\sigma}_0\ \sqrt{c_s}\ t^{3/2} \tag{1}$$

where $\alpha$ is a geometric coefficient equal to 1.12 for the penny-shaped crack, $c_s$ the shear wave velocity and t loading time. If $K_{IC}$ is regarded as a fracture criterion a relation between strain rate $\dot{\varepsilon}_0$ and fracture stress $\sigma_c$ can be established

$$\sigma_c = \left[\frac{9\pi\ E\ K_{IC}^2}{16\alpha^2\ c_s}\right]^{1/3} \dot{\varepsilon}_0^{1/3} \tag{2}$$

This cube root law (fig. 2) holds for high strain rates and/or sufficiently large cracks. By use of the known relation between stress intensity factor and stress $K_{IC} \sim \sigma_c\sqrt{\pi a}$, it can be shown that a relation exists as

$$\alpha \approx \left[\frac{c_s K_{IC}}{E\ \dot{\varepsilon}_0}\right]^{-2/3} \tag{3}$$

which links crack length a and strain rate $\dot{\varepsilon}_0$. For average values of concrete

$$a \approx 0.1\ \dot{\varepsilon}^{-2/3} \tag{4}$$

i.e. strain rate of 1 $s^{-1}$ requires a crack length of 0.1 m, strain rate of 100 $s^{-1}$ a one of 5 mm in order to make the theory applicable.

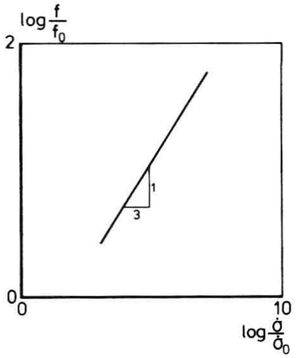



Fig. 2 Schematical prediction of strength vs. stress rate by Kipp e.a. [4]

Micro level

Linear fracture mechanics has been applied to concrete at a fictitious micro level by Weerheijm [5]. Concrete is shematized as a material containing penny-shaped cracks of single size and equal distance. Fig. 3 shows a concrete representative element with flaw diameter 2a and distance 2b.

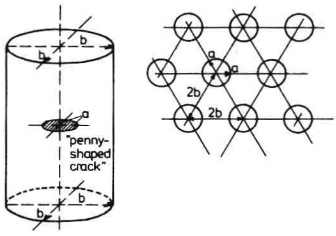



Fig. 3 Geometry of cracked concrete representative element [5]

At the beginning the ratio a/b is calculated from the pore volume n of the concrete; n accounts for the total pore volume making no difference between gel pores, capillary pores and initial shrinkage cracks. Assuming spherical pores of radius a in a fictitious sphere with radius b the ratio a/b becomes $n^{1/3}$. A concrete porosity of 10% leads to a/b = 0.46 and n = 20% leads to 0.58. This is of course a rough schematization. The absolute values of a and b are determined from the critical stress intensity factor which is taken from macroscopic experiments and a uniaxial tensile stress of 0.6 times the static tensile strength $f_t$. This means that one assumes this stress level to be a critical stress level where unstable crack propagation starts. From these two conditions a relation follows for the starting value of a. From

$$K_{IC} = \sigma\sqrt{\pi a} \ . \ f(\text{geom}, a/b) \tag{5}$$

follows with $\sigma = 0.6\ f_t$

$$a = \frac{1}{\pi}\left[\frac{K_{IC}}{0.6 f_t . f(a/b)}\right]^2 \tag{6}$$

After the initial values of a and b are known the dynamic aspect is treated by considering the kinetic energy during crack propagation.

The total energy consists of three parts, e.i.

1) the surface free energy at the crack faces;
2) the irrecoverable energy due to plastic deformation and friction;
3) the kinetic energy $E_{kin}$.

The parts 1 + 2 are equal to the fracture toughness $G_{IC}$ which is a material property. The sum of 1 + 2 + 3 is equal to the externally supplied energy $G_I$. As Weerheijm has shown $E_{kin}$ and $G_I$ depend upon stress, stress rate, initial crack length, crack velocity, Young's modulus and Poisson's ratio. From the condition

$$E_{kin} = \int_{a_1}^{a_2} (G_I - G_{IC})da \tag{7}$$

($a_1$ and $a_2$ are two states of cracking) a relation follows between stress and fracture time. For constant stress rate, the tensile strength can be calculated.

Since the mathematical formalism is complicated only a result of this study is shown in fig. 4 for a certain concrete mix: in the region between

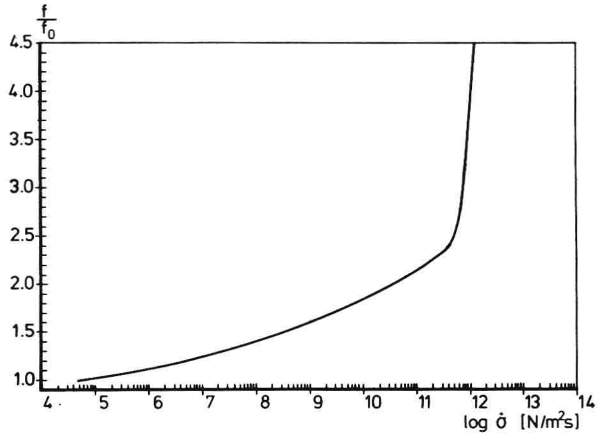



Fig. 4 Prediction of tensile strength as function of stress rate by Weerheijm [5]: $f_0$ = 5 N/mm², n = 6.4%, $K_{IC}^2$ = 0.3.10¹² N²/m³, E = 35 000 N/mm²

static testing and $\dot{\sigma} = 10^{11}$ N/m²s a slight continuous increase of strength is obvious and thereafter a steep increase within one order of magnitude of stress rate. Due to this model this increase is completely due to the fact that external energy is converted to kinetic energy. The attractive feature of this model is that the complete range of stress rates is covered by the same approach. As a matter of fact, this model is an appropriate application of Mott's [6] idea to concrete as an inhomogeneous precracked material.

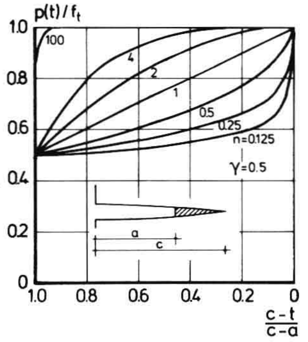
## 2.2 Non-Linear Fracture Mechanics

To apply non-linear fracture mechanics to concrete has proved to be successful in static loading. Various approaches are known the common feature of which is that they model concrete as a strain softening material. In the case of one discrete crack the strain field around the crack tip causes much higher stress than the tensile strength of concrete. The consequence is the development of a process zone ahead of the crack tip which is characterized by the Dugdale-Barenblatt model which treats the process zone as that part of the crack where cohesive stresses tend to close the crack [7,8]. Important parameters are the tensile strength and the stress crack-opening relation of concrete [9,10].

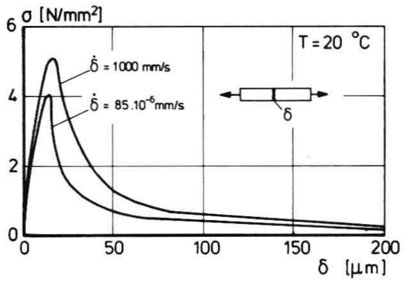
This last fact means that non-linear fracture mechanics is no means in order to establish the tensile strength but it is rather a tool in order to judge the behaviour of a cracked concrete element. If one knows the tensile strength and the strain softening behaviour as a function of loading rate one can make an estimate of a concrete element under a certain rate of loading. Let us try to illustrate this by an example. Assume a crack in a centrally cracked plate under a remote stress σ and a cohesive stress distribution p(t) in the process zone according to a power relation

$$\frac{p(t)}{f_t} = 1 - \gamma \left(\frac{c-t}{c-a}\right)^n \qquad (8)$$

with t the crack coordinate, c total crack length with process zone included, a the real crack length, γ a coefficient accounting for finite dimensions, n the stress distribution power, the size of the process zone can be calculated as function of n and $\gamma$ [9]. If $f_t$ increases with loading rate the size of the process zone decreases at the same stress, see fig. 5. That would mean that a cracked plate would behave more elastically in dynamic loading than it does in static loading. This is a qualitative statement which has not yet been treated quantitatively. Further research may be focussed on the non-linear fracture mechanics under dynamic loading in order to establish valuable knowledge as has been gained for static loading.




a)

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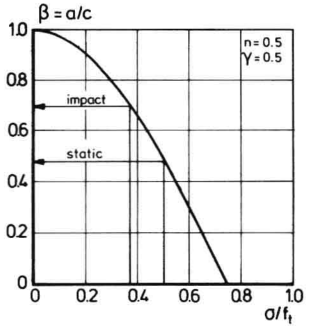


c)

Fig. 5 a) Stress distribution in process zone [9]
b) Tensile stress vs. deformation at two rates of deformation [11]
c) Process zone size as influenced by deformation rate

## 3. THERMODYNAMIC APPROACHES

To treat concrete by thermodynamics means to consider it on an atomic level. Atoms are in a state of continuous motion, attracting and repulsing forces are acting on them. Each atom is situated on a certain energy level. Due to continuous motion there is always a chance that an atom overcomes the inherent energy barrier and moves to another place in the system. If external energy is added to a system of atoms the atoms may overcome the energy barrier (activation energy) more easily. Energy can be supplied by mechanical loading, heating up or concentration gradients. The greater these external influences are the more likely that place changes occur. Place changes of atoms can be detected in an average way by deformations, cracks or chemical reactions.

Mihashi and Wittmann [12] used this approach to predict the loading rate influence on the strength of concrete. They combined the thermodynamic approach to some extent with fracture mechanics. They state that the fracture of concrete may be caused by a series of local failure processes in the phase a crack of hydration products of cement and interfaces between cement and aggregate. As soon as a failure criterion is satisfied in one part of the phase a crack is initiated. Extension of cracks and coalescence of cracks cause fracture. The concrete system consists of a group of elements linked in series, fig. 6. Each element contains a circular crack the length of which depends on the pore sizes of hardened cement paste (for the prediction of the rate influence, the absolute value of the crack length is not important). The distribution of the material defects and the characteristic properties of each element is statistically equal over the whole material.

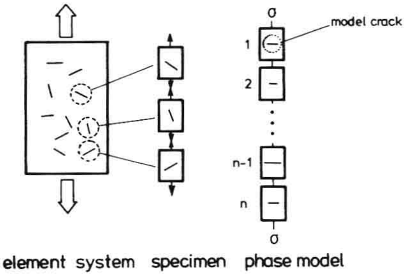


Fig. 6 Model of hardened cement with linked elements [12]

To this material schematization the rate theory is applied. The rate of crack initiation is a function of activation energy, stress and temperature. The rate of crack initiation is expressed by

$$r = \frac{kT}{h} \exp\left(-\frac{U_o}{kT}\right) (q\sigma)^{\frac{1}{n_b kT}} \qquad (9)$$

with k = Boltzmann constant, h = Planck constant, T = absolute temperature, $U_o$ = activation energy, q = local stress concentration factor, and $n_b$ = a material constant. Eq. (9) is a simple relation between crack initiation rate r and stress σ if all other parameters are taken constant:

$$r \sim \sigma^{\alpha} \qquad (10)$$

The authors calculate the mean value of probability of fracture during a time interval and end up with a relation between stress rate and tensile strength which can be simplified to