

Pijush K. Kundu

Ira M. Cohen

David R. Dowling

with contributions by P.S. Ayyaswamy and H.H. Hu



Fluid Mechanics

Fifth Edition

MULTIMEDIA FLUID MECHANICS



FLUID MECHANICS²

FIFTH EDITION

PIJUSH K. KUNDU
IRA M. COHEN
DAVID R. DOWLING



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Dedication

This revision to this textbook is dedicated to my wife and family who have patiently helped chip many sharp corners off my personality, and to the many fine instructors and students with whom I have interacted who have all in some way highlighted the allure of this subject for me.

D.R.D.

In Memory of Pijush Kundu



Pijush Kanti Kundu was born in Calcutta, India, on October 31, 1941. He received a BS degree in Mechanical Engineering in 1963 from Shibpur Engineering College of Calcutta University, earned an MS degree in Engineering from Roorkee University in 1965, and was a lecturer in Mechanical Engineering at the Indian Institute of Technology in Delhi from 1965 to 1968. Pijush came to the United States in 1968, as a doctoral student at Penn State University. With Dr. John L. Lumley as his advisor, he studied instabilities of viscoelastic fluids, receiving his doctorate in 1972. He began his lifelong interest in oceanography soon after his graduation, working as Research Associate in Oceanography at Oregon State University from 1968 until 1972. After spending a year

at the University de Oriente in Venezuela, he joined the faculty of the Oceanographic Center of Nova Southeastern University, where he remained until his death in 1994.

During his career, Pijush contributed to a number of sub-disciplines in physical oceanography, most notably in the fields of coastal dynamics, mixed-layer physics, internal waves, and Indian-Ocean dynamics. He was a skilled data analyst, and, in this regard, one of his accomplishments was to introduce the “empirical orthogonal eigenfunction” statistical technique to the oceanographic community.

I arrived at Nova Southeastern University shortly after Pijush, and he and I worked closely together thereafter. I was immediately impressed with the clarity of his scientific thinking and his thoroughness. His most impressive and obvious quality, though, was his love of science, which pervaded all his activities. Some time after we met, Pijush opened a drawer in a desk in his home office, showing me drafts of several chapters to a book he had always wanted to write. A decade later, this manuscript became the first edition of *Fluid Mechanics*, the culmination of his lifelong dream, which he dedicated to the memory of his mother, and to his wife Shikha, daughter Tonushree, and son Joydip.

Julian P. McCreary, Jr.,
University of Hawaii

In Memory of Ira Cohen



Ira M. Cohen earned his BS from Polytechnic University in 1958 and his PhD from Princeton University in 1963, both in aeronautical engineering. He taught at Brown University for three years prior to joining the University of Pennsylvania faculty as an assistant professor in 1966. He served as chair of the Department of Mechanical Engineering and Applied Mechanics from 1992 to 1997.

Professor Cohen was a world-renowned scholar in the areas of continuum plasmas, electrostatic probe theories and plasma diagnostics, dynamics and heat transfer of lightly ionized gases, low current arc plasmas, laminar shear layer theory, and matched asymptotics in fluid mechanics. Most of his contributions appear in the *Physics of Fluids* journal of the American

Institute of Physics. His seminal paper, "Asymptotic theory of spherical electrostatic probes in a slightly ionized, collision dominated gas" (1963; *Physics of Fluids*, 6, 1492–1499), is to date the most highly cited paper in the theory of electrostatic probes and plasma diagnostics.

During his doctoral work and for a few years beyond that, Ira collaborated with a world-renowned mathematician/physicist, the late Dr. Martin Kruskal (recipient of National Medal of Science, 1993) on the development of a monograph called "Asymptotology." Professor Kruskal also collaborated with Professor Cohen on plasma physics. This was the basis for Ira's strong foundation in fluid dynamics that has been transmitted into the prior editions of this textbook.

In his forty-one years of service to the University of Pennsylvania before his death in December 2007, Professor Cohen distinguished himself with his integrity, his fierce defense of high scholarly standards, and his passionate commitment to teaching. He will always be remembered for his candor and his sense of humor.

Professor Cohen's dedication to academics was unrivalled. In addition, his passion for physical fitness was legendary. Neither rain nor sleet nor snow would deter him from his daily bicycle commute, which began at 5:00 AM, from his home in Narberth to the University of Pennsylvania. His colleagues grew accustomed to seeing him drag his forty-year-old bicycle, with its original

three-speed gearshift, up to his office. His other great passion was the game of squash, which he played with extraordinary skill five days a week at the Ringe Squash Courts at Penn, where he was a fierce but fair competitor. During the final year of his life, Professor Cohen remained true to his bicycling and squash-playing schedule, refusing to allow his illness get in the way of the things he loved.

Professor Cohen was a member of Beth Am Israel Synagogue, and would on occasion lead Friday night services there. He

and his wife, Linda, were first married near Princeton, New Jersey, on February 13, 1960, when they eloped. They were married a second time four months later in a formal ceremony. He is survived by his wife, his two children, Susan Cohen Bolstad and Nancy Cohen Cavanaugh, and three grandchildren, Melissa, Daniel, and Andrew.

Senior Faculty
Department of Mechanical Engineering
and Applied Mechanics
University of Pennsylvania

About the Third Author



David R. Dowling was born in Mesa, Arizona, in 1960 but grew up in southern California where early practical exposure to fluid mechanics—swimming, surfing, sailing, flying model aircraft, and trying to throw a curve ball—dominated his free time. He attended the California Institute of Technology continuously for a decade starting in 1978, earning a BS degree in Applied Physics in 1982, and MS and PhD degrees in Aeronautics in 1983 and 1988, respectively. After graduate school, he worked at Boeing Aerospace and Electronics and then took a post-doctoral scientist position at the Applied Physics Laboratory of the University of Washington. In 1992, he started a faculty career in the Department of Mechanical Engineering at

the University of Michigan where he has since taught and conducted research in fluid mechanics and acoustics. He has authored and co-authored more than 60 archival journal articles and more than 100 conference presentations. His published research in fluid mechanics includes papers on turbulent mixing, forced-convection heat transfer, cirrus clouds, molten plastic flow, interactions of surfactants with water waves, and hydrofoil performance and turbulent boundary layer characteristics at high Reynolds numbers. From January 2007 through June 2009, he served as an Associate Chair and as the Undergraduate Program Director for the Department of Mechanical Engineering at the University of Michigan. He is a fellow of the American Society of Mechanical Engineers and of the Acoustical Society of America. He received the Student Council Mentoring Award of the Acoustical Society of America in 2007, the University of Michigan College of Engineering John R. Ullrich Education Excellence Award in 2009, and the Outstanding Professor Award from the University of Michigan Chapter of the American Society for Engineering Education in 2009. Prof. Dowling is an avid swimmer, is married, and has seven children.

About the DVD

We are pleased to include a free copy of the DVD *Multimedia Fluid Mechanics, 2/e*, with this copy of *Fluid Mechanics, Fifth Edition*. You will find it in a plastic sleeve on the inside back cover of the book. If you are purchasing a used copy, be aware that the DVD might have been removed by a previous owner.

Inspired by the reception of the first edition, the objectives in *Multimedia Fluid Mechanics, 2/e*, remain to exploit the moving image and interactivity of multimedia to improve the teaching and learning of fluid mechanics in all disciplines by illustrating fundamental phenomena and conveying fascinating fluid flows for generations to come.

The completely new edition on the DVD includes the following:

- Twice the coverage with new modules on turbulence, control volumes, interfacial phenomena, and similarity and scaling
- Four times the number of fluid videos, now more than 800
- Now more than 20 virtual labs and simulations
- Dozens of new interactive demonstrations and animations

Additional *new* features:

- Improved navigation via sidebars that provide rapid overviews of modules and guided browsing
- Media libraries for each chapter that give a snapshot of videos, each with descriptive labels
- Ability to create movie playlists, which are invaluable in teaching
- Higher-resolution graphics, with full or part screen viewing options
- Operates on either a PC or a Mac OSX

Preface

In the fall of 2009, Elsevier approached me about possibly taking over as the lead author of this textbook. After some consideration and receipt of encouragement from faculty colleagues here at the University of Michigan and beyond, I agreed. The ensuing revision effort then tenaciously pulled all the slack out of my life for the next 18 months. Unfortunately, I did not have the honor or pleasure of meeting or knowing either prior author, and have therefore missed the opportunity to receive their advice and guidance. Thus, the revisions made for this *5th Edition of Fluid Mechanics* have been driven primarily by my experience teaching and interacting with undergraduate and graduate students during the last two decades.

Overall, the structure, topics, and technical level of the *4th Edition* have been largely retained, so instructors who have made prior use of this text should recognize much in the *5th Edition*. This textbook should still be suitable for advanced-undergraduate or beginning-graduate courses in fluid mechanics. However, I have tried to make the subject of fluid mechanics more accessible to students who may have only studied the subject during one prior semester, or who may need fluid mechanics knowledge to pursue research in a related field.

Given the long history of this important subject, this textbook (at best) reflects one evolving instructional approach. In my experience as a student, teacher, and faculty member, a textbook is most effective when used as a supporting pedagogical tool for an effective lecturer. Thus my primary

revision objective has been to improve the text's overall utility to students and instructors by adding introductory material and references to the first few chapters, by increasing the prominence of engineering applications of fluid mechanics, and by providing a variety of new exercises (more than 200) and figures (more than 100). For the chapters receiving the most attention (1–9, 11–12, and 14) this has meant approximately doubling, tripling, or quadrupling the number of exercises. Some of the new exercises have been built from derivations that previously had appeared in the body of the text, and some involve simple kitchen or bathroom experiments. My hope for a future edition is that there will be time to further expand the exercise offerings, especially in Chapters 10, 13, 15, and 16.

In preparing this *5th Edition*, some reorganization, addition, and deletion of material has also taken place. Dimensional analysis has been moved to Chapter 1. The stream function's introduction and the dynamic-similarity topic have been moved to Chapter 4. Reynolds transport theorem now occupies the final section of Chapter 3. The discussion of the wave equation has been placed in the acoustics section of Chapter 15. Major topical additions are: apparent mass (Chapter 6), elementary lubrication theory (Chapter 8), and Thwaites method (Chapter 9). The sections covering the laminar shear layer, and boundary-layer theory from a purely mathematical perspective, and coherent structures in wall-bounded turbulent flow have

been removed. The specialty chapters (10, 13, and 16) have been left largely untouched except for a few language changes and appropriate renumbering of equations. In addition, some sections have been combined to save space, but this has been offset by an expansion of nearly every figure caption and the introduction of a nomenclature section with more than 200 entries.

Only a few notation changes have been made. Index and vector notation predominate throughout the text. The comma notation for derivatives now only appears in Section 5.6. The notation for unit vectors has been changed from bold \mathbf{i} to bold \mathbf{e} to conform to other texts in physics and engineering. In addition, a serious effort was made to denote two- and three-dimensional coordinate systems in a consistent manner from chapter to chapter. However, the completion of this task, which involves retyping literally hundreds of equations, was not possible in the time available. Thus, cylindrical coordinates (R, φ, z) predominate, but (r, θ, x) still appear in Table 12.1, Chapter 16, and a few other places.

And, as a final note, the origins of many of the new exercises are referenced to

individuals and other sources via footnotes. However, I am sure that such referencing is incomplete because of my imperfect memory and record keeping. Therefore, I stand ready to correctly attribute the origins of any problem contained herein. Furthermore, I welcome the opportunity to correct any errors you find, to hear your opinion of how this book might be improved, and to include exercises you might suggest; just contact me at drd@umich.edu.

David R. Dowling
Ann Arbor, Michigan
April 2011

COMPANION WEBSITE

An updated errata sheet is available on the book's companion website. To access the errata, visit www.elsevierdirect.com/9780123821003 and click on the companion site link. Instructors teaching with this book may access the solutions manual and image bank by visiting www.textbooks.elsevier.com and following the online instructions to log on and register.

Acknowledgments

The current version of this textbook has benefited from the commentary and suggestions provided by the reviewers of the initial revision proposal and the reviewers of draft versions of several of the chapters. Chief among these reviewers is Professor John Cimbala of the Pennsylvania State University. I would also like to recognize and thank my technical mentors,

Professor Hans W. Liepmann (undergraduate advisor), Professor Paul E. Dimotakis (graduate advisor), and Professor Darrell R. Jackson (post-doctoral advisor); and my friends and colleagues who have contributed to the development of this text by discussing ideas and sharing their expertise, humor, and devotion to science and engineering.

Nomenclature

NOTATION

\bar{f} = principle-axis version of f , background or quiescent-fluid value of f , or average or ensemble average of f
 \hat{f} = complex amplitude of f
 \tilde{f} = full field value of f
 f' = derivative of f with respect to its argument, or perturbation of f from its reference state
 f^* = complex conjugate of f , dimensionless version of f , or the value of f at the sonic condition
 f^+ = the dimensionless, law-of-the-wall value of f
 f_{cr} = critical value of f
 f_{CL} = centerline value of f
 f_0 = reference, surface, or stagnation value of f
 f_∞ = reference value of f or value of f far away from the point of interest
 Δf = change in f

SYMBOLS*

α = contact angle, thermal expansion coefficient (1.20), angle of rotation, angle of attack, Womersley number (16.12), angle in a toroidal coordinate system, area ratio
 a = triangular area, cylinder radius, sphere radius, amplitude

*Relevant equation numbers appear in parentheses

a_0 = initial tube radius
 \mathbf{a} = generic vector, Lagrangian acceleration (3.1)
 \mathbf{A} = generic second-order (or higher) tensor
 A, A = a constant, an amplitude, area, surface, surface of a material volume, planform area of a wing
 A^* = control surface, sonic throat area
 A_o = Avogadro's number
 A_0 = reference area
 A_{ij} = representative second-order tensor
 β = angle of rotation, coefficient of density change due to salinity or other constituent, variation of the Coriolis frequency with latitude, camber parameter
 \mathbf{b} = generic vector, control surface velocity (3.35)
 B, B = a constant, Bernoulli function (4.70), log-law intercept parameter (12.88)
 \mathbf{B}, B_{ij} = generic second-order (or higher) tensor
 Bo = Bond number (4.118)
 c = speed of sound (1.19, 15.6), phase speed (7.4), chord length (14.2), pressure pulse wave speed, concentration of solutes
 c_j = pressure pulse wave speed in tube j
 \mathbf{c} = phase velocity vector (7.8)
 c_g, \mathbf{c}_g = group velocity magnitude (7.68) and vector (7.144)
 χ = scalar stream function
 $^\circ\text{C}$ = degrees centigrade
 C = a generic constant, hypotenuse length, closed contour
 Ca = Capillary number (4.119)
 C_f = skin friction coefficient (9.32)
 C_p = coefficient of pressure (4.106, 6.32)

- C_p = specific heat capacity at constant pressure (1.14)
 C_D = coefficient of drag (4.107, 9.33)
 C_L = coefficient of lift (4.108)
 C_v = specific heat capacity at constant volume (1.15)
 C_{ij} = matrix of direction cosines between original and rotated coordinate system axes (2.5)
 d = diameter, distance, fluid layer depth
 \mathbf{d} = dipole strength vector (6.29), displacement vector
 δ = Dirac delta function (B.4.1), similarity-variable length scale (8.32), boundary-layer thickness, generic length scale, small increment, flow deflection angle (15.53), tube radius divided by tube radius of curvature
 $\bar{\delta}$ = average boundary-layer thickness
 δ^* = boundary-layer displacement thickness (9.16)
 δ_{ij} = Kronecker delta function (2.16)
 δ_{99} = 99% layer thickness
 D = distance, drag force, diffusion coefficient, Dean number (16.179)
 D_i = lift-induced drag (14.15)
 D/Dt = material derivative (3.4) or (3.5)
 D_T = turbulent diffusivity of particles (12.127)
 \mathcal{D} = generalized field derivative (2.31)
 ε = roughness height, kinetic energy dissipation rate (4.58), a small distance, fineness ratio h/L (8.14), downwash angle (14.14)
 $\bar{\varepsilon}$ = average dissipation rate of the turbulent kinetic energy (12.47)
 $\bar{\varepsilon}_T$ = average dissipation rate of the variance of temperature fluctuations (12.112)
 ε_{ijk} = alternating tensor (2.18)
 e = internal energy per unit mass (1.10)
 \mathbf{e}_i = unit vector in the i -direction (2.1)
 \bar{e} = average kinetic energy of turbulent fluctuations (12.47, 12.49)
 Ec = Eckert number (4.115)
 E_k = kinetic energy per unit horizontal area (7.39)
 E_p = potential energy per unit horizontal area (7.41)
 E = average energy per unit horizontal area (7.43), Ekman number (13.18), Young's modulus
 \bar{E} = kinetic energy of the average flow (12.46)
 \hat{E}_1 = total energy dissipation in a blood vessel
 f = generic function, Helmholtz free energy per unit mass, longitudinal correlation coefficient (12.38), Coriolis frequency (13.8), dimensionless friction parameter (15.45)
 ϕ = velocity potential (6.10), an angle
 \mathbf{f} = surface force vector per unit area (2.15, 4.13)
 F = force magnitude, generic flow field property, average energy flux per unit length of wave crest (7.44), generic or profile function
 \mathbf{F} = force vector, average wave energy flux vector
 Φ = body force potential (4.18), undetermined spectrum function (12.53)
 F_D = drag force
 F_L = lift force
 Fr = Froude number (4.104)
 γ = ratio of specific heats (1.24), velocity gradient, vortex sheet strength, generic dependent-field variable
 $\dot{\gamma}$ = shear rate
 \mathbf{g} = body force per unit mass (4.13)
 g = acceleration of gravity, undetermined function, transverse correlation coefficient (12.38)
 g' = reduced gravity (7.188)
 Γ = vertical temperature gradient or lapse rate, circulation (3.18)
 Γ_a = adiabatic vertical temperature gradient (1.30)
 Γ_a = circulation due to the absolute vorticity (5.33)

- G = gravitational constant, pressure-gradient pulse amplitude, profile function
 G_n = Fourier series coefficient
 G = center of mass, center of vorticity
 h = enthalpy per unit mass (1.13), height, gap height, viscous layer thickness, grid size, tube wall thickness
 η = free surface shape, waveform, similarity variable (8.25, 8.32), Kolmogorov microscale (12.50), radial tube-wall displacement
 η_T = Batchelor microscale (12.114)
 H = atmospheric scale height, water depth, shape factor (9.46), profile function, Hematocrit
 i = an index, imaginary root
 I = incident light intensity, bending moment of inertia
 j = an index
 J, J_s = jet momentum flux per unit span (9.61)
 J_i = Bessel function of order i
 \mathbf{J}_m = diffusive mass flux vector (1.1)
 φ = a function, azimuthal angle in cylindrical and spherical coordinates
 k = thermal conductivity (1.2), an index, wave number (7.2), wave number component
 κ = thermal diffusivity, von Karman constant (12.88), Dean number (16.171)
 κ_s = diffusivity of salt
 κ_T = turbulent thermal diffusivity (12.95)
 κ_m = mass diffusivity of a passive scalar in Fick's law (1.1)
 κ_{mT} = turbulent mass diffusivity (12.96)
 k_B = Boltzmann's constant (1.21)
 Kn = Knudsen number
 K = a generic constant, magnitude of the wave number vector (7.6), lift curve slope, Dean Number (16.178)
 K_p = constant proportional to tube wall bending stiffness
 K = compliance of a blood vessel, degrees Kelvin (16.48)
- \mathbf{K} = wave number vector, stiffness matrix
 l = molecular mean free path, spanwise dimension, generic length scale, wave number component (7.5, 7.6), shear correlation in Thwaites method (9.45), length scale in turbulent flow
 l_T = mixing length (12.98)
 L, L = generic length dimension, generic length scale, lift force
 L_M = Monin-Obukhov length scale (12.110)
 λ = wavelength (7.1, 7.7), laminar boundary-layer correlation parameter (9.44), flow resistance ratio
 λ_m = wavelength of the minimum phase speed
 λ_t = temporal Taylor microscale (12.19)
 λ_f, λ_g = longitudinal and lateral spatial Taylor microscale (12.39)
 Λ = lubrication-flow bearing number (8.16), Rossby radius of deformation, wing aspect ratio
 Λ_f, Λ_g = longitudinal and lateral integral spatial scales (12.39)
 Λ_t = integral time scale (12.18)
 μ = dynamic or shear viscosity (1.3), Mach angle (15.49)
 μ_v = bulk viscosity (4.37)
 m = molecular mass (1.22), generic mass, an index, two-dimensional source strength, moment order (12.1), wave number component (7.5, 7.6)
 M, M = generic mass dimension, mass, Mach number (4.111), apparent or added mass (6.108)
 M_w = molecular weight
 n = number of molecules (1.21), an index, generic integer number
 \mathbf{n} = normal unit vector
 n_s = index of refraction
 N = Brunt-Väisälä or buoyancy frequency (1.29, 7.128), number, number of pores in a sieve plate
 N_A = basis or interpolation functions
 ν = kinematic viscosity (1.4), cyclic frequency, Prandtl-Meyer function (15.56)

- ν_T = turbulent kinematic viscosity (12.94)
 $\hat{\nu}$ = Poisson's ratio
 O = origin
 p = pressure
 p_{atm} = atmospheric pressure
 p_i = inside pressure
 p_o = outside pressure
 p_0 = reference pressure at $z = 0$
 p_∞ = reference pressure far upstream or far away
 \bar{p} = average or quiescent pressure in a stratified fluid
 P = average pressure
 P = normalized pressure in a collapsible tube
 Π = wake strength parameter
 Pr = Prandtl number (4.116)
 \mathbf{q}, q_i = heat flux (1.2)
 q_n = generic parameter in dimensional analysis
 q = heat added to a system (1.10), volume flux per unit span, dimensionless heat addition parameter (15.45)
 Q = thermodynamic heat per unit mass, volume flux in two or three dimensions
 θ = potential temperature (1.31), unit of temperature, angle in polar coordinates, momentum thickness (9.17), local phase, an angle, angle in a toroidal coordinate system
 ρ = mass density (1.1)
 ρ_m = mass density of a mixture
 $\bar{\rho}$ = average or quiescent density in a stratified fluid
 ρ_θ = potential density (1.33)
 r = matrix rank, distance from the origin, distance from the axis
 \mathbf{r} = particle trajectory (3.1, 3.8)
 R = distance from the cylindrical axis, radius of curvature, gas constant (1.23), generic nonlinearity parameter, total peripheral resistance (16.9), tube radius of curvature
 R = viscous resistance per unit length, reflection coefficient (16.204), (16.153)
 R_u = universal gas constant (1.22)
 R_i = radius of curvature in direction i (1.5)
 \mathbf{R}, R_{ij} = rotation tensor (3.13), correlation tensor (12.13, 12.23)
 Ra = Rayleigh number (11.21)
 Re = Reynolds number (4.103)
 Ri = Richardson number, gradient Richardson number (11.66, 12.108)
 Rf = flux Richardson number (12.107)
 Ro = Rossby number (13.13)
 σ = surface tension (1.5), interfacial tension, vortex core size (3.28, 3.29), temporal growth rate (11.1), shock angle
 s = entropy (1.16), arc length, salinity, wingspan (14.1), dimensionless arc length
 σ_{ij} = viscous stress tensor (4.27)
 S = salinity, scattered light intensity, an area, dimensionless speed index, entropy
 S_e = one-dimensional temporal longitudinal energy spectrum (12.20)
 S_{11} = one-dimensional spatial longitudinal energy spectrum (12.45)
 S_T = one-dimensional temperature fluctuation spectrum (12.113, 12.114)
 \mathbf{S}, S_{ij} = strain rate tensor (3.12), symmetric tensor
 St = Strouhal number (4.102)
 t = time
 \mathbf{t} = tangent vector
 T, T = temperature (1.2), generic time dimension, period, transmission coefficient (16.153)
 Ta = Taylor number (11.52)
 T_o = free stream temperature
 T_w = wall temperature
 T_i = tension in the i -direction
 τ = shear stress (1.3), time lag
 $\boldsymbol{\tau}, \tau_{ij}$ = stress tensor (2.15)
 τ_0 = wall or surface shear stress
 v = specific volume = $1/\rho$
 u = horizontal component of fluid velocity (1.3)
 \mathbf{u} = generic vector, fluid velocity vector (3.1)

- u_i = fluid velocity components, fluctuating velocity components
 u_* = friction velocity (12.81)
 \mathbf{U} = generic uniform velocity vector
 U_i = ensemble average velocity components
 U = generic velocity, average stream-wise velocity
 ΔU = characteristic velocity difference
 U_e = local free-stream flow speed above a boundary layer (9.11), flow speed at the effective angle of attack
 U_{CL} = centerline velocity (12.56)
 U_∞ = flow speed far upstream or far away
 v = component of fluid velocity along the y axis
 \mathbf{v} = generic vector
 V = volume, material volume, average stream-normal velocity, average velocity, variational space, complex velocity
 V^* = control volume
 w = complex potential (6.42), vertical component of fluid velocity, function in the variational space, downwash velocity (14.13)
 W = thermodynamic work per unit mass, wake function
 \dot{W} = rate of energy input from the average flow (12.49)
- We = Weber number (4.117)
 ω = temporal frequency (7.2)
 $\boldsymbol{\omega}, \omega_i$ = vorticity vector (3.16)
 Ω = oscillation frequency, computational domain, rotation rate, rotation rate of the earth
 $\boldsymbol{\Omega}$ = angular velocity of a rotating frame of reference
 x = first Cartesian coordinate
 \mathbf{x} = position vector (2.1)
 x_i = components of the position vector (2.1)
 ξ = generic spatial coordinate, integration variable, similarity variable (12.84), axial tube wall displacement
 y = second Cartesian coordinate
 Y = mass fraction (1.1)
 Y_{CL} = centerline mass fraction (12.69)
 Y_i = Bessel function of order i , admittance
 ψ = stream function (6.3, 6.75), water potential
 Ψ = Reynolds stress scaling function (12.57), generic functional solution
 $\boldsymbol{\Psi}$ = vector potential, three-dimensional stream function (4.12)
 z = third Cartesian coordinate, complex variable (6.43)
 ζ = interface displacement, angular tube-wall displacement, relative vorticity
 Z = impedance (16.151)