Introduction to PROBABILITY and RANDOM PROCESSES

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To my mother and father
Haydée Prieto and Fernando Auñón

To my beloved wife
Margaret

To my daughters
Christine, Melissa, Serena, and Maria

J.A.

I dedicate this book
to my "GURUS"
(all my teachers).

V.C.

Preface

This book presents an introduction to the topics of statistics, random variables, and random processes. It is intended for junior and senior engineering students, and offers a unique, practical approach to the subject. With a carefully selected blend of both theoretical and real-data examples, it connects the classroom to real-world problem solving, and uses the computer to explore the subtleties of probability theory and its applications. The book includes a large number of computer examples using both Mathcad[1] and MATLAB.[2] It also contains traditional end-of-chapter problems. Rather than

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emphasizing programming sophistication, the book shows how the computer can be used as a live interface to visualize problem solving. The use of Mathcad and MATLAB demonstrate a natural progression through the solution of a problem. Students are able to change parameters in equations, or change entire equations and have results immediately available for their use. The computer examples available with each chapter are *not* designed as Mathcad or MATLAB tutorials. They are written in a fairly simplistic manner in order that students will be able to modify them to solve classroom type problems.

Chapters 1 and 2 present a general introduction to probability theory and to the basic statistical properties of a random variable. Chapter 3 departs from traditional textbooks by introducing students to the computer generation of gaussian-distributed random variables. The run test and the chi-square test are explained to test the equivalence of a probability density function of experimental data to a theoretical density function. Both Mathcad and MATLAB computer examples are presented at the end of the chapter.

Chapter 4 introduces the concept of random processes and the autocorrelation function. The coverage includes estimating the autocorrelation function of records of limited duration. Following the book's central pedagogical strategy, this section begins with an explanation of theory and a working, theoretical example. Then, in order to make the transition between a theoretical result and its practical use, real time-limited data is presented and analyzed.

Chapter 5 introduces students to the concept of the spectral density of a random process, and explores a number of techniques for the estimation of spectral density. First, the spectral density is estimated as the Fourier transform of the autocorrelation function. The autocorrelation function of a time-limited record is estimated using the FFT, and then the Fourier transform of the autocorrelation is found. The chapter also explores a second technique, the periodogram. In this technique, the data is divided into overlapping segments; the computation of the Fourier transform of each segment; and the average of all segments obtained. The model-based approach is the third technique explored. It exposes students to the theory that a random process may be described as an autoregressive process. The order of the model and coefficients of the model are found, and the estimate of the spectral density is calculated. A large collection of Mathcad and MATLAB computer examples are presented at the end of the chapter.

Chapter 6 introduces the analysis of linear systems when their inputs are random processes. It also considers the special case of matched filters. Chapter 7 presents a wide variety of useful applications, ranging from biomedical systems to radar systems. Finally, the Appendix provides a concise yet complete presentation of the fast Fourier transform (FFT).

In closing, we would like to thank the reviewers who aided in the development of the book: Dr. Ronald A. Iltis, University of California at Santa Barbara; Dr. V. Krishnan, University of Massachusetts at Lowell; Dr. Steven A. Tretter, University of Maryland; and, Jitendra K. Tugnait, Auburn University. We would also like to thank our editor, Lynn B. Cox, and editing supervisor, Terri Wicks.

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CHAPTER 1

Introduction to Probability

1.0 BASIC CONCEPTS OF PROBABILITY

The term *probability* is intimately involved with the term *uncertainty*. We say that it is improbable that there is life on the moon, that there is a 25 percent chance of rain tomorrow afternoon, etc. Practically, we are often confronted with situations where the exact truth is not known. We take chances and assume that a particular situation is likely to occur. Weather is a good example; the accurate prediction of the path of a hurricane has enabled many cities to adequately prepare for inclement weather. A tornado, however, is much more unpredictable, often having disastrous consequences. We are faced in everyday life with statements that have associated with them the word *probable*. For example, the probability of rolling a 1 using a fair die is 1/6; the probability of drawing a king of hearts out of a complete deck of cards is 1/52; etc. The first statement, addressing the probability of a 1's occurring when we toss a die, makes use of a number of assumptions. For example, it is assumed that the die is "fair"; we are also not counting other probable occurrences such as the loss of the die when it is tossed. It is usually assumed that when we toss a die, each side is equally likely to appear. One way of obtaining the number 1/6 is to toss the die many times and simply count the number of times in which a 1 appears. This concept leads to the well-known ratio

$$\frac{\text{Number of favorable occurrences}}{\text{Number of times the die was tossed}}$$

By evaluating this number we are actually evaluating the probability of a 1's occurring. This basic (and historic) definition of probability assumes that all possible outcomes, such as the occurrence of a specific face in the tossing of a die, are equally probable. Using this ratio, we conclude that every probability P is a number between 0 and 1:

$$0 \leq P \leq 1$$

Note that for P to be equal to 1, the numerator and denominator of the above ratio must equal each other. In such a case, when the probability of the event equals 1, we have a *certain* event. The relative frequency approach is essentially a counting approach. Basically, we are going to perform the experiment *many* times and then count the number of times that the particular event we are interested in occurs. For example, consider again the experiment of tossing a die. There are a number of possible events associated with this experiment. Let us mention just a few:

A 6 comes up.
An even number comes up.
A number less than 4 comes up.
A number between 1 and 6 (inclusive) comes up.

The fourth event is an interesting one, since there is nothing random associated with its outcome. Whenever we toss a die, the number *will be* between 1 and 6 (inclusive). This is what we call a certain event. Note that a certain event is not a random event, since we know ahead of time that it will occur. Therefore, the probability associated with this event is 1.

For any of the other events, the discrete probability associated with each may or may not be easy to calculate. The relative frequency approach provides us with a way to arrive at the probability associated with specific events. As mentioned earlier, this approach is essentially a counting approach. Suppose that we are interested in the probability that a 6 comes up when we toss a die. We perform the following experiment: We toss the die 1000 times and count each time that a 6 comes up. Then we approximate the probability associated with the event "a 6 comes up" as follows:

$$P(6) \approx \frac{\text{number of times a 6 appears}}{\text{total number of tosses}} = \frac{N_6}{N_{\text{trials}}} \tag{1.1}$$