

Paul
Dimo

NODAL
ANALYSIS
of POWER
SYSTEMS

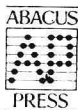
Nodal analysis of power systems

Paul Dimo

Nodal analysis of power systems



Editura Academiei
București
România



Abacus Press
Tunbridge Wells, Kent
England

1975

Revised and enlarged from
L'Analyse nodale des réseaux d'énergie
Eyrolles, 1971

Translated from the French by
Radu Georgescu

Translation edited by
John Hammel, Ph. D.
Chelsea College, London

This English edition first published in 1975
under the joint imprints of

EDITURA ACADEMIEI
str. Gutenberg 3 bis, Bucureşti, România
and

ABACUS PRESS
Abacus House, Speldhurst Road, Tunbridge Wells, Kent, England

© Abacus Press 1975

All rights reserved. No part of this publication may be reproduced,
stored in a retrieval system, or transmitted, in any form or by any
means, electronic, mechanical, photocopying, recording or otherwise,
without the prior permission of the publishers.

ISBN 0 85626 001 0

PRINTED IN ROMANIA

Contents

Author's foreword to the English edition	15
Notations and terms	19

1. Introduction

Aims and concepts of nodal analysis of power networks

1.1. Is it possible to use figurative concepts in the analysis of modern networks by digital computers?	21
1.2. Intuition and imagination of the operator in his dialogue with computers. The part played by diagrams	22
1.3. Figurative concepts of nodal analysis	23
1.3.1. First figurative concept, the REI net	23
1.3.2. The second figurative concept, the nodal image	23
1.3.3. Other associated concepts	24
1.4. Scope of nodal analysis	24

2. Physical nature of electric networks and their representation in nodal analysis

2.1. Linear part	26
2.1.1. Three-phase networks considered in the analysis as single-phase systems	26
2.1.2. Branches without mutual influences	27
2.1.3. Linear part of the network	27
2.2. Non-linear part	28
2.3. The mechanical part which generates or uses energy transmitted by the network	29
2.4. Essential part played by the non-linear branches and the "neutral node" in network configuration	30
2.5. Variation range of nodal voltages and node currents	30
2.6. Functional relations between nodal voltages and load currents	31
2.7. Initial configuration of the network in nodal analysis	31
2.7.1. Network scheme reduced to a single voltage, without coupling	31
2.7.2. Physical topology of the schemes used in nodal analysis	32

3. The REI net

3.1. Nodal equations and the representation of physical magnitudes in the special case of power networks	34
3.2. Synthesis of the (REI) net of the radial (R) type, equivalent (E) for a node and independent (I) of the rest of the network.	37
3.2.1. Successive transformations of circuits	37
3.2.2. Elimination of nodes by the Gauss method	38
3.3. "One-to-one" correspondence between the reduced nodal equations and the geometry of the equivalent networks (REI), a general method for the determination of schemes	39
3.4. Different types of REI nets classified according to the variation laws considered in the operations of reduction	42
3.4.1. REI net obtained by the linearization of the eliminated nodes	42
3.4.2. REI nets obtained by considering constant current in the eliminated nodes	42
3.4.3. REI nets obtained by considering constant active powers at the eliminated nodes	43
3.4.4. Mixed REI nets	44
3.4.5. Generalized use of the REI net obtained by the linearization of the eliminated nodes	45
3.5. Information obtained from an REI net	46
3.5.1. Reference state for which the REI nets have been constructed	46
3.5.2. Number of branches retained in the final net. The zone which affects the node under investigation	46
3.5.3. The admittance added between the investigated and the earth node	47
3.6. Recommended topology for REI nets	48

4. The nodal image

4.1. The nodal image: diagram of short-circuit currents	49
4.1.1. Information on currents	51
4.1.2. Information on the contribution of each generator to the total load	51
4.1.3. Information on voltages	53
4.2. Generalization of the theorems of J. Fallou	53
4.3. Networks extended to the internal reactances of generators; their REI nets and nodal images	54
4.3.1. Extension of networks over synchronous reactances	54
4.3.2. Extension of networks over transient reactances	55
4.3.3. Extension of networks to reactance equivalents to a definite regulation	55
4.3.4. Passage from the real network to different types of extended networks and the construction of REI nets and their images	55

5. Characterization of structures and operating conditions by nodal images

5.1. Visualization of the interdependence between the nodes of the network. The network "seen" from the nodes	56
5.2. Equivalent generators	56

5.2.1. Reduction of the number of branches of the REI net by the introduction of equivalent generators	58
5.2.2. Influence of certain parts of a network shown by the establishment of equivalent generators	58
5.3. Information on the voltages of the represented nodes and their interdependence. Direct reading of voltages on the nodal image	59
5.3.1. Measure of the node voltage	59
5.3.2. Interdependence between the voltage of the node investigated and the voltages of the nodes retained in the REI nets at the end of the radial branches	61
5.3.3. Voltage regulation by synchronous condensers	64
5.4. Extension of networks over the internal resistances of generators for the study of structures and operating conditions	65

6. Static stability of the network examined with the help of nodal images

6.1. REI net of "generator" nodes for the study of the static stability: net of reactances	67
6.2. Nodal image of the REI net of reactances in problems of static stability	68
6.2.1. Introduction in the image of a load assimilable to a synchronous motor	69
6.2.2. Introduction in the image of a synchronous motor representing the rest of the system	69
6.2.3. Introduction in the nodal image of the variation law of the load	70
6.3. Laws applied to the loads in problems of static stability	72
6.4. Criterion of the reactive power: dQ/dU	72
6.4.1. Formula of the dQ/dU criterion as a function of the components of the short-circuit currents	73
6.4.2. Direct reading of the criterion of reactive power on the nodal image . . .	74
6.5. Deterioration of operating conditions: "visualization" on the nodal image of the deterioration process	75
6.6. Some particular aspects of nodal images in problems of static stability . . .	76
6.6.1. Classical case of the generator-synchronous motor. The classical limiting angle of 90°	76
6.6.2. Co-linear vectors. Characterization of the optimal situation from the point of view of static stability and voltage regulation of a node	77
6.6.3. The characteristic angle of 45° for aligned vectors. Erroneous indications of equivalent generators	78
6.6.4. Geometric loci of nodal images which may be used in the deterioration of the operating conditions	79
6.7. Principal factors of static stability. Fundamental importance of the ways of deterioration	81
6.7.1. The variations occur only in the REI net	81
6.7.2. Essential part played by the deterioration of operating conditions in problems of static stability	81
6.8. Examples of static stability on nodal images. Intuitive "visualization" of behaviour	82

6.8.1. Effect of a synchronous condenser on static stability	83
6.8.2. Effect of static condensers	85
6.8.3. Deterioration of the operating conditions under constant voltage by changing the distribution of active powers between generators	85
6.8.4. Deterioration of operating conditions under variable voltage. "Critical" voltage of the node under consideration	86
6.8.5. Deterioration of operating conditions by the variation of the reactive power according to an arbitrary law	89
6.9. Conclusions. Scope of studies on static stability by the use of nodal images	91

7. Transient stability of the power system examined by REI nets and their nodal images

7.1. Present conception of problems of transient stability criticised	94
7.1.1. Absence of a method for the selection of the cases examined by numerical calculation	94
7.1.2. Absence of hierarchical criteria assigned to the results of numerical calculation	94
7.2. Principles and procedures for the constitution and transformation of the REI nets and their nodal images, after a disturbance followed by a transient state.	96
7.2.1. REI nets for visualizing transient states	96
7.2.2. Principles of the construction of nodal images during disturbances. "Dynamic" diagrams	97
7.2.3. Representation of structural changes. Nodal image containing the representation of the two states: before the disturbance (time 0^-) and after it (time 0^+)	99
7.2.4. Representation of vectors. Representative nodal image of a transient state for the same structure of the network	103
7.2.5. Nodal images of transient states corresponding to several structural changes	107
7.3. REI nets during transient states obtained with the help of a digital computer	109
7.3.1. Algorithm before the introduction of the disturbance	109
7.3.2. Algorithm for establishing the REI net after a disturbance	111
7.3.3. Algorithm for obtaining REI nets during transient states involving several structural changes of the network	113
7.4. Nodal images obtained during transient states with the help of the digital computer	114
7.4.1. Calculation of short circuits and their components	114
7.4.2. Position of the axes and determination of the vector movements by means of the computer	115
7.5. Equivalent generator in the calculations of transient states. The equivalent representative vector	116
7.5.1. Equivalent generator for the first calculation step	116
7.5.2. Equivalent generator in the transient state following the first calculation step	118
7.5.3. Index of transient states. Its use in establishing equivalent generators	120

7.6. Nodal images of reactance REI nets and their range of application for "visualizing" network states. Use of impedance REI nets	121
7.6.1. Two methods for passing from an impedance REI net to a reactance REI net	122
7.6.2. Additional precaution to be taken for the validity of the equivalent generator during the transient states of the network	124
8. Structure, state and behaviour characteristics of a network derived from REI nets and their nodal images	
8.1. Characteristics of structures "seen" from the nodes	126
8.1.1. Information about the structure	126
8.1.2. Information about the homogeneity of the network	126
8.1.3. Transient overvoltages resulting from the absence of homogeneity of the network.	127
8.1.4. Co-linear position of the vectors representing a perfect homogeneity of the network seen from the respective node	128
8.2. Characteristics of operating conditions concerning static stability and voltage regulation	128
8.2.1. "Natural" static stability of the network	129
8.2.2. "Artificial" static stability of the network	130
8.2.3. The dQ/dU criterion characterizing voltage regulation, as an absolute and relative index. Choosing the degraded state	131
8.3. Selection of the location of dangerous disturbances	132
8.3.1. Selection of the location of dangerous disturbances by the visualization of the nodal images of the resulting transient states	133
8.3.2. Selection of the location of dangerous disturbances by using the dynamic index $\Delta p/M$ and a single aspect of the nodal image	137
8.4. Correlation between static and transient stability	139
8.5. General organization of the studies concerning the service reliability of a network as regards stability	140
8.5.1. Succession of studies.	141
8.5.2. Numerical method	141
8.5.3. Particular aspects of unstable states by considering the spontaneous recovery of synchronism between generators	141
8.6. Studies on nodal images for the characterization of network behaviour. Examples	142
8.6.1. Selection of operating conditions from the viewpoint of static stability	142
8.6.2. Selection of the most unfavourable location of faults for transient stability	144
8.6.3. "Visualization" of the transient behaviour of the network for one of the selected cases	145
8.7. Remedies to be applied in dangerous cases deduced from nodal images. Symmetric and asymmetric faults	152
8.7.1. Symmetric faults.	152
8.7.2. Asymmetric faults	155
8.8. Conclusion. Use of the numerical method associated to the characteristic geometry of nodal images	157

9. The REI equivalent of a power system (the equivalent for a network as a whole)

9.1. The REI equivalent of the system for the extraction of the REI net of a node	159
9.2. The final REI equivalent	161
9.3. REI equivalent of a system containing a single load	163
9.3.1. Single-load REI equivalent deduced from the final equivalent	163
9.3.2. Single-load REI equivalent deduced from the initial network	166
9.4. Other types of REI equivalents of a system containing fictitious nodes	169
9.4.1. REI equivalent of the system with a single generator	170
9.4.2. REI equivalents of a system containing a certain number of load nodes or fictitious generator nodes	171
9.4.3. REI equivalent of a system reduced to two fictitious nodes	171
9.5. Validity for states other than the reference state of REI equivalents containing fictitious nodes	173
9.5.1. General method for building a fictitious node. The zero power balance network	173
9.5.2. The law of the homothety of currents included in the general method for building a fictitious node	174
9.5.3. Properties of the equivalents resulting from the introduction of the zero power balance network	174
9.6. Applications of REI models	175
9.6.1. Applications of the REI equivalent of the system built for the extraction of an REI net	176
9.6.2. The network structure characterized by the "final equivalent"	176
9.6.3. Network structure characterized by the "single-load equivalent"	178
9.6.4. Network structure characterized by a "two-node equivalent"	179
9.7. Partial study of a network on REI equivalents	179
9.8. Calculation of losses and formulae with constant coefficients (B) obtained with the help of the single-load equivalent. Economical operations of power plants by the consideration of losses	180
9.8.1. The nodal matrix [z] generally used in nodal equations from which the coefficients B are determined	181
9.8.2. Use of the matrix [y] and of the single-load equivalent	182
9.8.3. Approximate loss formulae deduced from the reference state. Example of calculation	184
9.8.4. Successive corrections of loss formulae indicated by the single-load equivalent	189
9.8.5. Examples of successive corrections of loss formulae with constant coefficients B	190
9.8.6. Corrections for obtaining a different series of coefficients B corresponding to a different total power	194
9.8.7. Conclusions concerning new ways of obtaining loss formulae	196
9.9. Information and suggestions resulting from single-load equivalents. REI net of the equivalent node	197

9.9.1. Information supplied by the admittances of the branches, the circulation of currents and the localized losses	197
9.9.2. Nodal image of the single-load equivalent node	198
9.9.3. Co-linearity of the vectors of the nodal image and of the single-load equivalent node. The ideal network	199

10. Specific algorithms of nodal analysis

10.1. Iteration method of the short-circuit currents for determining the operating conditions	202
10.1.1. Additional information obtained once the operating conditions have been established	204
10.1.2. Results and possible variant forms of the method	204
10.2. Determination of REI models by the method of multiple eliminaticns . .	207
10.2.1. Mathematical model for the determination of a single REI net	208
10.2.2. Elimination algorithm for obtaining a single REI net	209
10.2.3. Multiple eliminations for obtaining the REI net of an arbitrary node of a network	210
10.2.4. Results obtained by the method of multiple eliminations	213
10.3. Establishment of a unitary programme	215

11. Peripheral specialized devices

11.1. Specialized peripheral equipment for obtaining the nodal image	218
11.2. Nodal image obtained in real time for very large modern power plants . .	220
11.2.1. Information for the characterization of the operating conditions obtained at the operator's request	222
11.2.2. Information obtained at the operator's request for unstable hypothetical states starting from the state in real time represented on the screen of a high power generator	223
11.2.3. Information which may be obtained from known operating conditions in real time on the correlation between large size units and neighbouring power plants of the external network	226
11.2.4. Transient states and their effects examined on screens at the operator's request	227
11.3. Possibility of "visualization" starting from known operating conditions in real time. Behaviour of the network as a whole	229

12. Vast interconnected power systems and state indicators

12.1. Need of reduced representative models of large power systems	230
12.1.1. The Happ — Kron model	231
12.2. Reduced equivalent networks which maintain a correspondence with the nature of power systems: the "Daisy" and "REI" graphs	232
12.2.1. Characteristic elements pertaining to the nature and particular topology of power systems	232

12.2.2. Daisy graphs and their interconnection	233
12.2.3. The REI equivalents in interconnection problems	234
12.3. Single-load REI equivalents in interconnection problems	235
12.3.1. Useful characteristics.	235
12.3.2. Interconnection of REI equivalents	236
12.3.3. Multicomputer calculation of single-load REI equivalents of sub-networks	239
12.4. Various variants for determining the states of the interconnection on REI models	239
12.4.1. Torn boundary nodes and the integration of lines into the REI models of sub-networks	239
12.4.2. Double integration of interconnection lines in the two REI equivalents of two sub-networks	241
12.5. The most simple REI models	242
12.5.1. The importance of approximations and of dividing the computation between "off-line" and "on-line"	242
12.5.2. The most simple REI equivalents in problems of power flow on interconnection lines. "Gauging" the REI equivalent in such problems	243
12.5.3. The most simple REI equivalents in problems of economical power distribution between sub-networks	245
12.5.4. The most simple REI equivalents in regulation problems between sub-networks	249
12.5.5. Transformation of REI models following topological changes	250
12.5.6. Single-load REI equivalents or other REI equivalents in problems of load-flow	250
12.6. State "indicators" of REI models	251
12.6.1. New technique of indicators	251
12.6.2. Indicators of the consequences of a change of topology	252
12.6.3. The REI net of the single-load node in the establishment of indicators	252
12.6.4. Representation of boundary nodes for establishing the REI models of sub-networks	254
12.6.5. Indicators supplied by the single-load node	256
12.7. A new philosophy of stability problems? Consistency indicators	259
12.7.1. Indicators limited to the available information	259
12.7.2. "Consistency" indicators	261
12.8. Gauging and selecting the most reduced models that can be used "on-line". Example: checking the reduced models for the calculation of the coefficients of the synchronizing power ($dp/d\delta$)	261
12.9. General and unitary range of application of REI models to large interconnected networks	268
12.9.1. Structural correspondence between REI models and power systems	268
12.9.2. A reduced number of routine operations for obtaining REI models of various types	269
12.9.3. A single reduced model which can be adapted to any problem	271

APPENDIX

“REI” graphs and their nodal images (generalized nodal analysis)

A.1. REI graphs	273
A.2. “REI” net. Particular case of REI graphs	273
A.3. Obtainment of REI graphs in the general case	275
A.3.1. Information supplied by the REI graphs	278
A.4. Nodal image	278
A.5. Visualization of system states	279
A.5.1. Optimum and restrictions: loci of “nodal images” or diagrams leading to a certain geometry	279
A.5.2. The new type of man-computer dialogue in the study of linear systems . .	280
A.6. Example. Visualization with the aid of nodal analysis of the results of an economic contract	281
A.6.1. REI graph of the dependent variable	282
A.6.2. Nodal image. The most simple variant form by using an axis	282
A.6.3. Nodal image. Variant form using two axes	283
References	286

Author's foreword to the English edition

In 1958 a new method was presented under the title "Graphic Analyser" at the CIGRE conference [16] (Report N° 302, 1958); the new method had been already used by various Romanian research institutions (Institute of Energetics of the Romanian Academy and the I.S.P.E. Institute of the Department of Energy) with excellent results. A first monograph describing the new graphic method was published in 1959. It contained the unitary solutions which the new method supplied for problems of power system [17].

The method in its initial form was applicable to a network with a radial configuration. It was perfectly applicable to any network containing a reduced number of loops which would be always transformed in a radial equivalent network for the *study of a given node*. The behaviour of the central node of the radial network was then analysed with the help of a standard diagram — which we have subsequently called the "nodal image" — for any evolution of the state of the network.

The diagram was established by a vector representation of the components of the current at the central node. The diagram permitted complete graphic calculations for any state of the system, starting from a first representation effected for a reference state.

A new general and unitary (graphic) method which permitted in addition the characterisation of the state of a network by the simple inspection of a diagram was thus found for all the special problems concerning power systems (regulation, static and transient stability, etc.) and for which the literature gave only particular methods adapted to special problems.

We have proposed for the application of this method a mechanical device made up of small hinged rules known as a "Graphic Analyser" [16] and by means of which one can characterize any state of the network, without calculations, by simply varying the position and length of the rules.

The device, brought up to date and at present known as an "Anagraph", constitutes a peripheral device of the computer (see chapter 11 and references [29] and [41]).

The many computations effected by this graphic method have proved the possibility of introducing direct and simple reasoning and even intuition into problems whose complexity and large number of elements seemed to preclude rapid and intuitive comprehension of the physical phenomena.

But it is only after the advent of digital computers that the method underwent a spectacular evolution. In order to retain the possibility of direct reasoning and intuitive representation offered by graphical methods we have associated to the new methods of numerical calculation the graphical representation of two standardized “graphs”: an equivalent radial net which we have denoted by “REI” and which corresponds to the radial net of the graphical method, and a standardized diagram which we have called the “Nodal Image” and which corresponds well to the standard diagrams established by using the components of short-circuit currents.

The method was presented in 1944 at a CIGRÉ conference under the title “Simplification of network problems by the introduction of a new concept: the equivalent REI net and its image” [24].



Now — with the presentation of this work which follows the publication in other languages* of a number of monographs — it may be stated that “Nodal Analysis” and its new “philosophy” has acquired a certain degree of maturity [52], [85].



In this “maturity” stage where its methods are closely dependent on the revolution introduced by computers, the “Nodal Analysis” may be considered as an attempt to strengthen man’s position in his relations with the computer. Towards the year 2000 the relation man-computer will certainly be one of the important components of man’s activity. But at present this component is not completely defined, not even for the near future. Some authors have pointed out our ignorance concerning the trend of this new development.

Others [62] have remarked that if it can be proved that the computer cannot reproduce or transcend certain specifically human characteristics, *this will be one of the most important discoveries of the 20th century.*

On the new road of information theory, electronics and automation one may already distinguish the main lines of a dramatic difficulty namely that of “*not understanding the object of our investigation or inability to grasp the whole problem wherein it is placed, the absence of accessibility*”. This aspect has been recently underlined in an essay which tried to outline the future characteristics of the human brain activity [70], in a world where presumably the computer will be the master.

“Nodal Analysis” may be considered as an *endeavour to mitigate this adversity, to restore man in his traditional position of master and active participator* with his intuition and judgement in the processes which he oversees or analyses.



In “Nodal Analysis” the computer plays an auxiliary part. It only transforms an extended and complex system into a simple and standardized model (REI model) which the human mind comprehends easily. A simple diagram, also standardized,

* P. Dimo. *Nodal Analysis of power systems* (in Romanian), Editura Academiei, Bucharest, 615 p., 1968.

P. Dimo, *Analyse nodale des réseaux d'énergie*, Editions Eyrolles, Paris, 269 p., 1971.

P. Dimo, *Uslovy Analis Elektrichesky Sistem*, Edition MIR, Moscow, 264 p., 1973.