Structural Analysis of Regular Multi-Storey Buildings

KAROLY A. ZALKA

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Notations

CAPITAL LETTERS

A	cross-sectional area; area of plan of building; floor area; corner point
A_a	area of lower flange
A_b	cross-sectional area of beam
A_c	cross-sectional area of column
A_d	cross-sectional area of diagonal bar in cross-bracing
A_h	cross-sectional area of horizontal bar in cross-bracing
A_f	area of upper flange
A_g	area of web
A_o	area of closed cross-section defined by the middle line of the walls
B	plan breadth of the building (in direction y); constant of integration
B_l	local bending stiffness for sandwich model
B_o	global bending stiffness for sandwich model
C	centre of vertical load/mass; centroid; constant of integration
D	constant of integration
E	modulus of elasticity; constant of integration
E_c	modulus of elasticity of columns; modulus of elasticity of concrete
E_d	modulus of elasticity of diagonal bars in cross-bracing
E_h	modulus of elasticity of horizontal bars in cross-bracing
E_s	modulus of elasticity of steel
E_w	modulus of elasticity of shear wall
F	concentrated load (on top floor level); resultant of horizontal load
F_{cr}	critical concentrated load (on top floor level)
$F_{cr,\varphi}$	critical load for pure torsional buckling (for concentrated top load)
F_g	full-height (global) bending critical load (for concentrated top load)
F_l	full-height (local) bending critical load (for concentrated top load)
F_t	Saint-Venant torsional critical load (for concentrated top load)
F_ω	warping torsional critical load (for concentrated top load)
G	modulus of elasticity in shear
(GJ)	Saint-Venant torsional stiffness
$(GJ)_e$	effective Saint-Venant torsional stiffness
H	height of building/framework/coupled shear walls; horizontal force
I	second moment of area
I_{ag}	auxiliary constant
$I_{\omega g}$	auxiliary constant
I_b	second moment of area of beam
I_c	second moment of area of column

I_f sum of local and global second moments of area
I_g global second moment of area of the columns of the framework
$I_{g\omega}$ global warping torsional constant
I_o polar second moment of area
I_x, I_y second moments of area with respect to centroidal axes x and y
I_{xy} product of inertia with respect to axes x and y
I_w second moment of area of wall
I_ω warping (bending torsional) constant
J Saint-Venant torsional constant
$\overline{J}$ supplementary Saint-Venant torsional constant
K shear stiffness of framework; shear critical load
K^* shear stiffness/shear critical load of coupled shear walls
K_b full-height global shear stiffness; global shear critical load
K_b^* full-height global shear stiffness/shear critical load of coupled shear walls
K_c local shear stiffness related to the columns; local shear critical load
K_d shear stiffness representing the effect of the diagonal bars in cross-bracing
K_e effective shear stiffness
K_h shear stiffness representing the effect of the horizontal bars in cross-bracing
L width of structure; plan length of building (in direction x)
M bending moment
M_i concentrated mass at the ith floor level
M_t torsional moment
N total applied uniformly distributed vertical load; normal force
N_{cr} critical load
$N_{cr,x}$ critical load for sway buckling in direction x
$N_{cr,y}$ critical load for sway buckling in direction y
$N_{cr,\varphi}$ critical load for pure torsional buckling
N_f local bending critical load of framework
N_h homogeneous solution
N_g full-height global bending critical load
N_l full-height local bending critical load
N_p particular solution
N_t Saint-Venant torsional critical load
N_w local bending critical load of shear wall
$N_{y\varphi}$ coupled sway-torsional critical load
N_ω warping torsional critical load
$N(z)$ total vertical load at z
O shear centre
Q uniformly distributed floor load per square metre
S lateral stiffness; shear stiffness for sandwich model
S_ω torsional stiffness

SMALL LETTERS

a length of wall section; stiffness ratio
$\overline{a}$ stiffness ratio for a system of bracing units
a_i local bending stiffness ratio
a_0, a_1, a_2 coefficients for cubic equation

b length of wall section; stiffness ratio
$\overline{b}$ stiffness ratio for a system of bracing units
b_i shear stiffness ratio
b_w width of diagonal strip for infill
b_0, b_1, b_2 coefficients for cubic equation
c length of wall section
c_i global bending stiffness ratio
c_1 stability coefficient/critical load factor
d length of wall section; length of diagonal; depth of beam; deflection
d_{ASCE} maximum deflection recommended by ASCE
dz length of elementary section
e location of shear centre; distance of upper flange from centroid
e^* distance of lower flange from centroid (with bracing cores)
f frequency; auxiliary constant; number of frames and coupled shear walls
f_b lateral frequency associated with local bending stiffness
f_f lateral frequency of framework
f_g lateral frequency associated with global bending stiffness
f_s lateral frequency associated with the effective shear stiffness
$f_{s'}$ lateral frequency associated with the "original" shear stiffness
f_w lateral frequency of shear wall/core
f_x lateral frequency in direction x
f_y lateral frequency in direction y
$f_{y\varphi}$ coupled lateral-torsional frequency
f_φ frequency of pure torsional vibration
g gravity acceleration
h height of storey; length of wall section
h^* different storey height between ground floor and first floor
i summation index for columns/bracing units
i_p radius of gyration
j summation index
k non-dimensional parameter
k_s non-dimensional parameter for stability analysis
k_φ non-dimensional torsion parameter for frequency analysis
l width of bay
l^* distance between shear wall sections
m number of shear walls/cores/wall sections; mass; length of beam section
$\overline{m}$ torsional moment share on base unit
m_t total torsional moment on the bracing system
m_z torsional moment
n number of columns/walls; number of storeys
p intensity of uniformly distributed vertical load on beams
q intensity of uniform shear flow; intensity of axial load
q_i apportioner
q_ω torsional apportioner
q_1 apportioner for the base unit
r reduction factor for beam stiffness
r_f mass distribution factor for the frequency analysis
r_s load distribution factor for the stability analysis

s	non-dimensional stiffness ratio for bracing unit; effectiveness factor; distance of connecting beams with partially closed U-core
$\overline{s}$	non-dimensional stiffness ratio for bracing system
s_i	width of shear wall section
s_f	effectiveness factor for frequency analysis
s_φ	torsional effectiveness factor
t	wall thickness; distance of shear centre and centroid; time; perpendicular distance of bracing unit from shear centre; distance of column from centroid of cross-sections with frameworks
t_b	thickness of connecting beam with partially closed U-core
t_f	wall thickness
t_i	wall thickness
t_w	wall thickness
u	horizontal deflection of the shear centre in direction x
u_{max}	maximum horizontal deflection in direction x
u_1	horizontal motion
v	horizontal deflection in direction y
v_o	horizontal deflection of the shear centre in direction y
v_{max}	maximum horizontal deflection in direction y
v_φ	horizontal deflection caused by torsional moment around the shear centre
w	intensity of wind load
$\overline{w}$	intensity of wind load on base unit
x	horizontal coordinate axis; horizontal coordinate
$\overline{x}$	horizontal coordinate axis; coordinate in coordinate system $\overline{x}-\overline{y}$
x_c	coordinate of the centroid in the x-y coordinate system of the shear centre
x_i	coordinate of the shear centre of the ith bracing unit
x_{max}	location of maximum translation
$\overline{x}_i, \overline{y}_i$	coordinates of the shear centre of the ith bracing unit in the coordinate system $\overline{x}-\overline{y}$
$\overline{x}_o$	coordinate of the shear centre in coordinate system $\overline{x}-\overline{y}$
y	horizontal coordinate axis; horizontal coordinate
$\overline{y}$	horizontal coordinate axis; coordinate in coordinate system $\overline{x}-\overline{y}$
y_b	deflection due to bending deformation
y_c	coordinate of the centroid in the x-y coordinate system of the shear centre
y_i	coordinate of the shear centre of the ith bracing unit; deflection due to interaction
y_o	location of shear centre
$\overline{y}_o$	coordinate of the shear centre in coordinate system $\overline{x}-\overline{y}$
y_s	deflection due to shear deformation
z	vertical coordinate axis; vertical coordinate

GREEK LETTERS

α	eigenvalue; critical load parameter
α_s	eigenvalue; critical load parameter for the sandwich model with thin faces
β	part critical load ratio
β_s	part critical load ratio for the sandwich model with thin faces
Δ	displacement

η frequency parameter for lateral vibration
η_φ frequency parameter for pure torsional vibration
γ weight per unit volume
κ stiffness parameter for a single bracing unit
$\overline{\kappa}$ stiffness parameter for a system of bracing units
λ global critical load ratio
ν Poisson ratio
ω_1, ω_2 auxiliary constants
ω circular frequency
φ rotation
Ω_1, Ω_2 auxiliary constants
φ_{max} maximum rotation
Ψ auxiliary constant
ρ mass density per unit volume; cross-sectional constant
τ_x, τ_y eccentricity parameters for the three-dimensional analysis

Contents

1

Introduction

The book deals with the structural analysis of the bracing systems of multi-storey building structures and intends to offer useful tools to both researchers and practicing structural engineers. As a consequence, the material is divided into two parts: Part I presents the theoretical background and Part II gives worked examples.

A couple of decades ago approximate methods played a very important and normally dominant role in the structural design of large structures as often, because of the lack of computer power, it was not feasible, or practical, or sometimes possible, to carry out an "exact" analysis of big and complex structures. Then more and more powerful computers with more and more sophisticated programs started to become available to wider and wider structural engineering communities. Soon the debate started with questions like "Do we need old-fashioned approximate methods?" and "Should we rely on brainless number-crunching machines that cannot think?" and "Shall we just input all the data, press <Enter> and by tomorrow the structural analysis is done?" and "Computers in the design office: boon or bane" (Smart, 1997). This debate will perhaps go on for a long time. But one thing seems to be certain: simple analytical methods and closed-form approximate solutions do and will play an important role in practical structural engineering and theoretical research (Howson, 2006). Not only because they offer important independent checking possibilities to help to avoid CAD (Computer Aided Disaster) (Brohn, 1996) but also because the development and use of such methods help to understand the complex behaviour of large structures such as multi-storey buildings. They are also useful tools in developing structural engineering common sense and a feel for the behaviour of structures.

When multi-storey buildings are investigated, two main avenues are available for the structural engineer: sophisticated and powerful computer packages can be used or "conventional" calculations can be made. Perhaps the best way to tackle the task is to employ both approaches: at the preliminary design stage simple hand methods can quickly help to establish the main structural dimensions and to point to efficient bracing system arrangements. More detailed computer-based analyses can follow. Before the final decision is made, it is essential to check the results of the computer analysis and recheck the adequacy of the key elements of the bracing system. Here, again, suitable analytical methods can play a very useful part.

The fact that the methods in the book are all based on continuous models has another advantage. When the results of a finite element analysis (based on discrete models) are checked, it is advantageous to use a technique that is based on a different approach, i.e., on continuous media.

Structural analysis is normally carried out at two levels. The structural engineer has to ensure that a) the individual elements (beams, columns, floor slabs, etc.) are of adequate size and material to carry their load and b) the structure as a whole has adequate stiffness and the bracing system fulfils its main role to provide sufficient stability to the building.

The book does not deal with individual structural elements. Its aim is to present simple analytical methods for the complex global analysis of whole structural systems in the three main structural engineering areas. Closed-form solutions will be given for the maximum rotation and deflection, the fundamental frequency and the critical load of the building, assuming three-dimensional behaviour.

The continuum method will be used which is based on an equivalent medium that replaces the whole building. The discreet load and stiffnesses of the building will be modelled by continuous load and stiffnesses that make it possible to use analytical tools to produce relatively simple, closed-form solutions to the resulting differential equations and eigenvalue problems.

It will be assumed that the structures are

- at least four storeys high with identical storey heights
- regular in the sense that their characteristics do not vary over the height
- sway structures with built-in lower end at ground floor level and free upper end

and that

- the floor slabs have great in-plane and small out-of-plane stiffness
- the deformations are small and the material of the structures is linearly elastic
- P-delta effects are negligible.

Structural engineering research and practice often see researchers/structural designers who have specialized in one area with limited knowledge elsewhere. Designers are often reluctant to deal with theoretical matters; researchers often have little practical knowledge (or attitude); those dealing with stress analyses are sometimes ignorant of stability matters; people engaged in earthquake engineering may not be very good at the optimisation of bracing systems, etc.

This book offers a unified treatment for the different structures (frameworks, coupled shear walls, shear walls and cores) and also for the different types of investigation (deflection, rotation, frequency, stability). The same terminology will be used throughout, and it will be shown that these seemingly independent areas (deformations, frequencies, critical loads—or stress, dynamic and stability analyses) are in fact very closely related. In addition, the global critical load ratio links them to the performance of the bracing system in a rather spectacular manner.

Numerous approximate methods have been published for structural analyses. However, it is surprising how few, if any, have been backed up with comprehensive accuracy analysis. Here, in this book, dozens/hundreds of bracing units/systems are used to demonstrate the applicability and accuracy of the methods presented.

Although real multi-storey buildings seldom develop planer deformation only, Chapter 2 (dealing with the planar analysis of individual bracing units) is probably the key chapter of the book in the sense that it introduces all the characteristic stiffnesses that will be used for the three-dimensional investigations of whole systems later on. It is also shown here how the complex behaviour can be traced back to the local bending, global bending and shear deformation (and their torsional equivalents) of the bracing system. All the characteristic types of bracing unit are covered here: sway- and infilled frameworks, frameworks with cross-bracing, coupled shear walls, shear walls and cores.

Deflections and rotations are the subject of Chapter 3 where the main aim is to present simple, closed-form solutions for the maximum deflection and rotation of the building. The investigations spectacularly show the contribution of the two key (bending and shear) stiffnesses as well as the interaction between them. Chapter 4 deals with the frequency analysis of buildings. Closed form formulae and tables make it possible to calculate the lateral and torsional frequencies of the building. The coupling of the lateral and torsional modes can be taken into account by a simple summation formula or, if a more accurate result is needed, by calculating the smallest root of a cubic equation. The often neglected but very important area of stability is covered in Chapter 5. In using critical load factors, simple (Euler-like) formulae are presented for the lateral and torsional critical loads. The combined sway-torsional critical load is obtained as the smallest root of a cubic equation. Chapters 2, 3, 4 and 5 end with a demonstration of the accuracy of the method(s) presented in the chapter.

Chapter 6 introduces the global critical load ratio which is a useful tool for monitoring the "health" of the bracing system and indicates if the bracing system is adequate or more rigorous (second-order) analysis is needed. The global critical load ratio can also be used to assess different bracing system arrangements in minutes in order to chose the most economic one. The results of a comprehensive example illustrate the practical use of the global critical load ratio.

Part II presents sixteen examples worked out to the smallest details, with step-by-step instructions. The examples range from the deflection or frequency or stability analysis of individual bracing units to the complex deflection and frequency and stability analyses of bracing systems, considering both planar and spatial behaviour. Although most of the formulae in the book are of the back-of-the-envelope type, due to the complexity of global three-dimensional analyses, some of the calculations may still seem to be rather cumbersome to carry out by hand. It is very rare, however, that a structural engineer today would wish to do actual hand-calculations, however simple they may be. Convenient spreadsheets and calculation worksheets make it possible to do the structural analysis and document its result at the same time in minutes. All the methods presented in the book are suitable for this type of application; in fact the worksheet version of all the sixteen worked examples has been prepared and made available for download. These one-to-eight page long worksheets cover a very wide range of practical application and can also be used as templates for other similar structural engineering situations.

Part I: Theory

The widespread availability of powerful computers and sophisticated programs makes it possible to analyze even very large and complex structures with relatively little effort. This is very welcome. There is, however, a certain degree of danger that the structural engineer, in accepting the help of the computer, may get carried away and rely on the computer to a greater extent than would be desirable and pay less attention to the behaviour of the structure. It may be tempting to become complacent.

If the structural engineer's knowledge about the behaviour of complex structures is limited, then the temptation is even greater to accept the computer's solution to the structural engineering problem that has been fed to the computer. This is where "Part I: Theory" can be helpful. The continuum model of the multi-storey building is used repeatedly. The continuous medium approach makes it possible to handle complex structural engineering problems in a relatively simple way and to identify the key stiffness and geometrical characteristics that have a dominant role in shaping the behaviour of the structure.

In order for the accuracy analyses in Chapters 2, 3, 4 and 5 to correspond to the theoretical assumption that "the floor slabs have great in-plane and small out-of-plane stiffness" the floor slabs of the buildings are modelled using sets of bars interconnecting the vertical bracing units. The bars have very great cross-sectional areas and pinned ends. The shear walls are modelled by bar elements (cantilevers).