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Stochastic Processes and Functional Analysis

A Volume of Recent Advances in Honor of M. M. Rao

Alan C. Krinik Randall J. Swift

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Preface

An AMS Special Session in honor of M.M. Rao was held at the 2002 joint meetings of the American Mathematical Society and the Mathematical Association of America. That Special Session was on Stochastic Processes and Functional Analysis and was organized by Professors Alan C. Krinik and Randall J. Swift, both of California State Polytechnic University, Pomona.

Professor M.M. Rao has had a long and distinguished research career. His research spans the areas of probability, statistics, stochastic processes, Banach space theory, measure theory and differential equations - both deterministic and stochastic. The prolific published research of M.M. Rao impacts each of these broad areas of mathematics.

The purpose of the Special Session was to highlight the key role played by abstract analysis in simplifying and solving fundamental problems in stochastic theory. This notion is fundamental to the mathematics research of M.M. Rao, who uses functional analytic methods to bring questions in these diverse areas to light.

The Sessions were a great success, bringing together a diverse group of research mathematicians whose work has been influenced by M.M.'s work and who, in turn, have influenced his work. Not only did this diverse group of speakers benefit from the common unifying thread of the session, but also there were often lively discussions and questions from the session audience.

This volume contains most of the talks given at the Sessions as well as several that were contributed later. This collection of papers reflects the depth and enormous breadth of M.M. Rao's work. A major highlight of the Sessions was M.M.'s talk entitled "Stochastic analysis and function spaces", which was a remarkable unifying survey of recent work in the area. This volume features that talk as an article, which includes a broad bibliography of the important works in the area.

The volume begins with a biography of M.M. Rao, a complete bibliography of his published writings, a list of his Ph.D. students and, notably, a collection of essays about M.M. written by some of his Ph.D students. Many of M.M. students have remained devoted to him, decades after completing their degrees. Their loyal devotion arises from M.M.'s complete dedication

to them. He consistently put their concerns and welfare as his first priority. Their essays are a remarkable tribute.

This volume complements the Festschrift volume Stochastic Processes and Functional Analysis, which was published by Marcel Dekker, Inc. in 1997. That volume was in celebration of M.M.'s 65th birthday. As M.M. continues to work on, develop and expand mathematics, we look to future collections of articles that honor him and his love of mathematics.

R. J. Swift A. C. Krinik

Biography of M. M. Rao

M.M. Rao was born Malempati Madhusudana Rao in the village of Nimmagadda in the state of Andhra Pradesh in India on June 6, 1929. He came to the United States after completing his studies at the College of Andhra University and the Presidency College of Madras University. He obtained his Ph.D in 1959 at the University of Minnesota under the supervision of Monroe Donsker (as well as Bernard R. Gelbaum, Leonid Hurwicz, and I. Richard Savage).

His first academic appointment was at Carnegie Institute of Technology (now called Carnegie Mellon University) in 1959. In 1972, he joined the faculty at the University of California, Riverside where he remains today. He has held visiting positions at the Institute for Advanced Study (Princeton), the Indian Statistical Institute, University of Vienna, University of Strassbourg, and the Mathematical Sciences Research Institute (Berkeley).

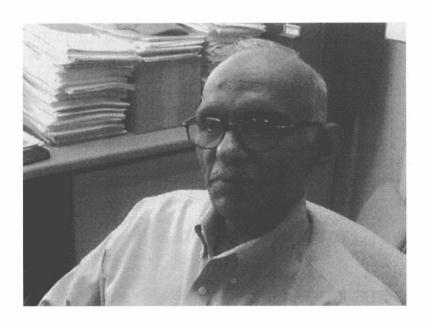
In 1966 he married Durgamba Kolluru in India. They have twin daughters Leela and Uma and one granddaughter.

M.M.'s research interests were initially in probability and mathematical statistics, but his intense mathematical interest and natural curiosity found him pursuing a wide range of mathematical analysis including stochastic processes, functional analysis, ergodic theory and related asymptotics, differential equations and difference equations. His breadth of interest is mirrored by his students, many of whom are recognized as experts in diverse fields such as measure theory, operator theory, partial differential equations and stochastic processes.

M.M. has always strived for complete understanding and generality in mathematics and rarely accepts less from others. This view of mathematics has played a central role in his teaching. M.M. Rao is truly a gifted lecturer and he has inspired many generations of students. He is a demanding Ph.D. advisor that expects the most from his students. The guidance and mentoring he provides them has led to many of his students becoming successful mathematicians. It is no wonder that he has had his share of the best available graduate students.

M.M. is a prolific writer. His first published writings were not on math-

ematics, but rather Indian poetry. He wrote poetry in is late teenage years and had a collection of his poems published when he was 21. His mathematical research publications are many and span five decades. He is active and vital as ever. He has just completed a second edition of his well-received measure theory text and is currently working upon revised and expanded second editions of his probability and conditional measures texts.



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Ph.D. Theses Completed Under the Direction of M.M. Rao

At Carnegie-Mellon University:

Dietmar R. Borchers (1964), "Second order stochastic differential equations and related Ito processes."

J. Jerry Uhl. Jr (1966), "Orlicz spaces of additive set functions and set martingales."

Jerome A. Goldstein (1967), "Stochastic differential equations and nonlinear semi-groups."

Neil E. Gretsky (1967), "Representation theorems on Banach function spaces." William T. Kraynek (1968), "Interpolation of sub-linear operators on generalized Orlicz and Hardy spaces."

Robert L. Rosenberg (1968), "Compactness in Orlicz spaces based on sets of probability measures."

George Y. H. Chi (1969), "Nonlinear prediction and multiplicity of generalized random processes."

At University of California, Riverside:

Vera Darlean Briggs (1973), "Densities for infinitely divisible processes."

Stephen V. Noltie (1975), "Integral representations of chains and vector measures."

Theodore R. Hillmann (1977), "Besicovitch - Orlicz spaces of almost periodic functions."

Michael D. Brennan (1978), "Planar semi-martingales and stochastic integrals."

James P. Kelsh (1978), "Linear analysis of harmonizable time series."

Alan C. Krinik (1978), "Stroock - Varadhan theory of diffusion in a Hilbert space and likelihood ratios."

Derek K. Chang (1983), "Bimeasures, harmonizable process and filtering." Marc H. Mehlman (1990), "Moving average representation and prediction for multidimensional strongly harmonizable process."

Randall J. Swift (1992), "Structural and sample path analysis of harmonizable random fields."

Michael L. Green (1995), "Multi-parameter semi-martingale integrals and boundedness principles."

Heroe Soedjak (1996), "Estimation problems for harmonizable random processes and fields."