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Applications in Multicriteria Decision Making, Data Envelopment Analysis, and Finance

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SECTION A
MULTICRITERIA APPLICATIONS

SELECTED MULTIOBJECTIVE METHODS FOR MULTIPERIOD PORTFOLIO OPTIMIZATION BY MIXED INTEGER PROGRAMMING

Bartosz Sawik

ABSTRACT

This chapter presents selected multiobjective methods for multiperiod portfolio optimization problem. Portfolio models are formulated as multicriteria mixed integer programs. Reference point method together with weighting approach is proposed. The portfolio selection problem considered is based on a multiperiod model of investment, in which the investor buys and sells securities in successive investment periods. The problem objective is to allocate the wealth on different securities to optimize the portfolio expected return, the probability that the return is not less than a required level. Multiobjective methods were used to find tradeoffs between risk, return, and the number of securities in the portfolio. In computational experiments the data set of daily quotations from the Warsaw Stock Exchange were used.

Keywords: multiobjective optimization; multiperiod portfolio management; mixed integer programming.

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1. INTRODUCTION

The portfolio problem, which involves computing the proportion of the initial budget that should be allocated in the available securities, is at the core of the field of financial management. A fundamental answer to this problem was given by Markowitz who proposed the mean-variance model which laid the basis of modern portfolio theory (Markowitz, 1952, 1997). In Markowitz's approach the problem is formulated as an optimization problem involving two criteria: the reward of portfolio, which is measured by the mean or expected value of return and should be maximized, and the risk of the portfolio, which is measured by the variance of return that should be minimized. In the presence of two criteria there is not a single optimal solution (portfolio structure), but a set of optimal portfolios, the so-called efficient portfolios, which tradeoff between risk and return. Since the mean-variance theory of Markowitz, an enormous amount of papers have been published extending or modifying the basic model in three directions. The first path goes to simplification of the type and amount of input data. The second direction concentrates on the introduction of alternative measure of risk. Finally, the third involves the incorporation of additional criteria and/or constraints (Anagnostopoulos & Mamanis, 2010).

The original Markowitz model forms as a quadratic program with continuous variables and some side constraints. Many attempts have been made to linearize the portfolio optimization procedure (Speranza, 1993; Young, 1998; Ogryczak, 2000; Benati & Rizzi, 2007; Sawik, 2006, 2008, 2009a, 2009b, 2009c, 2009d, 2009e, 2009f, 2009g, 2009h). The linear program solvability is very important for applications to real-life financial and other decisions where the constructed portfolios have to meet numerous side constraints. Examples of them are minimum transaction lots, transaction costs or mutual funds characteristics, etc. The introduction of these features leads to mixed integer program problems.

2. VALUE-AT-RISK

The formal definition of value-at-risk is the α-quantile of the return distribution function, $\alpha \in (0, 1)$ where α is usually chosen to be 0.01, 0.05, or 0.1.

The value-at-risk of the securities in portfolio for the selected multiperiod interval t and the probability level p is defined as an amount termed VaR, so that the variation of selected security price return observed for the security during the interval [0; t] will only be less than VaR with probability of $(1-p)$.

The advantage of using VaR measure in portfolio optimization is that this value of risk is independent of any distribution hypothesis (Benati & Rizzi, 2007). It concerns only downside risk, namely the risk of loss. This index measures the loss in question in a certain way. Finally, VaR is valid for all types of securities and therefore either involve the various valuation models or be independent of these models (Esch et al., 2005).

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3. REFERENCE POINT METHOD

The reference point method is based on the Tchebycheff metric (Alves & Climaco, 2007; Bowman, 1976). Let us denote by $||f(x)-\underline{f}||_\lambda$ the λ-weighted Tchebycheff metric, that is, $\min_{1\leq l\leq q}\{\lambda_l|f_l(x)-\underline{f}|\}$, where $\lambda_l\geq 0\;\forall l$, $\sum_{l=1}^{q}\lambda_l=1$, and $\underline{f}$ denotes a reference point of the criteria space. Considering $f(x)>\underline{f}$ for all $x\in X$, it has been proven (Bowman, 1976) that the parameterization on λ of $\min_{x\in X}||f_l(x)-\underline{f}||_\lambda$ generates the nondominated set.

The program $\min_{x\in X}||f_l(x)-\underline{f}||_\lambda$ may yield weakly nondominated solutions, which can be avoided by considering the *augmented weighted Tchebycheff* program:

$$\text{Minimize}\quad \delta+\gamma\sum_{l=1}^{q}f_l(x)$$

$$\text{Subject to}\quad \lambda_l\left(f_l(x)-\underline{f}\right)\leq\delta,\quad 1\leq l\leq q$$

$$x\in X$$

$$\delta\geq 0$$

where γ is a small positive value. It has been proven (Steuer, 1986) that there always exists γ small enough that enable to reach all the nondominated set for the finite-discrete and polyhedral feasible region cases (Alves & Climaco, 2007).

4. MULTIPERIOD EXTENSION OF THE MARKOWITZ PORTFOLIO WITH SINGLE CRITERION – M1

This chapter presents a single criterion extension of the Markowitz portfolio optimization model, in which the variance has been replaced with the *VaR*. The *VaR* is a quantile of the return distribution function.

This portfolio optimization problem is formulated as a single objective model by mixed integer programming (Sawik, 2009f). The portfolio selection problem considered is based on a multiperiod model of investment, in which the investor buys and sells securities in consecutive periods. The problem objective is to allocate wealth on different securities to maximize the portfolio expected return and the threshold of the probability that the return is not less than a required level.

The two types of decision variables are introduced in the model: a continuous wealth allocation variable that represents the percentage of wealth allocated to each security, a binary selection variable that prevents the choice of portfolios whose *VaR* is below the fixed threshold. The results of some computational experiments with the mixed integer programming approach modeled on a real data from the Warsaw Stock Exchange are reported.

In computational experiments the data set with time series of the daily quotation of returns of securities from the Warsaw Stock Exchange was used. The 18 years horizon from January 30, 1991 to January 30, 2009 – consist of 4020 historic daily quotations divided into 20 investment periods, with the selection of 241 input securities for portfolio, quoted each day in the historical horizon. Probability of realization for expected securities returns is the same for each day and summed up for whole period to one.

5. BI-OBJECTIVE DYNAMIC PORTFOLIO OPTIMIZATION PROBLEM – M2

This portfolio optimization problem is formulated as a bi-objective mixed integer program (Sawik, 2009d). The portfolio selection problem considered is based on a dynamic model of investment, in which the investor buys and sells securities in successive investment periods. The problem objective is to dynamically allocate the wealth on different securities to optimize the weighted difference of the portfolio expected return and the probability that the return is not less than a required level.

6. REFERENCE POINT METHOD TO BI-OBJECTIVE MIP PORTFOLIO – M3

This portfolio optimization problem is formulated as a bi-objective mixed integer program (Sawik, 2009a). The portfolio selection problem considered is based on a dynamic model of investment, in which the investor buys and sells securities in successive investment periods. The problem objective is to dynamically allocate the wealth on different securities to optimize by reference point method of the portfolio expected return and the probability that the return is not less than a required level.

The results of some computational experiments with the mixed integer programming approach modeled on a real data from the Warsaw Stock Exchange are reported. The input data set consist of time series of the daily quotation of returns of securities from the Warsaw Stock Exchange.

7. PROBLEM FORMULATION – M1

Suppose that n securities are available in the market with historical quotations in $t = 20$ intervals, each of $h = 201$ days, in total $m = 4020$ samples.

Let r_{ij} be the random variable representing the future daily return of jth security in ith historical time period.

The portfolio optimization problem with value-at-risk constraint is formulated as the classic Markowitz approach, but with value-at-risk instead of variance as risk measure.

The decision maker fixes two parameters, the probability α^{VaR} and lower bound r^{VaR} for successful returns – any investments whose value-at-risk is less than r^{VaR} will be not acceptable.

Let r^{Min} be the minimum return that can be observed in the market, for example the biggest possible loss of money invested in portfolio. In the worst case it is the whole amount of capital, so for instance r^{Min} can be equal – 100% (Table 1).

8. OPTIMIZATION MODEL – M1

The portfolio optimization model with maximization of expected portfolio return (1) is presented below. The single objective portfolio optimization

Table 1. Notation for Problem M1.

	Indices
i	= historical time period i, $i = 1, \ldots, m$
j	= security j, $j = 1, \ldots, n$
k	= historical multiperiod interval k, $k = 1, \ldots, t$ within m
	Input parameters
h	= number of historical quotations in each multiperiod interval
p_i	= probability assigned to the occurrence of past realization i
r_{ij}	= observed return of security j in historical time period i
r^{Min}	= minimum return observed in the market
r^{VaR}	= return value-at-risk
α^{VaR}	= the highest acceptable fraction (probability) of the failed investment in the considered multiperiod interval for $k = 1, \ldots, t$
	Variables
x_j^k	= percentage of capital invested in security j of multiperiod interval k
y_i^k	= 1, if return of portfolio in period i of multiperiod interval k is over r^{VaR}; 0 otherwise

model with value-at-risk is NP-hard problem even when future returns are described by discrete uniform distributions.

In the approach proposed in this chapter, the portfolio optimization problem is formulated as a single objective mixed integer program, which allows commercially available software (e.g., AMPL/CPLEX (Fourer, Gay, & Kernighan, 1990)) to be applied for solving medium size, yet practical instances.

Problem **M1** (k), $k = 1, \ldots, t$

$$\text{Maximize} \quad \sum_{i=(k-1)h+1}^{kh} p_i \sum_{j=1}^{n} r_{ij} x_j^k \tag{1}$$

$$\text{Subject to} \quad y_i^k \le \frac{\sum_{j=1}^{n} r_{ij} x_j^k - r^{Min}}{r^{VaR} - r^{Min}}, \quad i = (k-1)h + 1, \ldots, kh \tag{2}$$

$$y_i^k \ge \frac{\sum_{j=1}^{n} r_{ij} x_j^k - r^{Min}}{r^{VaR} - r^{Min}} - 1, \quad i = (k-1)h + 1, \ldots, kh \tag{3}$$

$$\sum_{i=(k-1)h+1}^{kh} p_i \left(1 - y_i^k\right) \le \alpha^{VaR} \tag{4}$$