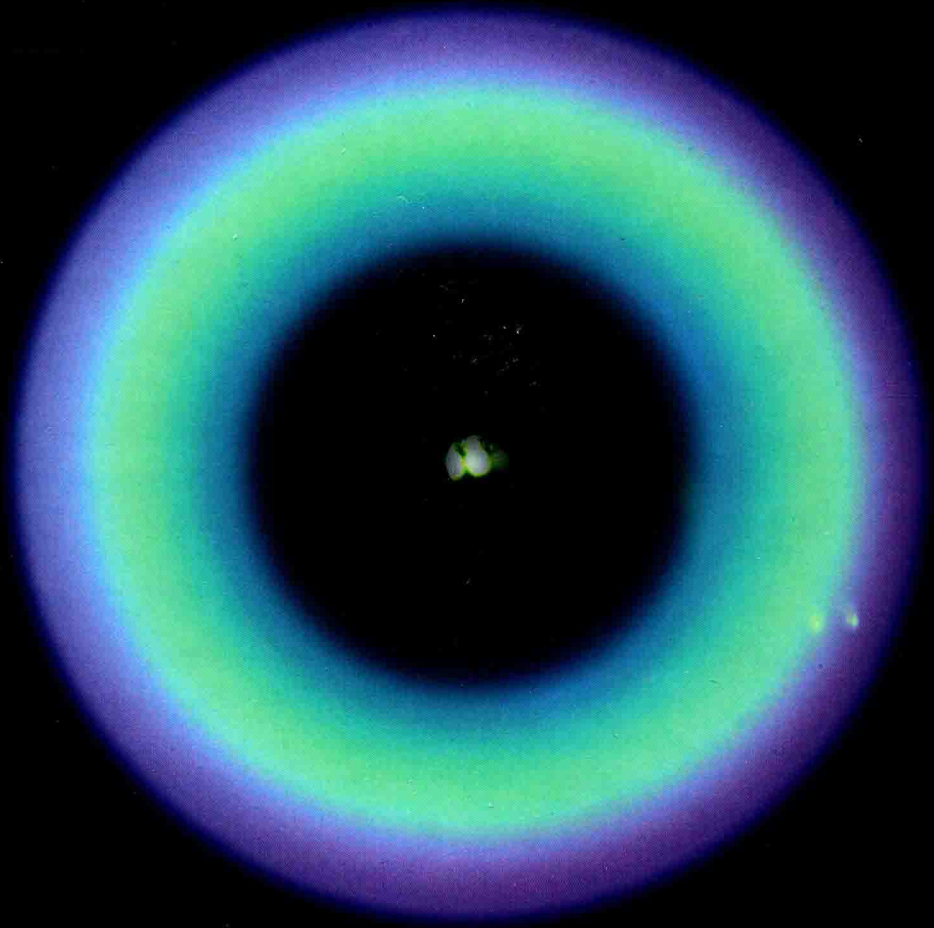


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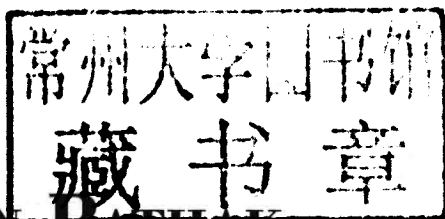


ANIRBAN PATHAK

 **CRC Press**
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ANIRBAN PATHAK



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*This book is dedicated to my mother, Mrs. Chandana Pathak,
for the elementary and fundamental lessons of life that she
taught me.*

Preface

This introductory textbook is written primarily for undergraduate students of physics, mathematics, computer science and other related disciplines. It is also expected to be valuable to teachers as well as to researchers working in other domains, who are interested in obtaining an understanding of quantum computation and quantum communication. I used to offer a course on quantum information theory from 2002-2006. Later I offered a few short courses in different summer schools and workshops. This book is prepared mainly from those lectures. There are many excellent textbooks on quantum information theory. However, most of those books are either too technical for beginners or they are not complete. This was one of my reasons for writing this book. But more importantly, every teacher has his/her own way to present the subject and teachers are usually biased on that. I belong to that class of biased teachers and this book is an initiative to present the subject in my way. Another fact that played a very important role in the present initiative is that there are engineering students who hardly know anything about quantum mechanics and there are physics students who do not know what a Turing machine is. But students from both groups are equally interested in quantum computing and often they join the same course. Keeping both kinds of students in mind, this book aims to give a brief idea of quantum computation and quantum communication in a self-contained manner. It does not demand any prior knowledge of quantum mechanics or computer science. It is written in a lucid manner, and a large number of problems with detailed solutions are provided in each chapter (especially in Chapter 3). In addition, a set of thought-provoking cartoons is included to make the subject more attractive. This is an introductory textbook so it will not be so thick that readers get afraid of the volume of the book and leave it before they begin. The field of quantum information and quantum computation is rapidly growing. I have tried to give a flavor of the new developments and open questions in the field, but I could not accommodate all the flavors of this interdisciplinary subject. Specifically, I could not do justice to experimental techniques. I'll consider the book successful if the readers find it easily understandable, interesting and encouraging enough to read more advanced texts and journal papers.

I understand that many students and researchers do not have access to all the journals. Keeping them in the mind, I have tried to mention all such sources where one can get access to interesting articles and courses for free. Especially in the bibliography I have provided several references from arxiv.org, and in the “further reading” sections of each chapter I have mentioned sources where seminal papers related to the field covered can be read for free. I hope readers with restricted library access will find these sources useful.

I have tried my best to avoid typos and errors. Still there may be a few present in the book. I request readers to communicate any errors, typos and suggestions by kindly sending an email to anirban.pathak@gmail.com.

This book has been written over a considerable amount of time and most of the book originates from my lecture notes. I have tried to properly cite all sources that are used here but there is a possibility that some are unintentionally omitted. I am extremely sorry for any such occurrence.

In the process of writing this book, I received help, support and encouragement from many individuals and institutes at various stages and in various forms. I would like to thank all of them. To begin with I must mention that the first few words of this book were written in 2003. It took much longer than expected. In this long period, I have lost many people who were very close to me and would have liked to see this book. I have lost my grandmother, my father, my aunts, a few of my teachers and friends who would have been very happy to see this book in existence but who passed away before it was completed. I thank them for their interest and encouragement in all my academic activities including this book-writing project. I am thankful to Dr. Anindita Banerjee and Ms. Chitra Shukla who have helped me considerably by creating the figures, correcting the typos and giving their feedback. The front cover of this book shows the image of the processed output of spontaneous parametric down conversion (SPDC) process. In the center of the image we can see the pump beam. The experiment was carried out by my colleagues at the Joint Laboratory of Optics, Olomouc. I am specially thankful to my colleagues Dr. Martin Hamar and Mr. Radek Machulka for producing this image. My old friend Mr. Anshuman Das has drawn the cartoons included in this book. His kind help has made the text more attractive. Prof. Avijit Pathak and Dr. Subhashish Banerjee have carefully read part of the manuscript and have provided their valuable feedback. I am thankful to them. Prof. Ajoy Ghatak's interest in the book was a constant encouragement. It was he who advised me to include as many examples as possible. As I see the final manuscript, it appears that his suggestion has really made it a textbook that can be used for classroom teaching. I am thankful to him for his valuable advice. The manuscript took its final form during my one-year stay at Palacky University, Czech Republic. This visit provided me ample time and the perfect ambiance to complete the manuscript. Prof. J. Peřina, Prof. V. Peřinová, Dr. O. Haderka, Dr. J. Peřina Jr. and Prof. M. Hrabovský, whose collaboration and help made this visit possible, deserve special words of thanks. Without their kind support it would have been impossible to complete the book. My special friend, Dr. R. Srikanth, who was always awake late at night to share my concerns about the book, has helped me in many ways. My long late-night conversations with him have many direct and indirect contributions to this book. No word of thanks is enough for his help. I would also like to thank all my collaborators and students together with whom I have learned the subject discussed in this

book. As I mentioned, to finish this book, I took a one-year leave from IIIT, India and visited Palacky University. During this period my wife Dr. Papia Chowdhury and my son Master Pustak Pathak were very cooperative. Their selfless encouragement and support made it possible. During most of the time while I was working on this manuscript, my research activities on quantum computing and quantum communication were supported by the Department of Science and Technology, India through project numbers SR/S2/LOP-0012/2010 and SR/FTP/PS-13/2004. My activities were also supported by the Operational Program Education for Competitiveness - European Social Fund project CZ.1.07/2.3.00/20.0017 and Operational Program Research and Development for Innovations - European Regional Development Fund project CZ.1.05/2.1.00/03.0058 of the Ministry of Education, Youth and Sports of the Czech Republic. Support obtained from these projects was the backbone of my book-writing project. I thank these agencies for their support. I am thankful to the administration of IIIT, Noida for granting me the sabbatical to complete the book. I am also thankful to IMSc, Chennai for offering me the associateship. I especially mentioned this because Ms. Aastha Sharma of CRC Press approached me with their proposal to write a textbook during my stay in IMSc. I had a half-written manuscript that had been gathering dust for a long time. This coincidence revived the project.

I am indebted to many more people for their indirect support to this book. I especially acknowledge the support and help of Mrs. Chandana Pathak, Mr. S. R. Chaudhuri, Ms. Dipti Ray, Mr. Kunal Jha, Mr. Sanjit Pathak, Mrs. Anindita Pathak, Dr. Gautam Sarkar, Dr. Y. Medury, Prof. K. C. Mathur, Prof. D. K. Rai, Prof. S. K. Kak, Prof. Swapan Mandal, Prof. P. K. Panigrahi, Prof. M. R. B. Wahiddin, Prof. Barry Sanders, Prof. Marco Genovese, Prof. J. Banerjee, Prof. Adam Miranowicz, Dr. Chiranjib Sur, Dr. Amit Verma, Dr. Biswajit Sen, Dr. B. P. Chamola, Dr. Somshubhro Bandyopadhyay, Mr. Aayush Bhandari, and Mr. Rishabh Jain.

Lastly many thanks to Ms. Aastha Sharma, Ms. Amy Rodriguez and their colleagues at CRC Press for their initiative to publish this book.

Olomouc, Czech Republic
December 15, 2012

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Contents

Preface	xiii
Author	xvii
1 Introduction and overview	1
1.1 What is information?	1
1.1.1 Is information physical?	2
1.1.2 Quantitative measure of information	5
1.1.3 Shannon's first coding theorem	12
1.1.4 More about the bit	14
1.2 Why do we need to know how to manage information?	15
1.2.1 Which technology?	16
1.2.2 The qubits	17
1.3 A brief history of quantum computation and quantum communication	19
1.4 Solved examples	25
1.5 Further reading	28
1.6 Exercises	30
2 Basic ideas of classical and quantum computational models and complexity classes	33
2.1 Elementary idea of complexity of an algorithm	33
2.2 Turing machine	35
2.2.1 Deterministic Turing machine	37
2.2.2 Probabilistic Turing machine	39
2.2.2.1 Reversible Turing machine	42
2.2.2.2 Quantum Turing machine	44
2.3 Circuit model of computation	46
2.4 Computational complexity and related issues	47
2.5 Solved examples	52
2.6 Further reading	54
2.7 Exercises	55

3	Mathematical tools and simple quantum mechanics required for quantum computing	57
3.1	A little bit of algebra required for quantum computing . . .	57
3.1.1	Vector space	58
3.1.1.1	The C^n vector space	58
3.1.1.2	Inner product space and Hilbert space . . .	59
3.1.1.3	Bases and linear independence	61
3.1.1.4	C^2 space: The space spanned by a single qubit	62
3.1.1.5	Outer product	63
3.1.2	Linear operators	63
3.1.3	Pauli matrices	64
3.1.4	Gram-Schmidt procedure	65
3.1.5	Eigenvalues and eigenvectors	66
3.1.6	Hermitian operators	66
3.1.7	Normal, unitary and positive operators	67
3.1.8	Diagonalizable operator and spectral decomposition	67
3.1.9	Tensor products	69
3.1.10	Trace	70
3.2	A little bit of quantum mechanics	71
3.2.1	Basic ideas of quantum mechanics	71
3.2.2	A little more of quantum mechanics	73
3.2.3	Density operator and density matrix	78
3.2.3.1	Density operator of pure states	79
3.2.3.2	Density operator of mixed states	80
3.2.4	The meaning of entanglement	81
3.2.5	Bell's inequality and nonlocality	84
3.3	A little more algebra for quantum computing	87
3.3.1	Bell measurement and entanglement	87
3.3.2	Partial trace	89
3.3.2.1	Quantum bit commitment and quantum coin tossing	90
3.3.3	Schmidt decomposition	92
3.3.4	Partial transpose and test of entanglement	98
3.3.5	Entanglement witness	102
3.3.6	State discrimination	104
3.3.6.1	Trace distance and fidelity	104
3.3.7	Measures of entanglement	108
3.3.7.1	Schmidt measure	108
3.3.7.2	Von Neumann entropy of the subsystem . . .	108
3.3.7.3	Negativity	111
3.3.7.4	Concurrence and entanglement of formation	113
3.3.8	State purification	117
3.3.9	Holevo bound	118
3.3.10	Bloch sphere	120

3.3.11	Nocloning theorem	124
3.3.11.1	Conclusions from nocloning theorem	125
3.3.11.2	Other no-go theorems	126
3.4	Solved examples	128
3.5	Further reading	133
3.6	Exercises	134
4	Quantum gates and quantum circuits	137
4.1	Single qubit gates	138
4.2	Two qubit gates	143
4.3	Three qubit gates	145
4.4	A little more on quantum gates	147
4.5	Quantum circuits	149
4.5.1	Quantitative measures of quality of a circuit	150
4.5.1.1	Gate count or circuit cost	150
4.5.1.2	Garbage bit	152
4.5.1.3	Quantum cost	152
4.5.1.4	Depth and width of a circuit	154
4.5.1.5	Total cost	154
4.5.2	Circuit optimization rules	155
4.5.2.1	Moving rule	155
4.5.2.2	Template matching	156
4.5.3	Let us visualize the quantum gate	158
4.6	Discussion	158
4.7	Solved examples	159
4.8	Further reading	170
4.9	Exercises	170
5	Quantum algorithms	173
5.1	Deutsch's algorithm	174
5.2	Deutsch Jozsa (DJ) algorithm	176
5.3	Grover's algorithm	178
5.4	Simon's algorithm	183
5.4.1	Classical approach	184
5.4.2	Quantum approach	184
5.4.3	Complexity analysis	186
5.5	Shor's algorithm	186
5.5.1	A little bit of number theory	187
5.5.1.1	Euclid's algorithm	188
5.5.1.2	Period of the modular exponential function	188
5.5.1.3	Continued fraction representation	189
5.5.2	The strategy	190
5.5.3	Quantum Fourier transformation	191
5.5.4	Main (quantum) part of the algorithm	194
5.6	Solution of Pell's equation and the principal ideal problem	198

5.7	Discussion	199
5.8	Solved examples	201
5.9	Further reading	204
5.10	Exercises	205
6	Quantum error correction	207
6.1	Quantum error correction	207
6.2	Basic idea of an error model	208
6.2.1	How to correct classical errors	209
6.2.2	Difference between classical error and quantum error	211
6.2.3	How to correct quantum errors	212
6.2.3.1	Correction of quantum bit flip error: A 3-qubit code	212
6.2.4	Correction of phase flip error: A 3-qubit code	213
6.2.5	Shor code: A 9-qubit code that can correct an arbitrary single qubit error	214
6.3	A little more on quantum error correction	219
6.3.1	Fault-tolerant quantum computation	221
6.3.2	Threshold theorem for quantum computation	223
6.4	Decoherence and decoherence free subspace	223
6.4.1	Decoherence free subspace	227
6.5	DiVincenzo criteria	229
6.6	Solved examples	232
6.7	Further reading	236
6.8	Exercises	236
7	Quantum teleportation and superdense coding	239
7.1	Different types of teleportation schemes	240
7.2	A simple scheme for perfect teleportation	243
7.3	Probabilistic teleportation	245
7.4	Controlled teleportation or quantum information splitting .	247
7.4.1	Hierarchical quantum information splitting	249
7.5	Modified teleportation schemes	252
7.5.1	Remote state preparation	252
7.5.1.1	Modified remote state preparation schemes	254
7.6	Superdense coding	255
7.6.1	Some more examples of dense coding	257
7.7	Solved examples	260
7.8	Further reading	263
7.9	Exercises	264
8	Quantum cryptography	265
8.1	Jargon related to cryptography	265
8.2	Some interesting classical ciphers	268
8.2.1	RSA and its limitations	269

8.3	Different aspects of quantum cryptography	271
8.3.1	Quantum cryptography: The art of getting positive results from the negative rules of quantum mechanics	273
8.4	Let us develop protocols of QKD	276
8.4.1	Let us attack Protocol 1	277
8.4.2	Protocol 4: BB84 protocol	280
8.4.2.1	Elementary idea of decoy state	281
8.5	Protocol 5: B92 protocol	283
8.6	GV protocol: QKD with orthogonal states	284
8.7	Ping-pong and modified ping-pong protocols	286
8.7.1	Protocol 7: Ping-pong protocol	286
8.7.2	The modified PP protocols	287
8.7.2.1	Protocol 8: PP protocol with full power of dense coding; CL protocol	287
8.7.2.2	Protocol 9: LM05 protocol	288
8.7.3	Protocol 10: PP ^{GV} protocol	289
8.8	DLL and modified DLL protocols	291
8.8.1	Protocol 12: DLL protocol	291
8.8.1.1	Protocol 13: The modified DLL protocol (DLL ^{GV})	292
8.9	DSQC protocol and its modifications	293
8.9.1	Protocol 14: The DSQC protocol with arbitrary state	293
8.9.1.1	Protocol 15: A QSDC protocol from Protocol 14	294
8.10	Protocols of quantum dialogue	295
8.10.1	Protocol 16: Ba An protocol	296
8.10.2	Protocol 17: Generalized protocol of quantum dialogue	296
8.10.2.1	Applications of quantum dialogue protocols in the socialist millionaire problem	298
8.11	Protocol 18: Quantum secret sharing	299
8.12	Discussion	301
8.13	Solved examples	303
8.14	Further reading	305
8.15	Exercises	307
	Bibliography	309
	Index	319

Chapter 1

Introduction and overview

Once upon a time there was a curious man. He knew nothing about the subject called “Information Technology.” One day he visited a library and suddenly saw a book entitled “*Information Technology: The Art of Managing Information.*” First, he thought: “This title is not for me. Let me ignore it and look at the next title.” But then the curious man started thinking: “What is it? What is information technology? What is information? Why do I need to know how to manage it?” Since a curious man lives in all of us, it would be tempting to follow the sequence of his thoughts and try to answer these questions. Let us start with a simple question: What is information technology? This question is very important as far as this book or any other text related to information theory is concerned. The simplest answer to this question is already provided in the title of that book as: Information technology is the art of managing information. As soon as we accept this particular definition, the other two questions that appeared in his mind become extremely relevant. In this chapter, we will try to answer those two questions and develop a quantitative perception of classical and quantum information. Once a basic perception is built in the first part of this chapter, we will describe a short history of quantum computation and quantum communication at the end of this chapter.

1.1 What is information?

To a large extent, our general perception of information is qualitative. For example, often after a lecture we say, “this talk was quite informative” or “there was not much new information in this talk.” This type of qualitative perception of information has been in existence from the beginning of human civilization, but a clear definition of information was not present until 1948. To begin with, we may define information as: *Information is something that we do not already know* [1]. Some simple examples may

help us to develop a perception about the meaning of this simple notion of information. Suppose you are watching a football (soccer) match with your friends and you have seen that Ronaldo has scored a goal. Immediately after that, one of your friends shouts: “Oh it’s a goal!” Here, when you see Ronaldo score, you gain some information, but you don’t gain any information from your friend’s shout because you already know that. So your friend’s shout only provides some data to you, but no information. Thus *information is useful data for a particular analysis or decision task*. It helps us to choose reliably between alternatives. Let us give another example. “Sholay” is a popular Hindi movie. In this movie there are two characters called Veeru and Jai. In the movie Jai often tosses a coin, which has the same symbol on both the sides. Jai knows it, but Veeru does not know. Now whatever the call of Veeru, Jai never gains any information from the outcome of the toss since he already knows the result.

There are many technical definitions of information, but here we have opted for a simple definition which states that information is what we do not already know. However, with just a good definition we cannot compare the amount of information. Suppose I want to compare the capacity of your pendrive with that of mine, then the above definition of information will not help us to do the comparison. However, I can conclude that my pendrive is better than your pendrive if my pendrive can store 50 units of information, but your pendrive can store only 30 units of information. To do so, we need a quantitative measure of information. Claude Shannon introduced such a measure of information in 1948 [2]. The existence of a quantitative measure of information implies that information is a quantity and that leads to a fundamental question: Is information a physical quantity? If yes, then we may be able to construct some new physical laws for information and existing laws of the physical world must be applicable to information, too. Further, since the physical world is quantum mechanical the essential nature of information should be quantum mechanical. Thus before we start talking about quantum information, we need to establish that the information is physical.

1.1.1 Is information physical?

Different views about the nature of information have co-existed for centuries. One of those views is that information is not an abstract entity and it is always tied to a physical representation. This particular view was strongly established by Rolf Landauer in the later part of the last century [3]. Landauer argued that since information is always tied to a physical representation the limitations and possibilities of the real physical world would be applicable to information, too. We can obtain a stronger perception of this particular notion of information if we try to understand how information is really stored, transferred or processed in the real world. For example, consider a situation in which we are in an auditorium and I am

delivering a lecture to convince you that information is physical. In this situation, how do you obtain information from me? The words spoken by me are conveyed by air pressure fluctuations which vibrate the membranes of your ears; nerves convert mechanical energy into electrical energy and finally the brain receives an electrical signal and you listen. So a physical process is involved in the communication of information. Similarly, writing on a piece of paper is essentially painting molecules of the paper in a certain meaningful fashion; in a magnetic hard disk we arrange magnetic dipoles in a certain meaningful fashion to store information. In brief, we cannot dissociate information from physical objects and consequently, information is not abstract and laws of physics are applicable to information. This fact that information is physical has a deeper meaning. It intrinsically implies that computer science is part of physics.

Since we need physical means to store, process and communicate information, the physical laws applicable to the physical resources used for the purpose of information processing, storage or communication would be applicable to information, too. A nice example is the following version of Einstein's postulate of the special theory of relativity: We cannot communicate information with velocity greater than that of light in vacuum. We know many things about the essential nature of physical observables. Let us list a few of them as examples and check whether these specific characteristics of physical observables are also observed in information or not.

- **Physical observable can be expressed in various ways without losing its essential nature:** For example, a cricket ball delivered by Kapil Dev and the sound coming out of a drum beaten by one of his excited fans can have the same energy. The same is true for information as it can also be expressed in various ways. For example, the following two statements: "I don't know where Malda is" and "I am completely unaware of the location of Malda" have something in common, although they share only one word in common. Loosely speaking the thing they have in common is their information content [4]. Essentially, the same information can be expressed in various ways, for example, you may substitute numbers for letters in a scheme such as $a=1$; $b=2$; $c=3$ and so on. The fact that information can be expressed in various ways without losing its essential nature is very useful in computation. This is so because it allows automatic manipulation of information. To be precise, it allows us to construct computing machines, which can process information by handling binary digits only [4].
- **Physical quantities can be transformed from one form to another:** For example, electrical energy can be converted to kinetic energy. The same is true for information because information is not sensitive to exactly how it is expressed and it can be easily trans-