HZ BOOKS
经 典 原 版 书 库

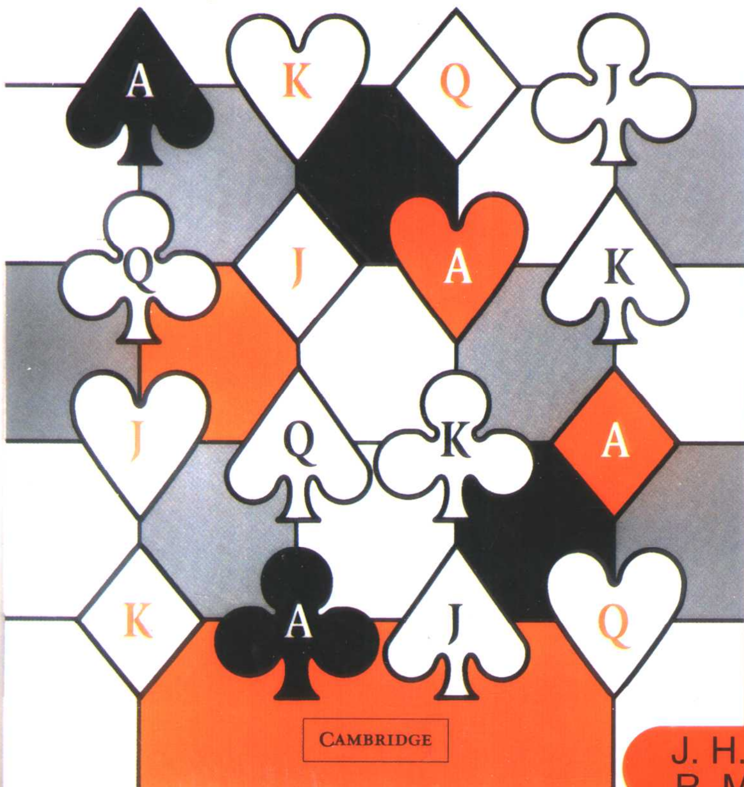
组合数学教程

（英文版 · 第2版）

A COURSE IN Combinatorics

SECOND EDITION

J.H. van Lint & R.M. Wilson



J. H. van Lint
R. M. Wilson 著

机械工业出版社
China Machine Press

经典原版书库

组合数学教程

（英文版·第2版）

A Course in Combinatorics

(Second Edition)

J. H. van Lint
R. M. Wilson 著

机械工业出版社
China Machine Press

J. H. van Lint and R. M. Wilson: A Course in Combinatorics, Second Edition (ISBN: 0-521-00601-5).

Originally published by Cambridge University Press in 1992, 2001.

This reprint edition is published with the permission of the Syndicate of the Press of the University of Cambridge, Cambridge, England.

本书原版由剑桥大学出版社出版。

本书英文影印版由英国剑桥大学出版社授权出版。

本书版权登记号：图字：01-2004-1207

图书在版编目（CIP）数据

组合数学教程（英文版·第2版）/范林特（van Lint, J. H.）等著. -北京：机械工业出版社，2004.4

（经典原版书库）

书名原文：A Course in Combinatorics, Second Edition

ISBN 7-111-13992-5

Ⅰ.组… Ⅱ.范… Ⅲ.组合数学－英文 Ⅳ.O157

中国版本图书馆CIP数据核字（2004）第009769号

机械工业出版社（北京市西城区百万庄大街22号　邮政编码　100037）

责任编辑：迟振春

北京中加印刷有限公司印刷·新华书店北京发行所发行

2004年4月第1版第1次印刷

787mm × 1092mm 1/16 · 38.5印张

印数：0 001-3 000册

定价：58.00元

A course in combinatorics

This is the second edition of a popular book on combinatorics, a subject dealing with ways of arranging and distributing objects, and which involves ideas from geometry, algebra and analysis. The breadth of the theory is matched by that of its applications, which include topics as diverse as codes, circuit design and algorithm complexity. It has thus become essential for workers in many scientific fields to have some familiarity with the subject. The authors have tried to be as comprehensive as possible, dealing in a unified manner with, for example, graph theory, extremal problems, designs, colorings and codes. The depth and breadth of the coverage make the book a unique guide to the whole of the subject. The book is ideal for courses on combinatorial mathematics at the advanced undergraduate or beginning graduate level. Working mathematicians and scientists will also find it a valuable introduction and reference.

J.H. VAN LINT is Emeritus Professor of Mathematics at the Technical University of Einhoven.

R.M. WILSON is Professor of Mathematics at the California Institute of Technology.

Preface to the first edition

One of the most popular upper level mathematics courses taught at Caltech for very many years was H. J. Ryser's course *Combinatorial Analysis*, Math 121. One of Ryser's main goals was to show elegance and simplicity. Furthermore, in this course that he taught so well, he sought to demonstrate coherence of the subject of combinatorics. We dedicate this book to the memory of Herb Ryser, our friend whom we admired and from whom we learned much.

Work on the present book was started during the academic year 1988–89 when the two authors taught the course Math 121 together. Our aim was not only to continue in the style of Ryser by showing many links between areas of combinatorics that seem unrelated, but also to try to more-or-less survey the subject. We had in mind that after a course like this, students who subsequently attend a conference on "Combinatorics" would hear no talks where they are completely lost because of unfamiliarity with the topic. Well, at least they should have heard many of the words before. We strongly believe that a student studying combinatorics should see as many of its branches as possible.

Of course, none of the chapters could possibly give a complete treatment of the subject indicated in their titles. Instead, we cover some highlights—but we insist on doing something substantial or nontrivial with each topic. It is our opinion that a good way to learn combinatorics is to see subjects repeated at intervals. For this reason, several areas are covered in more than one part of the book. For example, partially ordered sets and codes appear several times. Enumeration problems and graph theory occur throughout the book. A few topics are treated in more detail (because we like them) and some material, like our proof of the Van der Waerden permanent conjecture, appears here in a text book for the first time.

A course in modern algebra is sufficient background for this book, but is not absolutely necessary; a great deal can be understood with only a certain level of maturity. Indeed, combinatorics is well known for being "accessible". But readers should find this book challenging and will be expected to fill in details (that we hope are instructive and not too difficult). We mention in passing that we believe there is no substitute for a human teacher when trying to learn a subject. An acquaintance with calculus, groups, finite fields, elementary number theory, and especially linear algebra will be necessary for some topics. Both undergraduates and graduate students take the course at Caltech. The material in every chapter has been presented in class, but we have never managed to do all the chapters in one year.

The notes at the end of chapters often include biographical remarks on mathematicians. We have chosen to refrain from any mention of living mathematicians unless they have retired (with the exception of P. Erdős).

Exercises vary in difficulty. For some it may be necessary to consult the hints in Appendix 1. We include a short discussion of formal power series in Appendix 2.

This manuscript was typeset by the authors in AMS-TeX.

J. H. v. L., R. M. W.

Eindhoven and Pasadena, 1992

Preface to the 2nd edition

The favorable reception of our book and its use for a variety of courses on combinatorial mathematics at numerous colleges and universities has encouraged us to prepare this second edition. We have added new material and have updated references for this version. A number of typographical and other errors have been corrected. We had to change "this century" to "the last century" in several places.

The new material has, for the most part, been inserted into the chapters with the same titles as in the first edition. An exception is that the material of the later chapters on graph theory has been reorganized into four chapters rather than two. The added material includes, for example, discussion of the Lovász sieve, associative block designs, and list colorings of graphs.

Many new problems have been added, and we hope that this last change, in particular, will increase the value of the book as a text. We have decided not to attempt to indicate in the book the level of difficulty of the various problems, but remark again that this can vary greatly. The difficulty will often depend on the experience and background of the reader, and an instructor will need to decide which exercises are appropriate for his or her students. We like the idea of stating problems at the point in the text where they are most relevant, but have also added some problems at the end of the chapters. It is not true that the problems appearing later are necessarily more difficult than those at the beginning of a chapter. A number of the hints and comments in Appendix 1 have been improved.

Preparation of the second edition was done during a six-month visit to the California Institute of Technology by the first author as Moore Distinguished Scholar. He gratefully acknowledges the support of the Moore Foundation.

CONTENTS

1
Graphs

A *graph* G consists of a set V (or $V(G)$) of *vertices*, a set E (or $E(G)$) of *edges*, and a mapping associating to each edge $e \in E(G)$ an unordered pair x, y of vertices called the *endpoints* (or simply the *ends*) of e. We say an edge is *incident* with its ends, and that it *joins* its ends. We allow $x = y$, in which case the edge is called a *loop*. A vertex is *isolated* when it is incident with no edges.

It is common to represent a graph by a *drawing* where we represent each vertex by a point in the plane, and represent edges by line segments or arcs joining some of the pairs of points. One can think e.g. of a network of roads between cities. A graph is called *planar* if it can be drawn in the plane such that no two edges (that is, the line segments or arcs representing the edges) cross. The topic of planarity will be dealt with in Chapter 33; we wish to deal with graphs more purely combinatorially for the present.

edge	ends
a	x, z
b	y, w
c	x, z
d	z, w
e	z, w
f	x, y
g	z, w

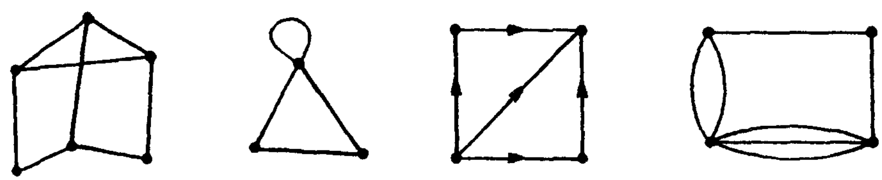
Figure 1.1

Thus a graph is described by a table such as the one in Fig. 1.1 that lists the ends of each edge. Here the graph we are describing has vertex set $V = \{x, y, z, w\}$ and edge set $E = \{a, b, c, d, e, f, g\}$; a drawing of this graph may be found as Fig. 1.2(iv).

A graph is *simple* when it has no loops and no two distinct edges have exactly the same pair of ends. Two nonloops are *parallel* when they have the same ends; graphs that contain them are called *multigraphs* by some authors, or are said to have 'multiple edges'.

If an *ordered* pair of vertices is associated to each edge, we have a *directed graph* or *digraph*. In a drawing of a digraph, we use an arrowhead to point from the first vertex (the *tail*) towards the second vertex (the *head*) incident with an edge. For a *simple* digraph, we disallow loops and require that no two distinct edges have the same ordered pair of ends.

When dealing with simple graphs, it is often convenient to identify the edges with the unordered pairs of vertices they join; thus an edge joining x and y can be called $\{x, y\}$. Similarly, the edges of a simple digraph can be identified with ordered pairs (x, y) of distinct vertices.



(i) graph (ii) graph with loop (iii) digraph (iv) multiple edges

Figure 1.2

There are several ways to draw the same graph. For example, the two graphs of Fig. 1.3 are essentially the same.

We make this more precise, but to avoid unnecessarily technical definitions at this point, let us assume that all graphs are undirected and simple for the next two definitions.

We say two graphs are *isomorphic* if there is a one-to-one correspondence between the vertex sets such that if two vertices are joined by an edge in one graph, then the corresponding vertices are joined by an edge in the other graph. To show that the two graphs in Fig. 1.3 are the same, find a suitable numbering of the vertices in both graphs (using $1, 2, 3, 4, 5, 6$) and observe that the edge sets are the same sets of unordered pairs.

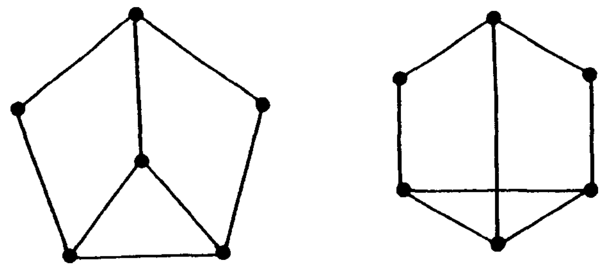


Figure 1.3

A permutation σ of the vertex set of a graph G with the property that $\{a, b\}$ is an edge if and only if $\{\sigma(a), \sigma(b)\}$ is an edge, is called an *automorphism* of G.

Problem 1A. (i) Show that the drawings in Fig. 1.4 represent the same graph (or isomorphic graphs).

(ii) Find the group of automorphisms of the graph in Fig. 1.4. Remark: There is no quick or easy way to do this unless you are lucky; you will have to experiment and try things.

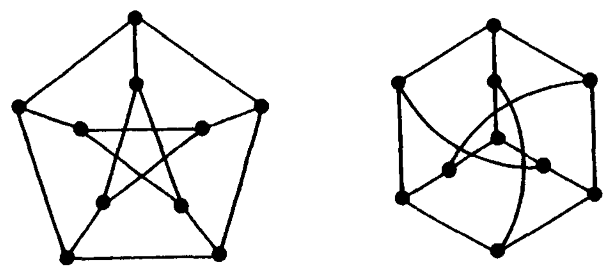


Figure 1.4

The *complete* graph K_n on n vertices is the simple graph that has all $\binom{n}{2}$ possible edges.

Two vertices a and b of a graph G are called *adjacent* if they are distinct and joined by an edge. We will use $\Gamma(x)$ to denote the set of all vertices adjacent to a given vertex x; these vertices are also called the *neighbors* of x.

The number of edges incident with a vertex x is called the *degree* or the *valency* of x. Loops are considered to contribute 2 to the valency, as the pictures we draw suggest. If all the vertices of a graph have the same degree, then the graph is called *regular.*

One of the important tools in combinatorics is the method of *counting* certain objects in two different ways. It is a well known fact that if one makes no mistakes, then the two answers are the same. We give a first elementary example. A graph is *finite* when both $E(G)$ and $V(G)$ are finite sets. We will be primarily concerned with finite graphs, so much so that it is possible we have occasionally forgotten to specify this condition as a hypothesis in some assertions.

Theorem 1.1. *A finite graph G has an even number of vertices with odd valency.*

PROOF: Consider a table listing the ends of the edges, as in Fig. 1.1. The number of entries in the right column of the table is twice the number of edges. On the other hand, the degree of a vertex x is, by definition, the number of times it occurs in the table. So the number of entries in the right column is

$$\sum_{x\in V(G)} \deg(x) = 2|E(G)|. \tag{1.1}$$

The assertion follows immediately. □

The equation (1.1) is simple but important. It might be called the 'first theorem of graph theory', and our Theorem 1.1 is its first corollary.

A *subgraph* of a graph G is a graph H such that $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$, and the ends of an edge $e \in E(H)$ are the same as its ends in G. H is a *spanning* subgraph when $V(H) = V(G)$. The subgraph of G *induced* by a subset S of vertices of G is the subgraph whose vertex set is S and whose edges are *all* the edges of G with both ends in S.

A *walk* in a graph G consists of an alternating sequence

$$x_0, e_1, x_1, e_2, x_2, \ldots, x_{k-1}, e_k, x_k$$