RANDOM VIBRATION OF MECHANICAL SYSTEMS

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Introduction

Great interest in the vibration of systems arises because nearly all technological devices are subjected to vibration of one form or another, and their response may or may not be desirable. For example, the undesirable effects of vibration include noise, the effects on the human body, decreased fatigue strength of machines and equipment, and decreased precision of measuring instruments and machine tools. Sometimes, however, vibration is desirable, as in vibratory conveyors, the lowering of dry friction coefficients, the delamination and consolidation of materials and the use of radio waves.

It is difficult to find a branch of the natural sciences in which vibration processes do not play a large and crucial role. The dynamic response of physical systems depends on the forcing loads. In many cases, especially where a non-linear model of the vibrating system is adopted, these loads are assumed to be non-random, or deterministic. This cannot always be justified; many dynamic physical systems are such that not only their excitation but also their parameters and their initial and boundary conditions are random.

A typical example of vibration caused by random excitation is that experienced by a vehicle moving along an uneven road; the unevenness of the road surface is clearly random. Another example is the action of the wind on engineering structures such as masts, overhead power lines, buildings, and bridges. The action of waves on a ship, of an aeroplane in a turbulent atmosphere and of a gyroscope on a moving object should all be considered to be random or probabilistic problems. Further examples of random motion are the vibrations caused by earthquakes, the vibration

in machines such as excavators and scrapers and the excitation which occurs in jet engines. These problems are described in detail in [10], [11], [12], [31], [48], [66] and [97].

The stochastic analysis of vibrations has as its main aim the determination of several of the characteristics of the vibrating system, such as the mean displacement, velocity or acceleration value, and the variance and correlation function of the appropriate quantities. It is also important to be able to accurately estimate the life and reliability of machines and structures; this is usually linked to the structure material fatigue properties [11].

The optimization and stability of mechanical systems subjected to random excitation are considered in [27], [30], [46], [61], [72], [75], [86], [94], [109] and [3], [8], [10], [26], [47], [53], [54], [96], 108].

In the theory of random vibrations, as with deterministic systems, both linear and non-linear vibrations and discrete and non-discrete systems have to be considered. It should be noted that for these systems the theory of Brownian motion was generalized and presented for the first time by Einstein in 1905 [57].

For linear mechanical systems, methods of analysis have been worked out sufficiently well both for discrete systems [29], [80], [87], [91], [99] and [105] and for systems with distributed parameters [9], [12], [13], [16], [36], [66] and [69]. The analysis of systems with constant coefficients has been worked out in much more detail, and several publications are available on the analysis of systems with random coefficients [4], [6], [10], [20], [26] and [108].

For non-linear systems some solutions exist for mechanical systems with particular types of non-linearity and random loads. These are generally systems with one or two degrees of freedom which are subjected to steady-state excitation. Non-linear systems with distributed parameters can be reduced to systems with a finite number of degrees of freedom by applying methods from the theory of deterministic vibrations. As far as the non-linear system with more than one but less than an infinite number of degrees of freedom is concerned, the desired results may only be obtained in the case of very particular types of non-linearity. Two basic methods of solving non-linear stochastic vibration problems are known; these are a) the method based on the kinetic Fokker–Planck–Kolmogorov diffusion equation, and b) correlational methods.

The kinetic Fokker–Planck–Kolmogorov equation plays a very important role in probabilistic methods. The equation may be utilized to obtain precise solutions, usually in those cases where the non-linearity is

related to the elastic forces. The application of kinematic equations to the analysis of vibrating systems may be found in many publications such as [4], [5], [6], [10], [13], [45], [50], [61], [68], [70], [79] and [80].

Despite the fact that they are more universally used, correlational methods require that the statistical linearization of the system be carried out initially; they are thus approximate methods. As such they utilize direct statistical linearization, Booton–Kazakov's first and second methods, harmonic and statistical linearization in the case in compound deterministic and stochastic excitation, Zajdenberg's linearization and the method of the small parameter. When deterministic and probabilistic methods are simultaneously applied, they complement each other well and form an excellent means for both the analysis and synthesis of non-linear systems. The fact that many investigators have applied a combination of Van der Pol's method, the method of the small parameter, or Bogolubov–Krylov's method with probabilistic methods testifies to this [6], [25], [26], [50], [51], [55], [78], [83] and [85].

It is the aim of the present work to present the theoretical bases of the vibrations of non-linear systems subjected to random loads. Using the methods presented, interesting characteristics of several vibrating systems have been calculated.

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PART I

Introduction to the Theory of Markovian Processes

CHAPTER 1

Random Processes

1.1 RANDOM PROCESS THEORY

In the theory of random processes certain classes of process are distinguished according to the degree of correlation between the random variables. The study of classes of processes is complicated, so that it constitutes a separate field of investigation.

One important class of random processes is the Gaussian process. We say that a random or stochastic process $X(t)$ is a Gaussian or a normal process if, for every instant of time $t_1, t_2, \ldots, t_n$ $(n \geqslant 1)$ the random variables $x_1 = x(t_1), x_2 = x(t_2), \ldots, x_n = x(t_n)$ have an n-dimensional probability density function

$$w_n(x_1, t_1; x_2, t_2; \ldots; x_n, t_n) = \frac{1}{\sigma_1 \sigma_2 \ldots \sigma_n \sqrt{(2\pi)^n \sigma}} \times \exp\left[-\frac{1}{2\sigma} \sum_{\mu,\nu=1}^{n} \sigma_{\mu\nu} \frac{(x_\mu - m_\mu)(x_\nu - m_\nu)}{\sigma_\mu \sigma_\nu} \right], \tag{1.1}$$

where

$$m_i = \langle x_i \rangle, \quad i = 1, 2, \ldots, n, \tag{1.2}$$

represents the mean value, and

$$\sigma_i^2 = \langle (x_i - m_i)^2 \rangle, \quad i = 1, 2, \ldots, n \tag{1.3}$$

is the variance. In addition, the standard deviation σ is given by

$$\sigma = \begin{vmatrix} 1 & \varrho_{12} & \cdots & \varrho_{1n} \\ \varrho_{21} & 1 & \cdots & \varrho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \varrho_{n1} & \varrho_{n2} & \cdots & 1 \end{vmatrix},$$

where

$$\varrho_{ij} = \frac{\langle (x_i - m_i)(x_j - m_j) \rangle}{\sigma_i \sigma_j}, \quad i, j = 1, 2, \ldots, n, \tag{1.4}$$

is the correlation coefficient of the two random variables x_i and x_j. $\sigma_{\mu\nu}$ is the algebraic co-factor of the element $\varrho_{\mu\nu}$ of the determinant σ.

It can be seen that the n-dimensional distribution of the Gaussian process is defined by the mean value of the process

$$m_x = m = m(t) = \langle X(t) \rangle \tag{1.5}$$

and the correlation function $R_x(t, t')$ of the two variables t and t' is given by

$$R_x(t, t') = \langle [X(t) - m(t)] [X(t') - m(t')] \rangle. \tag{1.6}$$

Thus the distribution of the normal process can be completely determined if we know the mean value of the process (1.5) and its eigen-correlation function (1.6), and for this it is sufficient to know the bivariant distribution $X(t)$.

Another class of stochastic processes which is of considerable practical significance is represented by stationary processes, both in the narrow and wider sense. A process is stationary in the narrow sense if the expressions for probability density (1.1) do not change with an arbitrary translation of the time axis. A process is stationary in the wider sense if its mean value is constant and the correlation function is only dependent on the time interval $\tau = t' - t$ (constant variance). If $X(t)$ is stationary in the narrow sense, then it is also stationary in the wider sense, but the reverse is not necessarily true. On the other hand, if a Gaussian process is stationary in the wide sense, then it is also stationary in the narrow sense.

The random variable X has a Poisson distribution if the probability that it assumes a specific value k is expressed by the relationship

$$P_k = P[X = k] = e^{-a} \frac{a^k}{k!}, \tag{1.7}$$

where

$$a > 0, \quad k = 0, 1, 2, \ldots$$

and

$$\sum_{k=0}^{\infty} P_k = 1. \tag{1.8}$$

The constant $a = \langle X \rangle$ is a parameter of the distribution.

The random process $X(t)$ with an analogous distribution to that of (1.7) is defined in a way similar to that of the random variable with a Poisson distribution.

For the n points placed randomly in the interval $(0, T)$, the proba-