

Selected Solutions for

Mathematical Applications

for Management, Life, and Social Sciences

SECOND EDITION

by Ronald J. Harshbarger and James J. Reynolds

JOHN P. RYAN



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The Pennsylvania State University

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Preface

This manual contains detailed solutions to the exercises contained in Mathematical Applications for Management, Life, and Social Sciences, Second Edition, by Ronald J. Harshbarger and James J. Reynolds. We have not repeated graphs that appear in the back of the textbook itself.

It is hoped that, when utilized in an appropriate manner, the solutions presented here will enable the student to develop a deeper understanding of both the mechanics and the style of the solution process. However, one does not learn to do mathematics by merely reading the mathematics that someone else has done. Only after a serious effort to solve a problem has been made should the solution be studied. In this manner, analysis of the solution will be the final step in the learning process, not the only one.

JPR

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Algebra Concepts

EXERCISE 0.1

1. $x \in \{x, y, z, a\}$
3. $12 \in \{1, 2, 3, 4, \dots\}$ (... means that the pattern of elements is to be continued)
5. $4 \in \{x: x \text{ is a natural number less than } 6\}$
7. $\{1, 2, 3, 4, 5, 6, 7\}$ (not 8, since 8 is not less than 8)
9. ϕ . There is no natural number satisfying the requirements
11. $\{x: x \text{ is a natural number greater than } 2 \text{ and less than } 8\}$
13. Yes. Each element of A is also a member of B.
15. No. $c \in A$, but $c \notin B$.
17. $D \subseteq C$.
19. $D \subseteq A$
21. $A \subseteq B$ and $B \subseteq A$
23. Yes
25. No. $4 \in E$, but $4 \notin D$, since 4 is not less than 4
27. A and B ($B = \{5, 6, 7, 8, \dots\}$). B & D; C & D.
29. $A \cap B = \{a, d, e\}$
31. Rewrite A as $\{1, 2, 3\}$. Then $A \cap B = \{3\}$
33. $A \cup B = \{1, 2, 3, 4, 5\}$
35. $A \cup B = \{a, e, i, o, u, b, c, d\}$ (do not list "a" twice)
37. $A \cup B = \{1, 2, 3, 4, 6, 7, \dots\}$
39. $B' = \{1, 2, 5, 6, 7, 9\}$

41. $A \cap B' = \{1, 2, 5, 7\}$
43. $A' \cup B' = \{1, 2, 4, 5, 6, 7, 9, 10\}$ ($A' = \{4, 6, 9, 10\}$)
45. $A - B = \{1, 7\}$ (start with 1, 3, 7 & 9, from A, and remove 3 and 9, which are also in B)
47. ϕ (Each member of A is removed, since each is also in B).
49. (a) $A \cup B = \{\text{Robbins, Shwartz, Gonzales, Barth, McRoberts}\}$
This is the set of all candidates.
- (b) $A \cup B = \{\text{Robbins}\}$. This is the set of all candidates who are both Republican and Democrat.
51. (a) See the figure in the back of the text.
- (b) 40
- (c) 85
- (d) Since $E \cap F' = E - F$, there are 25 aides who speak English, but not French.

EXERCISE 0.2

1. Commutative Property of Addition
 3. Associative Property of Addition
 5. Associative Property of Multiplication
 7. Distributive Property
 9. See the figure in the back of the text.
11. $\sqrt{16} < 5$
13. $\frac{3}{4} < \frac{5}{6}$
15. $-4 < -3$
17. $|-6| = 6$
19. -13
21. -16
23. 13
25. -17
27. 60
29. -22
31. $\pi < \sqrt{10}$
33. $|-3| + |5| > |-3 + 5|$
35. $-\frac{1}{13} > -0.08$
37. $-3^2 + 1.3 = -9 + 1.3 = -7.7$

$$39. -6 \div 2 \cdot 3 = -3 \cdot 3 = -9$$

$$41. \frac{4 + 2^2}{2} = \frac{8}{2} = 4.$$

EXERCISE 0.3

$$1. 2^4$$

$$3. \frac{1}{2^3}$$

$$5. \frac{x}{y^2} (z^0 = 1)$$

$$7. x^3 \cdot x^4 = x^{3+4} = x^7$$

$$9. x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \frac{1}{x^2}$$

$$11. 2^3 \cdot 2^4 = 2^7$$

$$13. x^8 \div x^4 = x^{8-4} = x^4$$

$$15. y^5 \div y^7 = y^{5-7} = y^{-2} = \frac{1}{y^2}$$

$$17. (x^4)^{\frac{1}{2}} = x^{(4)(\frac{1}{2})} = x^2$$

$$19. (xy)^{2/3} = x^{2/3} y^{2/3}$$

$$21. (2x^2y)^4 = 2^4(x^2)^4 y^4 = 16x^8y^4$$

$$23. \left(\frac{x^2}{y}\right)^5 = \frac{x^{10}}{y^5}$$

$$25. \sqrt[5]{-32x^5} = \sqrt[5]{-32} \cdot \sqrt[5]{x^5} = -2x$$

$$27. \sqrt[3]{-27} = -3$$

29. Not a real number.

$$31. (3x)^{-2} = \frac{1}{(3x)^2} = \frac{1}{9x^2}$$

$$33. (2x^2y)^{-3} = \frac{1}{(2x^2y)^3} = \frac{1}{8x^6y^3}$$

$$35. \quad (2x^{-2}y) \div (x^{-1}y^2) = \frac{2y}{x^2} \div \frac{y^2}{x} = \frac{2y}{x^2} \cdot \frac{x}{y^2} = \frac{2}{xy}$$

$$37. \quad (2a)^{1/3} \div (2a)^{2/3} = (2a)^{-1/3} = \frac{1}{\sqrt[3]{2a}}$$

$$39. \quad (2x + 1)^{1/2} [(2x + 1)^{3/2} - (2x + 1)^{-1/2}] = (2x + 1)^2 - (2x + 1)^0 = (2x + 1)^2 - 1 = 4x^2 + 4x$$

$$41. \quad 4\sqrt{x} + \frac{5}{x^3} = 4x^{1/2} + 5x^{-3}$$

$$43. \quad 4x^{2/3} + \frac{2}{3}x^{-4}$$

$$45. \quad \left(\frac{x}{y}\right)^{-n} = \frac{x^{-n}}{y^{-n}} = \frac{\frac{1}{x^n}}{\frac{1}{y^n}} = \frac{1}{x^n} \cdot \frac{y^n}{1} = \frac{y^n}{x^n} = \left(\frac{y}{x}\right)^n$$

EXERCISE 0.4

$$\begin{aligned} 1. \quad & 7x^2 + 8x + 4x^2 + 11x - 6 \\ & = 7x^2 + 4x^2 + 8x + 11x - 6 \\ & = (7 + 4)x^2 + (8 + 11)x - 6 \\ & = 11x^2 + 19x - 6 \end{aligned}$$

$$\begin{aligned} 3. \quad & 3x^3 + 4x^2y^2 + 3x^2y^2 - 7x^3 \\ & = 3x^3 - 7x^3 + 4x^2y^2 + 3x^2y^2 \\ & = (3 - 7)x^3 + (4 + 3)x^2y^2 \\ & = -4x^3 + 7x^2y^2 \end{aligned}$$

$$\begin{aligned} 5. \quad & (4a + 2b) - (3a + 3c) + (6b + 2c) \\ & = 4a + 2b - 3a - 3c + 6b + 2c \\ & = a + 8b - c \end{aligned}$$

$$\begin{aligned}
 7. & \quad (-3x^2y)(2xy^3)(4x^2y^2) \\
 & = (-3)(2)(4) x^2 \cdot x \cdot x^2 \cdot y \cdot y^3 \cdot y^2 \\
 & = -24 x^5 y^6
 \end{aligned}$$

$$\begin{aligned}
 9. & \quad -15m^3n \div 5mn^4 \\
 & = -3m^2n^{-3} \\
 & = \frac{-3m^2}{n^3}
 \end{aligned}$$

$$\begin{aligned}
 13. & \quad x^3 + [3x - (x^3 - 3x)] \\
 & = x^3 + [3x - x^3 + 3x] \\
 & = x^3 + [6x - x^3] \\
 & = x^3 + 6x - x^3 \\
 & = 6x
 \end{aligned}$$

$$\begin{aligned}
 17. & \quad (3y + 4)(2y - 3) \\
 & = 6y^2 - 9y + 8y - 12 \\
 & = 6y^2 - y - 12
 \end{aligned}$$

$$\begin{aligned}
 21. & \quad (2x + 1)(x - 3) \\
 & = 2x^2 - 6x + x - 3 \\
 & = 2x^2 - 5x - 3
 \end{aligned}$$

$$\begin{aligned}
 25. & \quad (x^2 - 3)(x^2 - 2) \\
 & = x^4 - 2x^2 - 3x^2 + 6 \\
 & = x^4 - 5x^2 + 6
 \end{aligned}$$

$$\begin{aligned}
 29. & \quad (3x - 4)(3x + 4) \\
 & = 9x^2 + 12x - 12x - 16 \\
 & = 9x^2 - 16
 \end{aligned}$$

$$\begin{aligned}
 11. & \quad \frac{15y^3z^2}{3yz^2} - \frac{8y^4z}{y^2z} \\
 & = 5y^2 - 8y^2 \\
 & = -3y^2
 \end{aligned}$$

$$15. \quad 2(x + y) = 2x + 2y$$

$$\begin{aligned}
 19. & \quad (x + 1)(x - 2) \\
 & = x^2 - 2x + x - 2 \\
 & = x^2 - x - 2
 \end{aligned}$$

$$\begin{aligned}
 23. & \quad (2x + 1)(3x + 1) \\
 & = 6x^2 + 2x + 3x + 1 \\
 & = 6x^2 + 5x + 1
 \end{aligned}$$

$$\begin{aligned}
 27. & \quad (2x - 3)^2 \\
 & = (2x - 3)(2x - 3) \\
 & = 4x^2 - 6x - 6x + 9 \\
 & = 4x^2 - 12x + 9
 \end{aligned}$$

$$\begin{aligned}
 31. \quad (3x - 4)^3 &= (3x - 4)(3x - 4)^2 \\
 &= (3x - 4)(9x^2 - 24x + 16) \\
 &= 27x^3 - 72x^2 + 48x - 36x^2 + 96x - 64 \\
 &= 27x^3 - 108x^2 + 144x - 64
 \end{aligned}$$

$$\begin{aligned}
 33. \quad (x - 3)(x^2 + 3x + 9) \\
 &= x^3 + 3x^2 + 9x - 3x^2 - 9x - 27 \\
 &= x^3 - 27
 \end{aligned}$$

$$\begin{aligned}
 35. \quad (x^2 + 3)^2 &= (x^2 + 3)(x^2 + 3) \\
 &= x^4 + 3x^2 + 3x^2 + 9 \\
 &= x^4 + 6x^2 + 9
 \end{aligned}$$

$$\begin{aligned}
 37. \quad (x^2 + 2)(x^2 - 2) \\
 &= x^4 - 2x^2 + 2x^2 - 4 \\
 &= x^4 - 4
 \end{aligned}$$

$$\begin{aligned}
 39. \quad (x + 1)(x^2 + 2x + 4) \\
 &= x^3 + 2x^2 + 4x + x^2 + 2x + 4 \\
 &= x^3 + 3x^2 + 6x + 4
 \end{aligned}$$

$$\begin{aligned}
 41. \quad (18m^2n + 6m^2n + 12m^4n^2) \div 6m^2n \\
 &= (24m^2n + 12m^4n^2) \div 6m^2n \\
 &= \frac{24m^2n}{6m^2n} + \frac{12m^4n^2}{6m^2n} \\
 &= 4 + 2m^2n
 \end{aligned}$$

$$\begin{array}{r}
 43. \quad x^2 - 1 \quad \overline{) \begin{array}{r} 3x^2 + 4x + 5 \\ 3x^4 + 4x^3 + 2x^2 - 4x - 5 \\ \underline{3x^4} - 3x^2 \\ 4x^3 + 5x^2 - 4x \\ \underline{4x^3} - 4x \\ 5x^2 - 5 \\ \underline{5x^2} - 5 \\ 0 \end{array}
 \end{array}$$

$$45. \quad 2x + 1 \quad \left| \begin{array}{r} x^2 \qquad - 3 \\ 2x^3 + x^2 - 6x + 1 \\ \underline{2x^3 + x^2} \\ - 6x + 1 \\ \underline{6x - 3} \\ 4 \end{array} \right.$$

$$47. \quad x - 2 \quad \left| \begin{array}{r} x^2 - 2x - 4 \\ x^3 - 4x^2 \\ \underline{x^3 - 2x^2} \\ -2x^2 \\ \underline{-2x^2 + 4x} \\ - 4x + 8 \\ \underline{- 4x + 8} \\ 0 \end{array} \right.$$

EXERCISE 0.5

$$1. \quad 9ab - 12a^2b + 18b^2 \\ = 3b(3a - 4a^2 + 6b)$$

$$3. \quad 4x^2 + 8xy^2 + 2xy^3 \\ = 2x(2x + 4y^2 + y^3)$$

$$5. \quad 5(y - 4) - x^2(y - 4) \\ = (y - 4)(5 - x^2)$$

$$7. \quad 6x - 6m + xy - my \\ = 6(x - m) + y(x - m)$$

$$9. \quad x^2 + 8x + 16 \\ = (x + 4)(x + 4) \\ = (x + 4)^2$$

$$= (x - m)(6 + y)$$

$$11. \quad 7x^2 - 10x - 8 \\ = (7x + 4)(x - 2)$$

$$13. \quad x^2 - 10x + 25 \\ = (x - 5)(x - 5) \\ = (x - 5)^2$$

$$15. \quad 10x^2 + 19x + 6 \\ = (2x + 3)(5x + 2)$$

$$17. \quad 10x^2 - 47x + 9 \\ = (5x - 1)(2x - 9)$$

$$19. \quad 12b^2 - 12b + 3 \\ = 3(4b^2 - 4b + 1) \\ = 3(2b - 1)(2b - 1) \\ = 3(2b - 1)^2$$

$$\begin{aligned}
 21. \quad & 4x^2 + 12x + 9 \\
 & = (2x + 3)(2x + 3) \\
 & = (2x + 3)^2
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & 2x^2 - 8x + 8 \\
 & = 2(x^2 - 4x + 4) \\
 & = 2(x - 2)(x - 2) \\
 & = 2(x - 2)^2
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & 63x^2 - 28 \\
 & = 7(9x^2 - 4) \\
 & = 7(3x + 2)(3x - 2)
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & 49a^2 - 144b^2 \\
 & = (7a + 12b)(7a - 12b)
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & x^4 + 6x^2 + 9 \\
 & = (x^2 + 3)(x^2 + 3) \\
 & = (x^2 + 3)^2
 \end{aligned}$$

$$\begin{aligned}
 31. \quad & y^4 - 16x^4 \\
 & = (y^2 - 4x^2)(y^2 + 4x^2) \\
 & = (y - 2x)(y + 2x)(y^2 + 4x^2)
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & 36x^2 - 49y^2 \\
 & = (6x + 7y)(6x - 7y)
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & 9x^2 - 30x + 25 \\
 & = (3x - 5)(3x - 5) \\
 & = (3x - 5)^2
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & 3x^2 - 24x + 48 \\
 & = 3(x^2 - 8x + 16) \\
 & = 3(x - 4)(x - 4) \\
 & = 3(x - 4)^2
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & 6y^2z^2 - 50yz^2 + 16z^2 \\
 & = 2z^2(3y^2 - 25y + 8) \\
 & = 2z^2(3y - 1)(y - 8)
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & x^4 - 3x^2 - 4 \\
 & = (x^2 - 4)(x^2 + 1) \\
 & = (x + 2)(x - 2)(x^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 43. \quad & x^6 - x^4y^2 \\
 & = x^4(x^2 - y^2) \\
 & = x^4(x + y)(x - y)
 \end{aligned}$$

EXERCISE 0.6

$$1. \frac{18x^3y^3}{9x^3z} = \frac{2y^3}{z}$$

$$3. \frac{x-3y}{3x-9y} = \frac{x-3y}{3(x-3y)} = \frac{1}{3}$$

$$5. \frac{x^2 - 2x + 1}{x^2 - 4x + 3} = \frac{(x-1)(x-1)}{(x-3)(x-1)} = \frac{x-1}{x-3}$$

$$7. \frac{6x^3y^3 - 15x^2y}{3x^2y^2 + 9x^2y} = \frac{3x^2y(2xy^2 - 5)}{3x^2y(y+3)} = \frac{2xy^2 - 5}{y+3}$$

$$9. \frac{6x^3}{8y^3} \cdot \frac{16x}{9y^2} \cdot \frac{15y^4}{x^3} = \frac{2^5 \cdot 3^2 \cdot 5 \cdot x^4 \cdot y^4}{2^3 \cdot 3^2 \cdot x^3 \cdot y^5} = \frac{20x}{y}$$

$$11. \frac{8x-16}{x-3} \cdot \frac{4x-12}{3x-6} = \frac{32(x-2)(x-3)}{3(x-3)(x-2)} = \frac{32}{3}$$

$$13. (x^2 - 4) \cdot \frac{2x-3}{x+2} = \frac{(x+2)(x-2)(2x-3)}{(x+2)} = (x-2)(2x-3)$$

$$15. \frac{x^2 - x - 2}{2x^2 - 18} \cdot \frac{18 - 2x^2}{x^2 - 5x + 4} \cdot \frac{x^2 - 2x - 8}{x^2 - 6x + 9}$$

$$= \frac{2(x-2)(x+1)(3-x)(3+x)(x-4)(x+2)}{2(x+3)(x-3)(x-4)(x-1)(x-3)(x-3)}$$

$$= \frac{3-x}{x-3} \cdot \frac{(x+1)(x-2)(x+2)}{(x-1)(x-3)^2}$$

$$= -\frac{(x+1)(x^2-4)}{(x-1)(x-3)^2}$$

$$17. \frac{15ac^2}{7bd} \div \frac{4a}{14b^2d} = \frac{15ac^2}{7bd} \cdot \frac{14b^2d}{4a}$$

$$= \frac{15bc^2}{2}$$

$$\begin{aligned}
 19. \quad & \frac{y^2 - 2y + 1}{7y^2 - 7y} \div \frac{y^2 - 4y + 3}{35y^2} \\
 &= \frac{(y-1)^2}{7y(y-1)} \cdot \frac{35y^2}{(y-3)(y-1)} \\
 &= \frac{5y}{y-3}
 \end{aligned}$$

$$21. \quad (x^2 - x - 6) \div \frac{x^2 - 9}{x^2 - 3x}$$

$$= \frac{x(x-3)(x+2)(x-3)}{(x+3)(x-3)}$$

$$= \frac{x(x+2)(x-3)}{x+3}$$

$$23. \quad \frac{a}{a-2} - \frac{a-2}{a}$$

$$= \frac{a^2}{a(a-2)} - \frac{(a-2)^2}{a(a-2)}$$

$$= \frac{a^2 - (a^2 - 4a + 4)}{a(a-2)}$$

$$= \frac{4(a-1)}{a(a-2)}$$

$$25. \quad \frac{x-1}{x+1} - \frac{2}{x^2+x}$$

$$= \frac{x-1}{x+1} - \frac{2}{x(x+1)}$$

$$= \frac{x(x-1) - 2}{x(x+1)}$$

$$= \frac{x^2 - x - 2}{x(x+1)}$$

$$= \frac{(x+1)(x-2)}{x(x+1)}$$

$$= \frac{x-2}{x}$$

$$\begin{aligned}
27. \quad & \frac{4a}{3x+6} + \frac{5a^2}{4x+8} \\
&= \frac{4a}{3(x+2)} + \frac{5a^2}{4(x+2)} \\
&= \frac{16a + 15a^2}{12(x+2)} \\
&= \frac{a(15a+16)}{12(x+2)}
\end{aligned}$$

$$\begin{aligned}
29. \quad & \frac{x-7}{x^2-9x+20} + \frac{x+2}{x^2-5x+4} \\
&= \frac{x-7}{(x-5)(x-4)} + \frac{x+2}{(x-4)(x-1)} \\
&= \frac{(x-7)(x-1) + (x+2)(x-5)}{(x-1)(x-4)(x-5)} \\
&= \frac{2x^2 - 11x - 3}{(x-1)(x-4)(x-5)}
\end{aligned}$$

$$\begin{aligned}
31. \quad & \frac{3x-1}{2x-4} + \frac{4x}{3x-6} - \frac{4}{5x-10} \\
&= \frac{3x-1}{2(x-2)} + \frac{4x}{3(x-2)} - \frac{4}{5(x-2)} \\
&= \frac{15(3x-1) + 40x - 24}{30(x-2)} \\
&= \frac{85x-39}{30(x-2)}
\end{aligned}$$

$$\begin{aligned}
 33. \quad & \frac{1}{x^2 - 4y^2} - \frac{1}{x^2 - 4xy + 4y^2} \\
 &= \frac{1}{(x - 2y)(x + 2y)} - \frac{1}{(x - 2y)(x - 2y)} \\
 &= \frac{(x - 2y) - (x + 2y)}{(x + 2y)(x - 2y)^2} \\
 &= \frac{-4y}{(x + 2y)(x - 2y)^2}
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & \frac{x}{x^2 - 4} + \frac{4}{x^2 - x - 2} - \frac{x}{x^2 + 3x + 2} \\
 &= \frac{x}{(x - 2)(x + 2)} + \frac{4}{(x - 2)(x + 1)} - \frac{x}{(x + 2)(x + 1)} \\
 &= \frac{x(x + 1) + 4(x + 2) - x(x - 2)}{(x + 1)(x - 2)(x + 2)} \\
 &= \frac{7x + 8}{(x + 1)(x - 2)(x + 2)}
 \end{aligned}$$

REVIEW EXERCISES

1. Yes. Each member of A is a member of B.
3. $A \cap B = \{1, 3\}$
5. $A - B = \{2, 9\}$
7. See the figure in the back of the text.
9. See the figure in the back of the text.
11. $-2^2 + 1 = -4 + 1 = -3$
13. $(-3)^2 - (-1)^3 = 9 - (-1) = 10$
15. $x^{-2/3} \cdot x^{1/3} = x^{-1/3} = \frac{1}{x^{1/3}}$