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Mathematics

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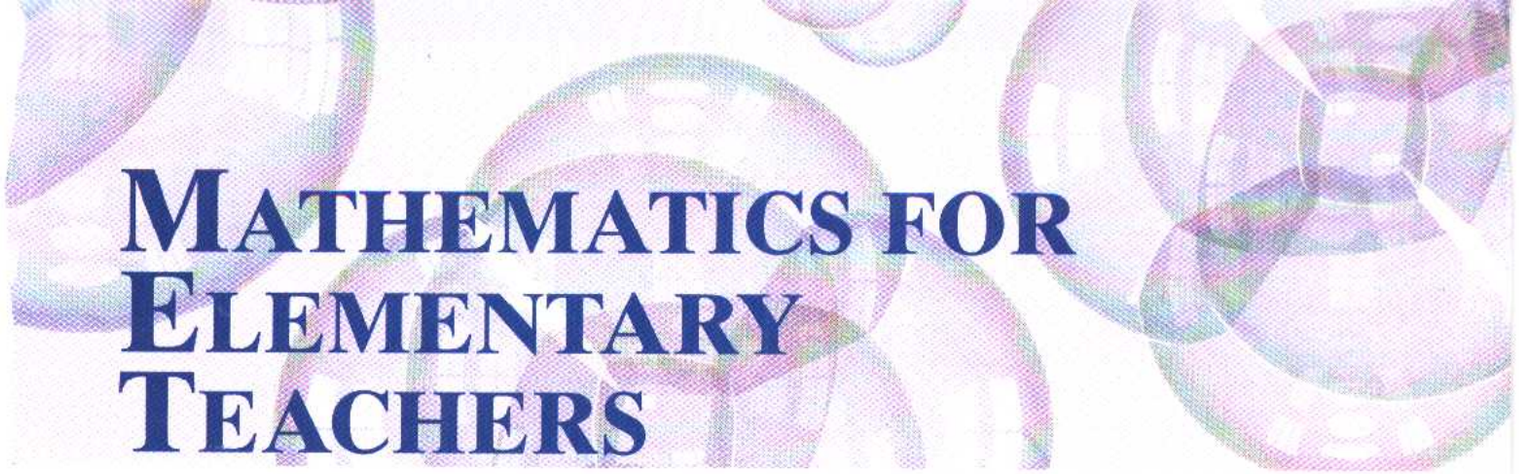
Mathematics

FOR ELEMENTARY TEACHERS

Gary L. Musser
William F. Burger

Fourth Edition

A CONTEMPORARY APPROACH



MATHEMATICS FOR ELEMENTARY TEACHERS

a Contemporary Approach

Fourth Edition

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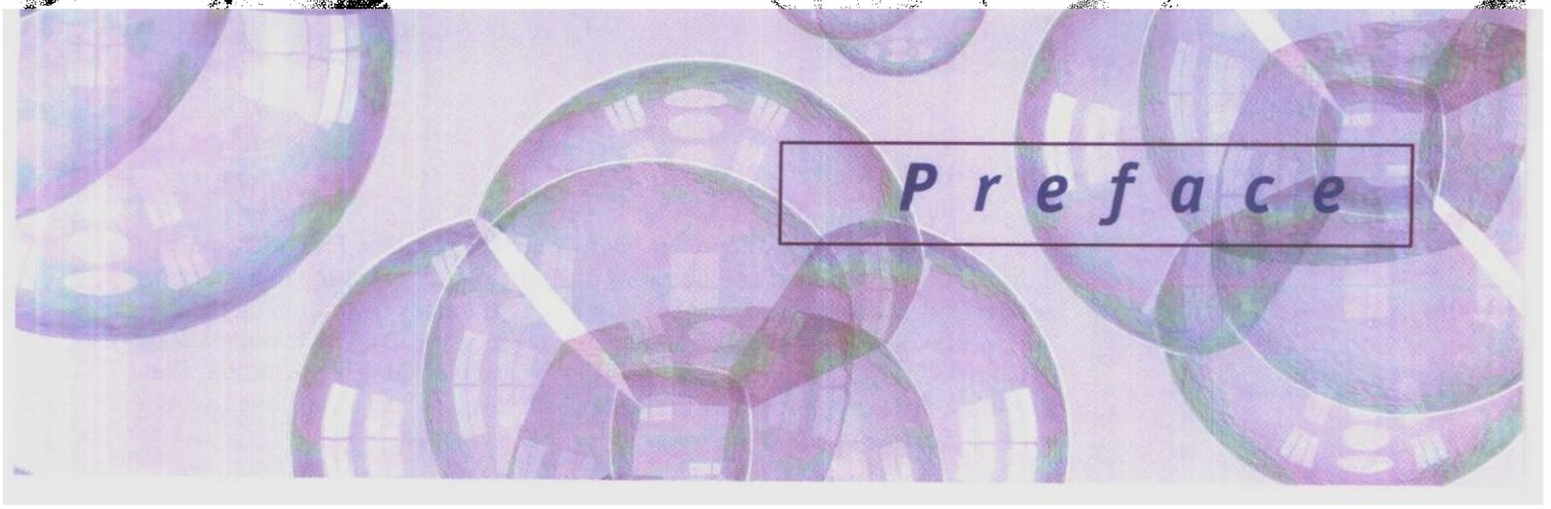
A Special Dedication to Bill

My co-author, and friend, Bill Burger, passed away March 23, 1991, at the young age of 46. Bill was a superb teacher, researcher, colleague, and a pillar of the community. A strong family man, he left his wife, Adrienne, and daughter, Mary, who were very, very dear to him.

I was fortunate to have collaborated with Bill for over 13 years. We had many discussions about students, mathematics, and pedagogy that I will treasure forever. Bill was a class act—integrity and quality were always foremost in his mind. I continue to profit from my many years of work with him. He is missed greatly by all of his students and colleagues, but especially by his family and me. Thanks, Bill, for our many great conversations and for your outstanding contribution to mathematics education.

To:

Irene, my wonderful wife of 35 years; Greg, my great son, for valuing education; Marge, my mother, the best one could hope for; G. L., my father who passed away before I could thank him enough; and Mary, Bill's daughter, she's the greatest.



Preface

There is nothing more satisfying for a teacher than seeing a student experience, master, and apply the skills and knowledge learned in the course. Teaching prospective teachers is even more gratifying, knowing that future generations of students will have capable and well educated teachers guiding them through the inviting world of mathematics. With these goals in mind, this edition was created especially for the students of today.

Interestingly, many of the recommendations of the **National Council of Teachers of Mathematics** (NCTM) and the **Mathematical Association of America** (MAA) reflect the continuing strengths of *Mathematics for Teachers A Contemporary Approach.* In this edition, those features are highlighted for students and teachers alike. Striving to always provide the most current and useable text encouraged even more great ideas and pedagogy to be integrated throughout the text. Thanks go out to all of my colleagues around the country who have provided insightful ideas and advice through questionnaires, reviews, focus groups, and personal visits and communications.

I encourage you to spend a few moments reviewing the following few pages in order to get a better sense of all the exciting changes to the new edition. In addition, all ordering information is included.

As always, I look forward to hearing from you about your experiences with the text. The process of education is never finished. . . .

Gary L. Musser

Content Features

Number Systems: Insofar as possible, number topics are covered sequentially to parallel their development in the school curriculum. Fractions and integers are each treated as extensions of whole numbers. Rational numbers are developed briskly as extensions of both the fractions (by adjoining their opposites) and the integers (by adjoining their reciprocals). The mathematical structure of an ordered field continues to serve to unify this presentation. The important applications of statistics and probability serve as a capstone to the study of number systems.

Approach to Geometry: Geometry is organized from the point of view of the five-level van Hiele model of a child's development in geometry. After studying shapes and measurement, geometry is approached more formally through Euclidean congruence and similarity, coordinates, and transformations. The Epilogue provides an eclectic approach by solving geometry problems using a variety of techniques.

Underlying Themes

Problem-Solving: An extensive collection of problem-solving strategies is progressively developed; these strategies can be applied to a generous supply of problems in the exercise/problem sets. The depth of problem-solving coverage can be varied by the number of strategies selected throughout the book and by the problems assigned.

Deductive Reasoning: The use of deduction is promoted throughout the book. The approach is gradual, with later chapters having more multistep problems. In particular, the last sections of Chapters 14, 15, and 16 and the Epilogue offer a rich source of interesting theorems and problems in geometry.

Technology: The Math Explorer, which is used in many schools, is used to show its capability to do long division with remainder and calculate using fractions as well as to perform the many functions of scientific calculator. The Technology Section includes a study of Logo, computer exploration software for geometry, and graphics calculators.

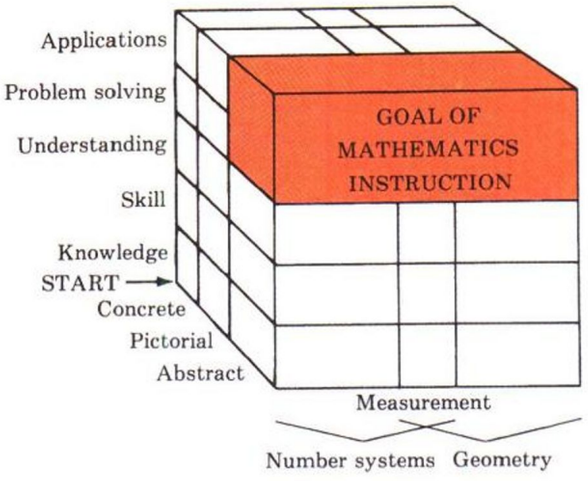
Additional Topics

Since reviewers were split concerning where the following topics should appear in text, they are placed near the end of the book to allow for maximum flexibility.

Topic One, *Elementary Logic,* may be used anywhere in a course.
Topic Two, *Relations,* requires the language of sets in Chapter 2.
Topic Three, *Advanced Counting Techniques: Permutations and Combinations,* may be used to extend Chapter 11—*Probability.*
Topic Four, *Clock Arithmetic—A Mathematical System,* uses the concepts of opposite and reciprocal, and hence may be most instructive after Chapter 6—*Fractions* and Chapter 8—*Integers* have been completed. This last section also contains an introduction to modular arithmetic.

Pedagogical Features

The general organization of the book was motivated by the following "mathematics learning cube.":



The three dimensions of the cube—cognitive levels, representational levels, and mathematical content—are integrated throughout the textual material as well as in the problem sets and chapter tests. Problem sets are organized into exercises (to support knowledge, skill, and understanding) and problems (to support problem solving and applications).

Course Options

The material in this book has been organized to allow for a wide variety of courses. At Oregon State University, all preservice elementary teachers are required to take 12 quarter-hours of mathematics. This book is used for the first 9 quarter-hours, a one-year sequence. Each week, students attend 3 one-hour lectures and a 1-hour lab in which materials from the Student Resource Handbook are covered. This book, together with the Problem-Solving Study Guide, is also used in a 3 quarter-hour junior level course in problem solving for elementary teachers. This course is devoted entirely to developing the students' facilities in applying Polya's four-step process and the strategies introduced at the beginning of each chapter.

Since many schools have special mathematical requirements, below are suggested chapters and topics to fit any particular course.

Basic course: Chapters 1–7.

Basic course with Logic: Topic 1, Chapters 1–7.

Basic course with Informal Geometry: Chapters 1–7, 12.

Basic course with Introduction to Geometry and Measurement: Chapters 1–7, 12, 13.

Course Through Real Numbers: Chapters 1–9.

Course Through Real Numbers with applications: Chapters 1–9, 10, and/or 11.

Course Through Real Numbers with Geometry: Chapters 1–9, 12, 13, 14 (Sections 1 and 2), 15 (Sections 1 and 2), 16 (Sections 1 and 2), Epilogue.

Course in Geometry: Chapters 12–16 (with or without Programming in Logo—Turtle Geometry and exploration software.)

Summary of Changes in the Fourth Edition

- *Exercise/problem sets* have been thoroughly revised. The A and B *exercises* are matched and arranged to follow the sequential development in each section.
- *Chapter reviews* have been organized to help students identify weaknesses. These reviews can also be used as a diagnostic preview to each chapter.
- An *Epilogue* has been added to encourage students to apply a variety of techniques to solving grometry problems.
- *'Strategy' problems* appear in exercise/problem sets throughout to provide for continual review of the strategies.
- Margins contain references to the *NCTM Standards* and problem-solving strategies at relevant points.
- *'Reflections on Research'* appear in margins throughout to add credence to our approach.
- Representative student questions have been added to the *Student Page Snapshots.*
- *Writing/discussion and project problems* have been added to each section.

Acknowledgments

I would like to acknowledge the following people for their assistance in the preparation of the first three editions of this book: Ron Bagwell, Julie Borden, Sue Borden, Tommy Bryan, Christie Gilliland, Dale Green, Kathleen Seagraves Higdon, Lilian Brady, Roger Maurer, David Metz, Naomi Munton, Tilda Runner, Karen Swenson, Donna Templeton, Lynn Trimpe, Rosemary Troxel, and Kris Warloe.

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text preview

The following pages highlight the special features of the text and its supplements.

chapter one

1 INTRODUCTION TO PROBLEM SOLVING

focus on: GEORGE PÓLYA: THE FATHER OF MODERN PROBLEM SOLVING

George Pólya was born in Hungary in 1887. He received his Ph.D. at the University of Budapest. In 1940 he came to Brown University and then joined the faculty at Stanford University in 1942.

In his studies, he became interested in the process of discovery which led to his famous four-step process for solving problems:

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back.

He died in 1985 leaving mathematics with the important legacy of teaching problem solving. His "Ten Commandments for Teachers" are:

1. Be interested in your subject.
2. Know your subject.
3. Try to read the faces of your students; try to see their expectations and difficulties; put yourself in their place.
4. Realize that the best way to learn anything is to discover it by yourself.
5. Give your students not only information, but also know-how, mental attitudes, the habit of methodical work.
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving the problems to come—try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once—let the students guess before you tell it—let them find out by themselves as much as is feasible.
10. Suggest it; do not force it down their throats.

George Pólya

Pólya wrote over 250 mathematical papers and three books that promote problem solving. His most famous book, *How to Solve It*, which has been translated into 15 languages, introduced his four-step approach together with heuristics, or strategies, which are helpful in solving problems. Other important works of Pólya are *Mathematical Discovery*, Volumes 1 and 2, and *Mathematics and Plausible Reasoning*, Volumes 1 and 2.

1

Focus On: Each chapter opens with an interesting historical feature which introduces ideas and concepts central to the chapter.

enrichment features

Problem-Solving Strategies

1. Guess and Test
2. Use a Variable
3. Draw a Picture
4. Look for a Pattern
5. Make a List
6. Solve a Simpler Problem
7. Draw a Diagram
8. **Use Direct Reasoning**

Problem Solving Strategies: Each chapter introduces new Problem Solving Strategies; a comprehensive list is included at the beginning of each chapter.

Strategy 8: Use Direct Reasoning

The Use Direct Reasoning strategy is used virtually all the time in conjunction with other strategies when solving problems. Direct reasoning is used to reach a valid conclusion from a series of statements. Often, statements involving direct reasoning are of the form "If A then B." Once this statement is shown to be true, statement B will hold whenever statement A does. (An expanded discussion of reasoning is contained in the Logic section near the end of the book.) In the following initial problem, no computations are required. That is, a solution can be obtained merely by using direct reasoning, and perhaps by drawing pictures.

Initial problem

In a group of nine coins, eight weigh the same and the ninth is heavier. Assume that the coins are identical in appearance. Using a pan balance, what is the smallest number of weighings needed to identify the heavy coin?

Initial Problem: At the beginning of each chapter, an initial problem is posed that can be solved by using the strategy introduced in that chapter.

NCTM Standard

Analyze functional relationships to explain how a change in one quantity results in a change in another.

NCTM Standards: NCTM Standards are called out in the margins where the Standard listed is being used in the text.

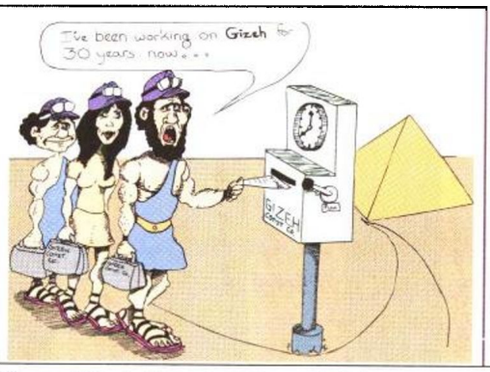
Reflection from Research

Second- and third-grade students tend to use repeated addition to solve simple multiplication AND division problems. In a division problem, such as $15 \div 5$, they will repeatedly add the divisor until they reach the quotient ($5 + 5 = 10$; $10 + 5 = 15$) often using their fingers to keep track of the number of times they use 5 (Mulligan & Mitchelmore, 1995).

Reflections on Research: Encouraging the recognition of the research that is being drawn on throughout the text, these marginal notes provide a great portfolio of current research.

Mathematical morsel

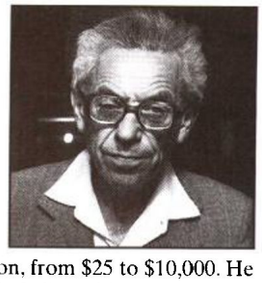
Around 2900 B.C. the great Pyramid of Gizeh was constructed. It covered 13 acres and contained over 2,000,000 stone blocks averaging 2.5 tons each. Some chamber roofs are made of 54-ton granite blocks, 27 feet long and 4 feet thick, hauled from a quarry 600 miles away and set into place 200 feet above the ground. The relative error in the lengths of the sides of the square base was $\frac{1}{14,000}$ and in the right angles was $\frac{1}{27,000}$. This construction was estimated to have required 100,000 laborers working for about 30 years.



Mathematical Morsels: These tidbits are the reward for completing a section. All of these have been illustrated by a former student who is now a fifth grade teacher.

People in Mathematics

EVELYN BOYD GRANVILLE (1924–) Evelyn Boyd Granville was a mathematician in the Mercury and Apollo space programs, specializing in orbit and trajectory computations and computer techniques. She says that if she had foreseen the space program and her role in it, she would have been an astronomer. Evelyn grew up in Washington, D.C., at a time when the public schools were racially segregated. She was fortunate to attend a black high school with high standards and was encouraged to apply to the best colleges. In 1949 she graduated from Yale with a Ph.D. in mathematics, one of two black women to receive doctorates in mathematics that year and the first ever to do so. Following her work with the space program, she joined the mathematics faculty at California State University. She has written (with Jason Frand) the text *Theory and Application of Mathematics for Teachers.* "I never encountered any problems in combining career and private life. Black women have always had to work."



PAUL ERDOS (1913–) Paul Erdos is one of the most prolific of mathematicians. Erdos (pronounced "air-dish") has authored or coauthored approximately 900 research papers. He has been called an "itinerant mathematician" because of his penchant for traveling to mathematical conferences around the world. His achievements in number theory are legendary. At one mathematical conference, he was dozing during a lecture of no particular interest to him. When the speaker mentioned a problem in number theory, Erdos perked up and asked him to explain the problem again. The lecture then proceeded, and a few minutes later Erdos interrupted to announce that he had the solution! Erdos is also known for posing problems and offering monetary awards for their solution, from $25 to $10,000. He also is known for the many mathematical prodigies he has discovered and "fed" problems.

People in Mathematics: Near the end of each chapter, stories highlight many of the giants in mathematics throughout history as well as others who have contributed to mathematics in various ways. Hopefully, the accomplishments of these people will help you see how you, too, will be making a significant contribution through your own teaching.

Student page snapshot

INVESTIGATING
PROBLEM SOLVING

THINK
EXPLORE
SOLVE
LOOK BACK

How Much Antifreeze?

Dr. Spellman is preparing antifreeze and water to add to her car radiator. The radiator holds 8 quarts of liquid. Last winter the lowest temperature was −20 degrees F. She used this chart to determine the amount of antifreeze and water to add to the radiator. The numbers indicate the percent of the solution made up of glycol or isopropyl alcohol.

Freezing Points of Different Antifreeze Mixtures

Degree F	+10	0	−10	−20	−30	−40	−50
% of Glycol in Solution	24	32	38	43	48	52	56
% of Isopropyl Alcohol in Solution	33	42	61	72	78	81	

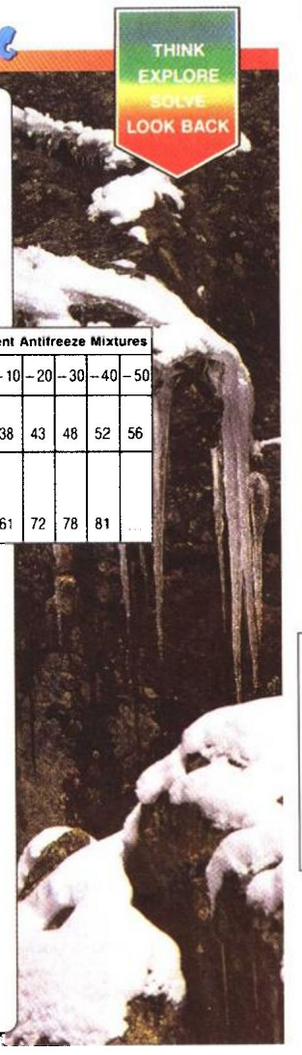
Thinking Critically

To what temperature should Dr. Spellman protect her car? How much antifreeze should she add to her radiator?

Analyzing and Making Decisions

1. What was the coldest temperature last winter?
2. Which type of antifreeze does she need more of to protect the car to 0 degrees F? Explain.
3. **What if** she wanted to protect her car to −20 degrees F? What percentage of glycol would she need? What percentage of water would she need? How many quarts of each would she need? What percentage of isopropyl alcohol and of water would she need? How many quarts of each would she need?
4. To what temperature would you recommend she protect her car? If she used glycol and water, what percentage of each would she need? How many quarts of each would that be? What if she used isopropyl alcohol and water? What percentage would she need? How many quarts of each would that be?

Look Back **What if** Dr. Spellman wanted to protect her car to −60 degrees F? Estimate how she could do that with each kind of antifreeze.



Student Page Snapshot: Each chapter has a page from an elementary school textbook together with questions you as a teacher will need to consider.

1. What are some issues that Dr. Spellman should consider when preparing her car for winter?
2. Answer question 4 above.
3. Describe how you could estimate the percentage of glycol or isopropyl alcohol needed to protect the car down to −60° F as suggested in the Look Back question.

study features

TABLE 2.6

number of splits	number of amoebas
1	2
2	4
3	8
4	16
.	.
.	.
.	.
n	2^n

4. Amoebas regenerate themselves by splitting into two amoebas. Table 2.6 shows the relationship between the number of splits and the number of amoebas after that split, starting with one amoeba. Notice how the number of amoebas grows rapidly. The rapid growth as described in this table is called **exponential growth.**

Although the preceding examples are different in some ways, underlying each of them is the concept of function.

DEFINITION

Function

A **function** is a rule that assigns to each element of a first set an element of a second set in such a way that no element in the first set is assigned to two different elements in the second set.

Key Terms: Key terms appear in boldface — they are also contained in the chapter review to remind you of their importance.

NCTM Standard

Represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations.

TABLE 9.1

sales, s	earnings, $E(s)$
1000	1250
2000	1300
3000	1350
4000	1400
5000	1450

Reflection from Research

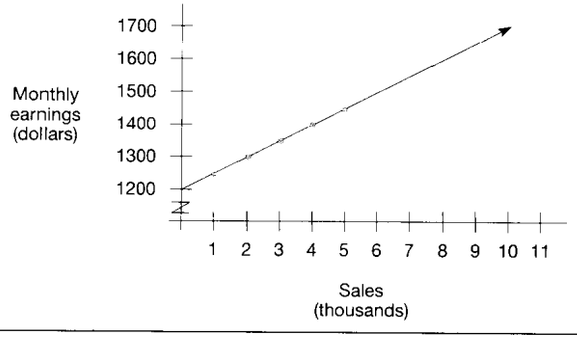
Having students describe a possible relationship to real-world phenomena modeled by a graph may encourage students' understanding of the purpose of graphs (Narode, 1986).

Example 9.16

A salesperson is given a monthly salary of $1200 plus a 5% commission on sales. Graph the salesperson's total earnings as a function of sales.

Solution

Let s represent the dollar amount of the salesperson's monthly sales. The total earnings can be represented as a function of sales, s, as follows: $E(s) = 1200 + (0.05)s$. Several values of this function are shown in Table 9.1. Using these values we can plot the function $E(s)$ (Figure 9.16). The mark on the vertical axis below 1200 is used to indicate that this portion of the graph is not the same scale as on the rest of the axis.



Examples: Many examples are provided to illustrate major ideas — working these while reading each section will clarify concepts.

DEFINITION

Division of Rational Numbers

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any rational numbers where $\frac{c}{d}$ is nonzero. Then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

Definitions, Theorems, and Properties: Main definitions, theorems and Properties in each section are highlighted by boxes for quick review. You must fully *understand* the material contained in these boxes to master the material in this book.

THEOREM

Fundamental Theorem of Arithmetic

Each composite number can be expressed as the product of primes in exactly one way (except for the order of the factors).

PROPERTIES

Properties of Integer Multiplication

Let a, b, and c be any integers.

closure property for integer multiplication

ab is an integer.

commutative property for integer multiplication

$ab = ba$.

associative property for integer multiplication

$(ab)c = a(bc)$.

identity property for integer multiplication

1 is the unique integer such that $a \cdot 1 = a = 1 \cdot a$ for all a.