

Nuclear Reactor **PHYSICS**

RAYMOND L. MURRAY

Nuclear Reactor Physics

RAYMOND L. MURRAY

*Professor of Physics
North Carolina State College
of Agriculture and Engineering*

Englewood Cliffs, N.J.

PRENTICE-HALL, INC.

© COPYRIGHT 1957 BY
PRENTICE-HALL, INC.
ENGLEWOOD CLIFFS, N.J.

ALL RIGHTS RESERVED. NO PART OF THIS BOOK
MAY BE REPRODUCED IN ANY FORM, BY MIMEO-
GRAPH OR ANY OTHER MEANS, WITHOUT PERMIS-
SION IN WRITING FROM THE PUBLISHERS.

Library of Congress Catalog Card No.: 57-8234

First printing March, 1957
Second printing September, 1957

Nuclear Reactor Physics

A SELECTIVE LIST OF OTHER PRENTICE-HALL TECHNICAL BOOKS

Elements of Atomic Physics, Peaslee and Mueller
Experimental Spectroscopy, 2nd ed., Sawyer
Fundamental Formulas of Physics, Menzel, editor
Fundamentals of Quantum Mechanics, Persico
Heat and Temperature Measurement, Weber
High Energy Particles, Rossi
Industrial Electronic Engineering, Davis and Weed
Introduction to Experimental Physics, Fretter
Introduction to Nuclear Engineering, Murray
Mathematical Methods for Scientists and Engineers, Smith
Modern Physics, Van Name
Practical Spectroscopy, Harrison, Lord, and Loofbourow
Principles of Automatic Controls, Nixon
Procedures in Experimental Physics, Strong, Cartwright, et al.
Quantum Chemistry, Pitzer
Quantum Theory, Bohm
Solid State Physics, Dekker
Thermodynamics of Fluid Flow, Hall

PREFACE

NUCLEAR REACTOR PHYSICS has come to connote the analysis of the behavior of an assembly of fissionable material. This book is intended to serve as an introduction to the physical concepts and calculation methods in this new branch of applied physics. It is designed for use by the first-year graduate student in science or engineering and the design engineer in the nuclear energy field. The principal emphasis is placed on the distributions in energy and space of neutron flux, the determination of the critical amount of fissionable material, and the transient behavior and control of the reactor as a heat source. It is assumed that the reader has familiarity with the fundamental facts of nuclear physics, but it is recognized that most of the description of the chain reactor is in terms of classical models. The goals that have been sought are to present the theory simply and logically, and to provide enough detail in the many numerical illustrations to achieve a degree of practical utility. Problems, with answers, are given at the end of each chapter.

The material presented is based on a series of undergraduate and graduate courses given by the author since 1950 in the Nuclear Engineering curriculum at North Carolina State College.

Valuable suggestions and assistance were provided by a number of people. Professor William M. Breazeale, Professor Richard A. Fayram, and Dr. Alfred M. Perry reviewed the first draft. Thanks are due also to Arthur Banister, Gene Baraff, Gerald Katzin, Harold Lamonds, Joseph Lundholm, Chreston Martin, and Charles Terrell of North Carolina State College for reading certain sections. The encouragement and assistance from the author's wife, Ilah Mae Rengler Murray, is gratefully recognized.

RAYMOND L. MURRAY

Raleigh, North Carolina

CONTENTS

Chapter 1. <i>The Nuclear Reactor</i>	1
1.1. Nuclear reactions with neutrons	1
1.2. Fission	4
1.3. The nuclear chain reactor	6
1.4. Reactor types and examples	8
1.5. Criticality and neutron economy	11
1.6. Design problems	13
1.7. Nuclear reactor theory	15
 Chapter 2. <i>Neutron Motion</i>	 19
2.1. Cross sections	19
2.2. Neutron flux	23
2.3. Neutron current	25
2.4. Maxwellian flux distribution	30
2.5. Neutron energy losses	33
2.6. Epithermal and fast flux distributions	37
 Chapter 3. <i>Flux Distributions and Critical Mass</i>	 47
3.1. The diffusion equation	47
3.2. Solution of diffusion equation in a thermal reactor	50
3.3. Fermi age theory	57
3.4. One-group theory	64
3.5. Modified one-group theory	69
3.6. Integral formulation of neutron motion	73
 Chapter 4. <i>The Heterogeneous Reactor</i>	 82
4.1. Flux distribution and thermal utilization, f	84
4.2. Flux in cylindrical system	89
4.3. Resonance escape probability, p	92
4.4. Fast fission factor, ϵ	95

4.5. Multiplication factor and critical dimensions	98
4.6. The exponential pile	102
Chapter 5. Two-Group Theory	110
5.1. Formulation of differential equations and general solutions	110
5.2. Critical condition	118
5.3. Flux distributions	120
5.4. Reactor design calculation	121
5.5. Modified two-group theory	133
5.6. Matrix-method solution for multiple reflectors	138
Chapter 6. The Time-Dependent Reactor	145
6.1. Neutron accumulation	145
6.2. Factors affecting transient response	150
6.3. Differential equations of time-dependent reactor	152
6.4. Accident conditions	154
6.5. Inhour equation	156
6.6. One-delayed-neutron group approximation	160
6.7. Response to the ramp function	163
6.8. Sub-critical multiplication—reactor startup	164
6.9. Reactor transient simulators	167
Chapter 7. Temperature Effects on Multiplication	174
7.1. Temperature coefficient and thermal expansion	174
7.2. Cross section variations	177
7.3. Power and temperature distributions	182
7.4. Distributed fuel and coolant	188
7.5. Temperature-reactivity-power coupling	196
Chapter 8. Reactor Control	210
8.1. Control rods	210
8.2. Reactivity values of control rods by the two-group model	215
8.3. Reactivity changes with time in a converter reactor	220
8.4. Control by burnable poison	224
8.5. Transient fission product poisons	226
8.6. Calculation of reactivity by perturbation methods	231
Chapter 9. Transport Theory	239
9.1. Distribution of flux in direction	239
9.2. Derivation of Boltzmann or transport equation	242

CONTENTS

ix

9.3. Rigorous solution for heavy element	245
9.4. Spherical harmonics solution	247
9.5. Spherical harmonics treatment of heterogeneous reactors	250
9.6. Integral form of transport equation	259
9.7. Applications to neutron absorption problems	262
 Chapter 10. Neutron Slowing and Multigroup Methods	268
10.1. The energy-dependent transport equation	268
10.2. Age theory	271
10.3. Solution for hydrogenous mixture	272
10.4. Improvements in age theory for hydrogenous mixtures	275
10.5. Multigroup methods for intermediate reactors	280
10.6. Analogue computation of space-dependent flux	290
 <i>Appendices</i>	297
A. Bessel Functions	299
B. Physical Constants	308
C. Laplace Transforms	309
 <i>Index</i>	311

Chapter 1

THE NUCLEAR REACTOR

A nuclear reactor may be defined as a device in which nuclear energy is liberated as a result of a chain reaction involving neutrons and fissionable elements. It is a source of thermal and radiant energy that utilizes nuclear reactions rather than chemical or electrical processes. As in other systems, the factors that must be considered in design and operation are the components or ingredients, the arrangement, and the method of control. The nuclear aspect introduces problems and methods of analysis that are unique in industrial practice. The behavior of the system often may be treated by the classic kinetic theory of gases and by methods conventional in the analysis of heat transfer and electric circuits. The interactions of component particles, however, are described by the concepts of nuclear physics. On the assumption that the reader has studied the equivalent of an introductory course in nuclear physics, we shall restrict attention to the reactions of special application to the theory of nuclear reactors.

1.1. Nuclear reactions with neutrons

The neutron plays a central role in the nuclear reactor since it serves as the agent by which nuclear fission occurs. Having no electric charge, it is not influenced by the presence of matter unless it comes within a distance of about 10^{-12} cm of the nucleus. Once within this range, it is subject to one of two events—scattering or absorption. The conventional classification of such neutron reactions follows.

(a) *Elastic collision.* The collision of a neutron with a nucleus may be elastic, with momentum and kinetic energy conserved. This will result simply in a transfer of part of the neutron kinetic energy to the target nucleus, with a change in the direction of neutron motion. Details of the relations between energy, speed, and angles in the elastic scattering process will be given in the next chapter.

(b) *Inelastic collision.* In heavy elements such as iron or uranium, a neutron with energy in the vicinity of 1 mev may produce excitation of the nucleus. The neutron thus may lose a large fraction of its initial energy. The nucleus returns to the ground energy state by the emission of a gamma ray.

(c) *Radiative capture.* Neutron absorption may convert the nucleus into a different isotope. The formation of Co^{60} from Co^{59} is typical. Excess energy resulting from the absorption of the neutron is released almost instantaneously by the nucleus as a capture gamma ray; if the product isotope is radioactive, it will emit beta particles and additional gamma rays according to its half-life.

(d) *Capture with charged particle emission.* If the neutron energy is high enough, a transmutation with the ejection of a proton or alpha particle will occur. The production of N^{16} , by the reaction with O^{16} of neutrons with energy above 10 mev, is an important example.

(e) *Fission.* The neutron may induce fission. In the isotopes U^{235} , Pu^{239} , and U^{233} , fission can be produced by either low or high energy neutrons, with the probability of fission particularly high for slow neutrons. U^{238} will fission only with neutrons of energy above 1 mev. This event results in the emission of several fast neutrons, which may be used to sustain a chain reaction.

(f) *Fissionable isotope production.* Radiative capture of neutrons in the isotopes ${}_{92}\text{U}^{238}$ and ${}_{90}\text{Th}^{232}$ leads to new fissionable elements, plutonium ${}_{94}\text{Pu}^{239}$ and ${}_{92}\text{U}^{233}$, according to the sequence of events below:

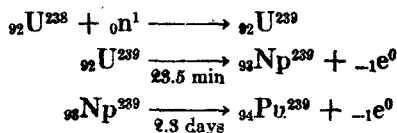
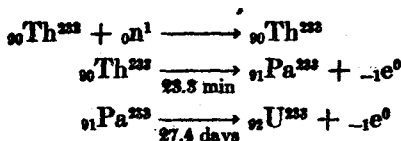


Table 1.1

Neutron Reactions and Radioactive Products

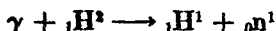
Unless otherwise noted, the neutron energy is 0.0253 ev. Beta energies are maximum values.

Target Isotope	Cross Section (barns)	Maximum Capture Gamma Energy (mev)	Product Isotope(s)	Product Radiations and Energy (mev)	Half-Life	
^1_1H	0.380	2.23	$^1_1\text{H}^+$		Stable	Secondary radiation in hydrogenous shields
$^4_2\text{He}^+$	0.010	6.81	$^4_2\text{He}^{10}$	β (0.56)	2.9×10^8 yr	Reactor construction material
$^3_{10}\text{B}$	4010	0.478	$^3_{10}\text{He}^4$ $^3_{10}\text{Li}^7$		Stable	Control elements, neutron detectors, shielding
$^{12}_{13}\text{C}$	0.0032	4.95	$^{12}_{13}\text{C}^{13}$		Stable	Capture gammas in graphite thermal columns
$^{14}_7\text{N}$	(>10.75 mev)		$^{14}_7\text{H}^+$ $^{14}_6\text{C}^{14}$	β (0.155)	5568 yr	Production of radioactive tracer
$^{16}_8\text{O}$	0.07 at 14 mev		$^{16}_8\text{H}^+$ $^{16}_7\text{N}^{16}$	80% γ (0.2-7)	7.53 sec	Water activation, shielding
$^{18}_8\text{O}$	2.1×10^{-4}		$^{18}_8\text{O}^{18}$	90% β (4.5) 70% β (2.9) γ (1.6)	29.4 sec	Water activation, shielding
$^{23}_{11}\text{Na}$	0.505	6.41	$^{23}_{11}\text{Na}^{24}$	β (1.39) γ (2.76, 1.38)	15.1 hr	Shielding
$^{27}_{13}\text{Al}$	0.250	7.72	$^{27}_{13}\text{Al}^{28}$	β (3.0) γ (1.8)	2.27 min	Structural activation, shielding
$^{40}_{18}\text{Ar}$	0.62		$^{40}_{18}\text{Ar}^{41}$	β (1.13) γ (1.37)	109 min	Radioactivity in air
$^{59}_{27}\text{Co}$	37.0	7.85	$^{59}_{27}\text{Co}^{60}$	β (0.31) γ (1.17, 1.33)	5.27 yr	Radioactive isotope for medical applications
$^{113}_{48}\text{Cd}$	20,800	9.05	$^{113}_{48}\text{Cd}^{114}$		Stable	Reactor control elements
$^{115}_{49}\text{In}$	145	5.86	$^{115}_{49}\text{In}^{116}$	β (1.0) γ (0.1-2.1)	58.9 min	Neutron detecting foils

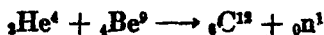


These reactions relate to the *converter* and *breeder* reactors, in which useful fissionable materials not found abundantly in nature are formed.

Table 1.1 lists a number of other important neutron reactions, with the characteristics of the resulting radioactive isotopes. In this table, the cross section, to be discussed in detail in Chapter 2, is a measure of the probability of absorption. Two other non-fission nuclear reactions that provide neutrons are important in the reactor field. The photodisintegration of heavy hydrogen is given by

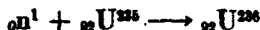


The alpha particles from radium and its products react with beryllium according to



1.2. Fission

The *fission* process consists of division of a nucleus such as U^{235} into two or more heavy fragments of much lower mass and atomic numbers than the original element. The first step is the absorption of a neutron:



After emission of a capture gamma ray, the U^{236} nucleus may remain intact. In 16 per cent of all absorptions the result is essentially stable (2.4×10^7 yr) U^{236} ; in the remaining 84 per cent the result is fission. The U^{236} may split in many ways. As a typical example,



The fission fragments have two important properties, kinetic energy and radioactivity. They are highly unstable, since they have a large neutron excess over the stable element of the same

atomic number. They decay radioactively by a chain of beta and gamma ray emissions. The total *sensible* energy release from fission is around 190 mev on the average, divided among the products approximately as follows:

167	kinetic energy of fission products
5	fast neutrons
7	instantaneously emitted gamma rays
5	fission product decay beta particles
6	fission product decay gamma rays
190	

An additional energy of about 11 mev, released in the form of highly penetrating neutrinos, does not contribute to the practical utilization of fission heat. In spite of being large, the total energy release is less than 0.1 per cent of the total mass-energy value of the uranium nucleus. The exact correlation of fission rate and heat power in a nuclear reactor depends on the fraction of gamma rays that escape from the system, as well as on the degree of equilibrium of fission product production and decay. Fortunately, precise numbers are not necessary. Convenient relations for estimates based on 190 mev/fission are

3.3×10^{10}	fissions/watt-sec
1.3 gm	U^{235} consumed per megawatt-day
10^7 kwh/lb	of fuel fissioned

From 1 to 6 neutrons may be emitted in fission. Table 1.2 shows

Table 1.2
Neutrons from Fissionable Elements

	ν	η (slow)	η (fast)
^{235}U	2.46	2.06	2.33
^{239}Pu	2.88	2.03	2.70
^{233}U	2.54	2.31	...

the *average* number of neutrons per fission (ν) and the average number of neutrons per absorption (η) in the principal fissionable elements activated by low and high energy neutrons. For natural uranium, with isotopic composition 0.7205 per cent U^{235} , 99.274 per cent U^{238} , the value of η is 1.34.

1.3. The nuclear chain reactor

If the number of neutrons in an assembly of fissionable materials can be maintained constant, a self-sustaining *chain reaction* exists. This is possible only because more than one neutron is produced for each neutron that sets off the fission process.

The basic interactions and components in a typical nuclear chain reactor are now presented. The reactor will be assumed to contain U^{235} as *fuel* and to operate with low-energy neutrons. The fission of a nucleus of U^{235} gives rise to around 2.5 neutrons on the average and 190 mev of useful energy. The kinetic energy of the fission fragments is the primary source of potentially useful heat, which can be removed by a circulating *coolant*. In a typical

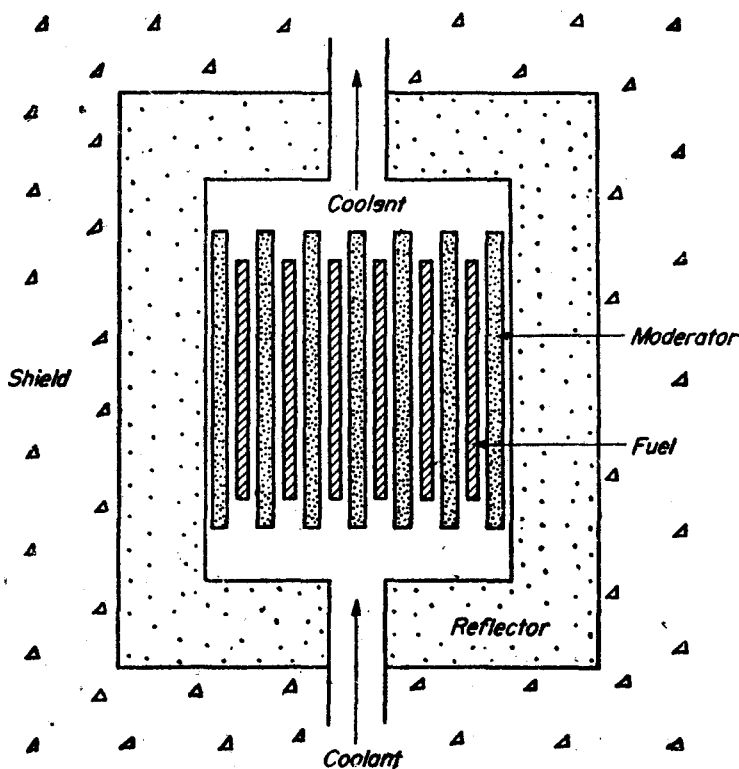


Figure 1.1. Schematic reactor

reactor, a *moderator* containing a light element such as hydrogen or carbon is intermixed with the fuel. Successive collisions with moderator nuclei serve to reduce the energy of fission neutrons (2 mev average) to the thermal* level. In the course of slowing down, many neutrons escape through the boundaries of the cen-

Table 1.3

Classification of Reactors

Energy of neutrons that produce fission

fast
intermediate (or epithermal)
thermal

Nuclear fuel

natural U (0.7% U^{235})
slightly enriched U (1-2% U^{235})
highly enriched U ($\approx 90\%$ U^{235})
 Pu^{239}
 U^{233}

Method of heat removal, by circulation of

coolant only
fuel mixed with coolant
moderator-coolant
fuel, moderator, and coolant

Purpose

research
prototype
propulsion
heat source
electric power generation
isotope production (fissionable or for industrial use)

Arrangement of fuel and moderator

heterogeneous
homogeneous

Materials used in the following reactor components

moderator
coolant
structure
reflector
shield

* Thermal neutrons are those in equilibrium with a substance having thermal energy corresponding to temperature T . Since room temperature corresponds to 0.025 ev neutron energy, the latter is a commonly quoted thermal value.

tral portion or *core*—a process called “leakage.” A surrounding *reflector* has the function of reducing the number lost in this manner, while contributing additional moderation. Relatively few of the higher energy neutrons are removed from the cycle by absorption, unless an appreciable amount of U^{235} with its resonance absorption peaks is present. The choice of structural and moderating materials normally eliminates other capture losses at energies above thermal. Slow neutrons remain in the system for a relatively long time, since the chance of scattering is greater than that of absorption. Some are eventually absorbed by moderator, structure, U^{238} , or fission product isotopes, while a few escape from the assembly. The rest are absorbed by U^{235} . The reaction is self-sustaining or *critical* if the neutrons released from the fission of one U^{235} nucleus eventually produce one more fission (or if an initial fast neutron provides another to replace it at the end of the foregoing cycle). The sub-critical reactor is one sustained only by a separate supply of neutrons; in the super-critical reactor, neutrons will accumulate.

Figure 1.1 shows a schematic diagram of a reactor, with structural details omitted for simplicity. The radiation *shield* is not essential to the chain reaction, but must be provided for the protection of personnel from neutrons and gamma rays.

Reactors may be classified according to their function, materials of construction, and arrangement, as shown in Table 1.3.

1.4. Reactor types and examples

If one formed all the possible combinations of reactor features according to the classification in Table 1.3, about 1000 reactor types would be found. Many would not be feasible at all; others would be inordinately expensive. Some of the types that have been operated or show the most promise are now described briefly, to assist in orientation. First, consider six *power reactor* systems.

Heterogeneous, natural uranium, converter reactors. A chain reaction cannot be sustained in a mass of natural uranium metal, no matter how large, because of the unfavorable competition between neutron capture in U^{238} with fission. However, fuel may be arranged in lumps or rods separated by a material such