Numerical and Computer Methods in Structural Mechanics

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Preface

The impact of modern digital computers on the field of structural mechanics has consisted of far more than mere application of existing numerical techniques to problems of ever increasing complexity. Indeed, a far-ranging, yet often subtle, interaction has grown up between the methods employed for the static and dynamic analysis of structures and structural elements on the one hand and the capabilities of computer hardware and software on the other. It is important for the progress of structural mechanics that the implications of this interaction be recognized by practitioners of structural mechanics and by computer software designers.

The objective of this Office of Naval Research Symposium was to summarize the present and probable future status of numerical methods in structural mechanics and of related computer techniques and computer capabilities. The concern for exploration of these separate areas and their interaction is reflected in seven aspects which can be distinguished in these Proceedings.

First, the analytical basis of the finite element method—the computer technique most widely implemented in software—is examined broadly in four papers. Cowper presents a general introduction to the finite element procedure and a discussion of convergence in terms of variational principles. Isoparametric and related elements are treated by Zienkiewicz; incompatible displacement models by Wilson, Taylor, Doherty, and Ghaboussi; and hybrid models by Pian.

Second, the paper by Schrem and the one by Irons and Kan explore in depth two fundamental and interrelated aspects of the numerical and computer implementation of finite element procedures, namely the storage and retrieval of data and the selection of equation-solving algorithms.

Third, an entire session, consisting of the papers by Meijers; Tocher and Herness; Yates, Sable, and Vinson; Dainora; Chu; and Ayres, was devoted to a critical review of some of the significant general-purpose structural mechanics programs. In selecting the papers, the organizers of the Symposium were attempting to concentrate on those programs which have gained a measure of practical use outside the organization which originated the program. While novel, this attempt to obtain constructive critical information from knowledgeable users must be viewed as a token one. To provide much-needed program information exchange among users, greatly expanded efforts in this direction are warranted.

As a fourth aspect, a session concentrated on the presentation of alternatives to and extensions of the usual finite element approaches and on relations and comparisons among various analytical outlooks. The paper by Wright and Baron presents a comprehensive survey of finite difference methods. The contributions by Key and Krieg and by Bushnell provide useful comparisons and combinations of finite difference and finite element methods. The difficulties of nonlinear, dynamic finite element problems are explored by McNamara and Marcal. The paper by Schnobrich and Pecknold describes a direct physical interpretation to aid in the selection of finite difference models.

Fifth, the Symposium addressed itself to a major future trend in structural mechanics computing, namely the abandonment of individual, unconnected programs in favor of large, integrated data bases. The paper by Fenves is devoted to general questions of large interactive systems. The role of computer graphics, an important tool in such information systems, is discussed by Batdorf and Kapur. Two outstanding accomplishments in the actual implementation of integrated data bases are the subjects of papers by Moe and by McCormick, Baron, and Perrone.

Sixth, the organizers of the Symposium felt that it was their obligation to bring to the attention of workers in structural mechanics new software and hardware capabilities which have a major potential impact on computer use, both in the nature of problems to be tackled and the magnitude of problems that can be handled. The papers by Wong and by Graham discuss such outstanding new capabilities, as well as the challenges they pose.

The final session of the Symposium paralleled the last part in that it dealt specifically with new applications that seem likely to affect the content of the discipline of structural mechanics itself in the next decade. The paper by Kamel, Liu, and White extrapolates the problems which relate to ship design. Gallagher's paper points out the trends which can be perceived in numerical analysis, while the contribution of Rice and Tracey indicates the analytical problem-solving capability needed for an effective approach to problems of fracture mechanics. Technically important new areas which apply structural mechanics, biomechanics and crash safety, are discussed by Bugliarello and Desjardins.

It is hoped that the exposition of problems and interests in this Symposium will illustrate the degree of interaction existing now between numerical and computer methods in structural mechanics and will foster an even broader symbiosis between these areas in the future.

Contents

Part I. FINITE ELEMENTS—FUNDAMENTALS

Hybrid Models

Theodore H. H. Pian

Computer Implementation of the Finite–Element Procedure

Ernst Schrem

Part II. CRITICAL REVIEW OF GENERAL-PURPOSE STRUCTURAL MECHANICS PROGRAMS

Review of the ASKA Program

P. Meijers

A Critical View of NASTRAN

James L. Tocher and Ervin D. Herness

The DAISY Code

D. N. Yates, W. W. Sable, and T. J. Vinson

An Evaluation of the STARDYNE System

John Dainora

Analysis and Design Capabilities of STRUDL Program

S. L. Chu

Integrated Design of Tanker Structures

Johannes Moe

The STORE Project (The Structures Oriented Exchange)

J. M. McCormick, M. L. Baron, and N. Perrone

Part V. NEW CAPABILITIES FOR COMPUTER-BASED ANALYSIS

Symbolic Computing

Andrew Ka-Ching Wong

A Review of the Capabilities and Limitations of Parallel and Pipeline Computers

William R. Graham

Equation-Solving Algorithms for the Finite-Element Method

Bruce M. Irons and David K. Y. Kan

FLING—A FORTRAN Language for Interactive Graphics

W. J. Batdorf and S. S. Kapur

Part VI. NUMERICAL METHODS FOR A CHANGING TECHNOLOGY

Trends and Directions in the Applications of Numerical Analysis

Richard H. Gallagher

Vehicle Crashworthiness

S. P. Desjardins

Computational Fracture Mechanics

J. R. Rice and D. M. Tracey

Biomechanics

George Bugliarello

The Computer in Ship Structure Design

H. A. Kamel, D. Liu, and E. I. White

PART I

Finite Elements—Fundamentals

Variational Procedures and Convergence of Finite-Element Methods

G. R. Cowper

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When the finite-element method first appeared on the scene in the mid-1950s, stiffness matrices were derived on what might be termed an "intuitive" or "direct" basis. Continuous structures were regarded in the same light as those structures such as trusses and frames, which were actually made up of physically discrete elements. Thus a continuous structure was regarded as an assembly of elements which were joined only at nodes, and one spoke of the forces applied at nodes and of the equilibrium of nodes. Interelement continuity considerations did not go beyond the matching of displacements at nodes. As the method developed it was realized that the method could be regarded as an application of the variational principles of structural mechanics, especially of the principle of minimum potential energy. This realization greatly stimulated the development of the method. Certainly, variational principles are now generally used as the foundation of the finite element method, as is evident from a perusal of the proceedings of several recent symposia on the subject.

There are several advantages to basing the finite-element method on variational principles. One is that the method is thereby put on a sound theoretical foundation. Another is that the requirements for interelement continuity are clarified, as these requirements are quite explicit in the statement of the variational principles. Furthermore, greater flexibility in the formulation of elements is possible because generalized displacements need not be restricted to quantities that are conjugate to physical forces and moments. Indeed the entire concept of nodal forces as actual physical forces can be dispensed with. Higher derivatives of displacement, for example curvatures and twists in the case of a bent plate, become acceptable as generalized displacements. Greater flexibility in the formulation of elements also comes about because a variety of variational principles are available. In addition to the classical principles of minimum potential energy and minimum complementary energy there is the more general principle of Reissner, and also a number of modified principles which permit the relaxation of various requirements, particularly requirements of interelement continuity of displacements or stresses. A final advantage is that the scope of the finite element method is broadened to include nonstructural problems which can be expressed in terms of a variational principle. The method has been applied to problems of heat flow, potential and viscous fluid flow, seepage of ground water, and others.

The classical variational principles of linear structural mechanics are the principles of minimum potential energy and of minimum complementary energy. The former principle may be stated thus: The potential energy is stationary with regard to all kinematically admissible variations of displacements from the state of equilibrium. For stable equilibrium the stationary value of the potential energy is an absolute minimum. In symbols,

$$\delta\Pi = 0, \qquad \Pi = U - V \tag{1}$$

where Π is the potential energy, U is the strain energy of the body, and V is the virtual work of the applied loads. An alternative statement is: Among all kinematically admissible displacements, those satisfying the equilibrium conditions make the potential energy an absolute minimum. Note that the displacements must be kinematically admissible. This means that they must satisfy sufficient continuity conditions within the structure and must satisfy the kinematic boundary conditions. There is, however, no requirement that the stress boundary conditions be satisfied.

The principle of minimum complementary energy is concerned with stress fields that satisfy the conditions of equilibrium but not necessarily the requirements of compatibility. It may be stated thus: Among all statically admissible stress fields, the one which satisfies the stress-strain relations in the interior of the structure and the displacement boundary conditions makes the complementary energy an absolute minimum.

Some analysts have found that the requirements of kinematic admissibility or static admissibility are rather troublesome to achieve with finite elements, especially in plate and shell problems. As a result, a number of modified variational principles have been proposed which allow relaxed continuity conditions. These principles have been used effectively, chiefly by Pian and

Tong [2], as a basis for so-called hybrid elements. Reissner's variational principle has also been used as a basis for finite elements. Reissner's principle is more general than either of the principles of minimum potential or complementary energy, in that it allows simultaneous variations in both stresses and displacements. This permits independent assumptions as to the distributions of displacements and stresses, and is claimed to thus lead to more accurate approximations for the stresses. The hybrid principles and Reissner's principle, unlike the principles of minimum potential and complementary energy, are not minimum principles but state only that certain functionals are stationary.

The variational principles and their application to the finite element method have been discussed and classified by a number of writers. Mention may be made of the excellent surveys by Pian [1] and Pian and Tong [2] and the conference papers of Hansteen [3] and Tottenham [4]. Mention should also be made of the earlier papers of de Veubeke [5], [6], which point out the dual nature of the principles of minimum potential energy and minimum complementary energy and emphasize how these two principles can be used to set bounds on stiffness coefficients. The survey of Pian is particularly useful in illustrating and classifying the many different possibilities for formulating finite elements based on the various variational principles. In view of these surveys it would be superfluous to go further into the details of variational principles, and I would like to turn instead to the question of convergence.

Does an approximate solution, obtained by means of finite elements, converge to the correct solution as the mesh of finite elements is uniformly refined? For compatible displacement elements and for equilibrium elements a relatively simple proof of convergence can be given, based on the minimum property of the potential or complementary energies. A convergence proof is of more than academic interest. For one thing, it contributes to the confidence with which finite elements can be used, because the user has a guarantee that his results must approach the correct answer as the mesh of elements is refined. This has not always been the case, and the early days of the method provided examples of ill-chosen elements which did not converge to the right answer. In addition the convergence proof points up the conditions necessary for convergence and good accuracy, and thus provides useful guidance in constructing elements. The essential points of the proof have been given in a number of papers [7]–[10], of which the paper by McLay is particularly notable.

We consider the convergence as it applies to compatible displacement elements which are formulated on the basis of the principle of minimum potential energy. Using this principle, approximate solutions to a structural analysis problem can be constructed by the Rayleigh–Ritz procedure. The