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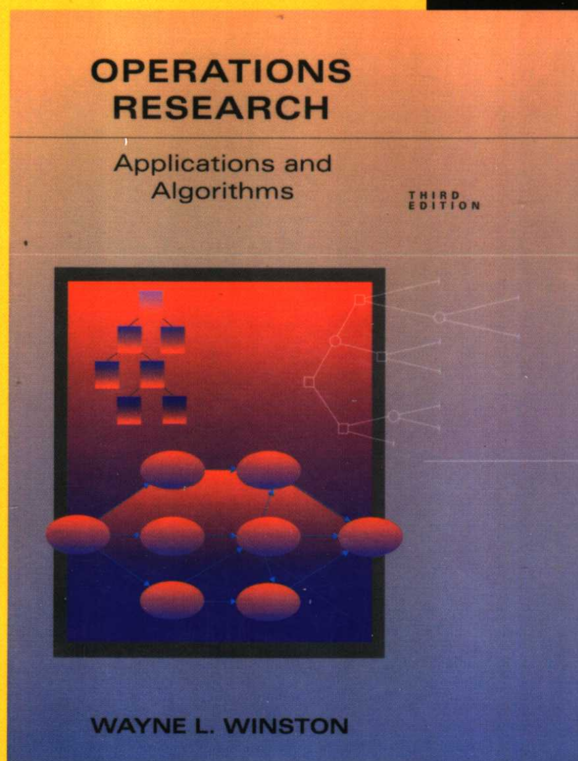
国外大学优秀教材——工业工程系列（影印版）

WAYNE L. WINSTON

运筹学 决策方法（第3版）

OPERATIONS RESEARCH
Decision Making
(THIRD EDITION)



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北京

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Forward

This textbook series is published at a very opportunity time when the discipline of industrial engineering is experiencing a phenomenal growth in China academia and with its increased interests in the utilization of the concepts, methods and tools of industrial engineering in the workplace. Effective utilization of these industrial engineering approaches in the workplace should result in increased productivity, quality of work, satisfaction and profitability to the cooperation.

The books in this series should be most suitable to junior and senior undergraduate students and first year graduate students, and to those in industry who need to solve problems on the design, operation and management of industrial systems.

Gavriel Salvendy

Department of Industrial Engineering, Tsinghua University

School of Industrial Engineering, Purdue University

April, 2002

前　　言

本教材系列的出版正值中国学术界工业工程学科经历巨大发展、实际工作中对工业工程的概念、方法和工具的使用兴趣日渐浓厚之时。在实际工作中有效地应用工业工程的手段将无疑会提高生产率、工作质量、合作的满意度和效果。

该系列中的书籍对工业工程的本科生、研究生和工业界中需要解决工程系统设计、运作和管理诸方面问题的人士最为适用。

加弗瑞尔·沙尔文迪

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2002年4月

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1 Network Models

2 Decision Making Under Uncertainty

3 Decision Making with Multiple Objectives

†Indicates that section may be omitted without loss of continuity.
‡Indicates that material on use of the computer has been added to this edition.

1

Network Models

Many important optimization problems can best be analyzed by means of a graphical or network representation. In this chapter, we consider four specific network models—shortest path problems, maximum flow problems, CPM-PERT project-scheduling models, and minimum-spanning tree problems—for which efficient solution procedures exist. We also discuss minimum-cost network flow problems (MCNFPs), of which transportation, assignment, transshipment, shortest path, and maximum flow problems and the CPM project-scheduling models are all special cases. Finally, we discuss a generalization of the transportation simplex, the network simplex, which can be used to solve MCNFPs. We begin the chapter with some basic terms used to describe graphs and networks.

1.1 Basic Definitions

A **graph**, or **network**, is defined by two sets of symbols: nodes and arcs. First, we define a set (call it V) of points, or **vertices**. The vertices of a graph or network are also called **nodes**.

We also define a set of arcs A.

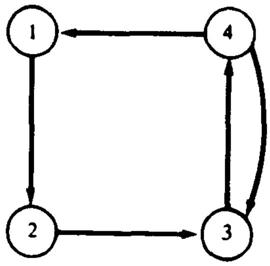
DEFINITION ▪ An **arc** consists of an ordered pair of vertices and represents a possible direction of motion that may occur between vertices. ▪

For our purposes, if a network contains an arc (j, k), motion is possible from node j to node k. Suppose nodes 1, 2, 3, and 4 of Figure 1 represent cities, and each arc represents a (one-way) road linking two cities. For this network, $V = \{1, 2, 3, 4\}$ and $A = \{(1, 2), (2, 3), (3, 4), (4, 3), (4, 1)\}$. For the arc (j, k), node j is the **initial node**, and node k is the **terminal node**. The arc (j, k) is said to go from node j to node k. Thus, the arc (2, 3) has initial node 2 and terminal node 3, and it goes from node 2 to node 3. The arc (2, 3) may be thought of as a (one-way) road on which we may travel from city 2 to city 3. In Figure 1, the arcs show that travel is allowed from city 3 to city 4, and from city 4 to city 3, but that travel between the other cities may be one way only.

Later, we often discuss a group or collection of arcs. The following definitions are convenient ways to describe certain groups or collections of arcs.

DEFINITION ▪ A sequence of arcs such that every arc has exactly one vertex in common with the previous arc is called a **chain**. ▪

FIGURE 1
Example of a Network



DEFINITION ▪ **A path is a chain in which the terminal node of each arc is identical to the initial node of the next arc.** ▪

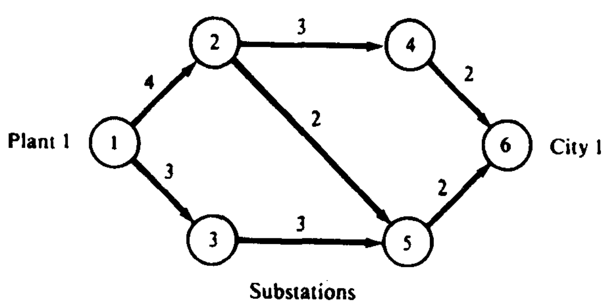
For example, in Figure 1, (1, 2)–(2, 3)–(4, 3) is a chain but not a path; (1, 2)–(2, 3)–(3, 4) is a chain *and* a path. The path (1, 2)–(2, 3)–(3, 4) represents a way to travel from node 1 to node 4.

1.2 Shortest Path Problems

In this section, we assume that each arc in the network has a length associated with it. Suppose we start at a particular node (say, node 1). The problem of finding the shortest path (path of minimum length) from node 1 to any other node in the network is called a **shortest path problem**. Examples 1 and 2 are shortest path problems.

EXAMPLE 1 Let us consider the Powerco example (Figure 2). Suppose that when power is sent from plant 1 (node 1) to city 1 (node 6), it must pass through relay substations (nodes 2–5). For any pair of nodes between which power can be transported, Figure 2 gives the distance (in miles) between the nodes. Thus, substations 2 and 4 are 3 miles apart, and power cannot be sent between substations 4 and 5. Powerco wants the power sent from plant 1 to city 1

FIGURE 2
Network for Powerco



to travel the minimum possible distance, so it must find the shortest path in Figure 2 that joins node 1 to node 6.

If the cost of shipping power were proportional to the distance the power travels, then knowing the shortest path between plant 1 and city 1 in Figure 2 (and the shortest path between plant i and city j in similar diagrams) would be necessary to determine the shipping costs for the transportation version of the Powerco problem.

EXAMPLE 2 I have just purchased (at time 0) a new car for $12,000. The cost of maintaining a car during a year depends on the age of the car at the beginning of the year, as given in Table 1. To avoid the high maintenance costs associated with an older car, I may trade in my car and purchase a new car. The price I receive on a trade-in depends on the age of the car at the time of trade-in (see Table 2). To simplify the computations, we assume that at any time, it costs $12,000 to purchase a new car. My goal is to minimize the net cost (purchasing costs + maintenance costs − money received in trade-ins) incurred during the next five years. Formulate this problem as a shortest path problem.

Solution Our network will have six nodes (1, 2, 3, 4, 5, and 6). Node i is the beginning of year i. For $i < j$, an arc (i, j) corresponds to purchasing a new car at the beginning of year i and keeping it until the beginning of year j. The length of arc (i, j) (call it c_{ij}) is the total net cost incurred in owning and operating a car from the beginning of year i to the beginning of year j if a new car is purchased at the beginning of year i and this car is traded in for a new car at the beginning of year j. Thus,

$$
\begin{aligned}
c_{ij} &= \text{maintenance cost incurred during years } i, i+1, \ldots, j-1 \\
&\quad + \text{cost of purchasing car at beginning of year } i \\
&\quad - \text{trade-in value received at beginning of year } j
\end{aligned}
$$

TABLE 1
Car Maintenance Costs

Age of Car (years)	Annual Maintenance Cost
0	$2,000
1	$4,000
2	$5,000
3	$9,000
4	$12,000

TABLE 2
Car Trade-in Prices

Age of Car (years)	Trade-in Price
1	$7000
2	$6000
3	$2000
4	$1000
5	$0

Applying this formula to the information in the problem yields (all costs are in thousands)

$$c_{12} = 2 + 12 - 7 = 7$$
$$c_{13} = 2 + 4 + 12 - 6 = 12$$
$$c_{14} = 2 + 4 + 5 + 12 - 2 = 21$$
$$c_{15} = 2 + 4 + 5 + 9 + 12 - 1 = 31$$
$$c_{16} = 2 + 4 + 5 + 9 + 12 + 12 - 0 = 44$$
$$c_{23} = 2 + 12 - 7 = 7$$
$$c_{24} = 2 + 4 + 12 - 6 = 12$$
$$c_{25} = 2 + 4 + 5 + 12 - 2 = 21$$
$$c_{26} = 2 + 4 + 5 + 9 + 12 - 1 = 31$$
$$c_{34} = 2 + 12 - 7 = 7$$
$$c_{35} = 2 + 4 + 12 - 6 = 12$$
$$c_{36} = 2 + 4 + 5 + 12 - 2 = 21$$
$$c_{45} = 2 + 12 - 7 = 7$$
$$c_{46} = 2 + 4 + 12 - 6 = 12$$
$$c_{56} = 2 + 12 - 7 = 7$$

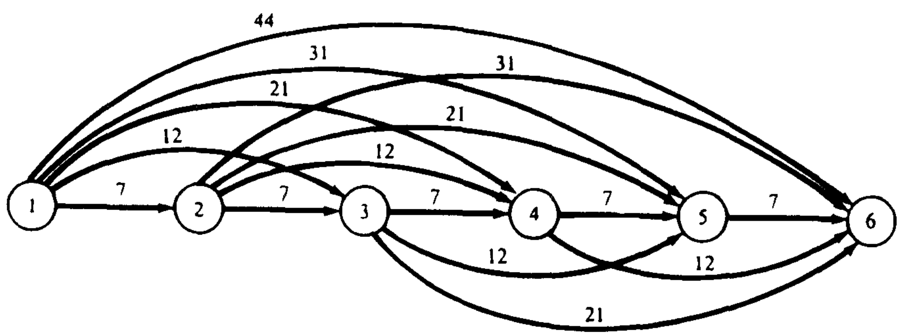
We now see that the length of any path from node 1 to node 6 is the net cost incurred during the next five years corresponding to a particular trade-in strategy. For example, suppose I trade in the car at the beginning of year 3 and next trade in the car at the end of year 5 (the beginning of year 6). This strategy corresponds to the path 1–3–6 in Figure 3. The length of this path ($c_{13} + c_{36}$) is the total net cost incurred during the next five years if I trade in the car at the beginning of year 3 and at the beginning of year 6. Thus, the length of the shortest path from node 1 to node 6 in Figure 3 is the minimum net cost that can be incurred in operating a car during the next five years.

Dijkstra's Algorithm

Assuming that all arc lengths are nonnegative, the following method, known as **Dijkstra's algorithm,** can be used to find the shortest path from a node (say, node 1) to all other nodes. To begin, we label node 1 with a permanent label of 0. Then we label each node i that is connected to node 1 by a single arc with a "temporary" label equal to the length of the arc joining node 1 to node i. Each other node (except, of course, for node 1) will have a temporary label of ∞. Choose the node with the smallest temporary label and make this label permanent.

Now suppose that node i has just become the $(k + 1)$th node to be given a permanent label. Then node i is the kth closest node to node 1. At this point, the temporary label of any node (say, node i') is the length of the shortest path from node 1 to node i' that passes only through nodes contained in the $k - 1$ closest nodes to node 1. For each node j that now

FIGURE 3
Network for Minimizing Car Costs



has a temporary label and is connected to node i by an arc, we replace node j's temporary label by

$$\min \begin{cases} \text{node } j\text{'s current temporary label} \\ \text{node } i\text{'s permanent label } + \text{ length of arc } (i, j) \end{cases}$$

(Here, $\min\{a, b\}$ is the smaller of a and b.) The new temporary label for node j is the length of the shortest path from node 1 to node j that passes only through nodes contained in the k closest nodes to node 1. We now make the smallest temporary label a permanent label. The node with this new permanent label is the $(k + 1)$th closest node to node 1. Continue this process until all nodes have a permanent label. To find the shortest path from node 1 to node j, work backward from node j by finding nodes having labels differing by exactly the length of the connecting arc. Of course, if we want the shortest path from node 1 to node j, we can stop the labeling process as soon as node j receives a permanent label.

To illustrate Dijkstra's algorithm, we find the shortest path from node 1 to node 6 in Figure 2. We begin with the following labels (a * represents a permanent label, and the ith number is the label of the node i): $[0^* \quad 4 \quad 3 \quad \infty \quad \infty \quad \infty]$. Node 3 now has the smallest temporary label. We therefore make node 3's label permanent and obtain the following labels:

$$[0^* \quad 4 \quad 3^* \quad \infty \quad \infty \quad \infty]$$

We now know that node 3 is the closest node to node 1. We compute new temporary labels for all nodes that are connected to node 3 by a single arc. In Figure 2 that is node 5.

$$\text{New node 5 temporary label} = \min\{\infty, 3 + 3\} = 6$$

Node 2 now has the smallest temporary label; we now make node 2's label permanent. We now know that node 2 is the second closest node to node 1. Our new set of labels is

$$[0^* \quad 4^* \quad 3^* \quad \infty \quad 6 \quad \infty]$$

Since nodes 4 and 5 are connected to the newly permanently labeled node 2, we must change the temporary labels of nodes 4 and 5. Node 4's new temporary label is $\min\{\infty, 4 + 3\} = 7$ and node 5's new temporary label is $\min\{6, 4 + 2\} = 6$. Node 5 now has the smallest temporary label, so we make node 5's label permanent. We now know that node 5 is the third closest node to node 1. Our new labels are

$$[0^* \quad 4^* \quad 3^* \quad 7 \quad 6^* \quad \infty]$$

Since only node 6 is connected to node 5, node 6's temporary label will change to $\min\{\infty, 6 + 2\} = 8$. Node 4 now has the smallest temporary label, so we make node 4's label permanent. We now know that node 4 is the fourth closest node to node 1. Our new labels are

$$[0^* \quad 4^* \quad 3^* \quad 7^* \quad 6^* \quad 8]$$

Since node 6 is connected to the newly permanently labeled node 4, we must change node 6's temporary label to $\min\{8, 7 + 2\} = 8$. We can now make node 6's label permanent. Our final set of labels is $[0^* \quad 4^* \quad 3^* \quad 7^* \quad 6^* \quad 8^*]$. We can now work backward and find the shortest path from node 1 to node 6. The difference between node 6's and node 5's permanent labels is 2 = length of arc (5, 6), so we go back to node 5. The difference between node 5's and node 2's permanent labels is 2 = length of arc (2, 5), so we may go back to node 2. Then, of course, we must go back to node 1. Thus, 1–2–5–6 is a shortest path (of length 8) from node 1 to node 6. Observe that when we were at node 5, we could also have worked backward to node 3 and obtained the shortest path 1–3–5–6.

The Shortest Path Problem as a Transshipment Problem

Finding the shortest path between node i and node j in a network may be viewed as a transshipment problem. Simply try to minimize the cost of sending one unit from node i to node j (with all other nodes in the network being transshipment points), where the cost of sending one unit from node k to node k' is the length of arc (k, k') if such an arc exists and is M (a large positive number) if such an arc does not exist. The cost of shipping one unit from a node to itself is zero. This transshipment problem may be transformed into a balanced transportation problem.

To illustrate the preceding ideas, we formulate the balanced transportation problem associated with finding the shortest path from node 1 to node 6 in Figure 2. We want to send one unit from node 1 to node 6. Node 1 is a supply point, node 6 is a demand point, and nodes 2, 3, 4, and 5 will be transshipment points. Using $s = 1$, we obtain the balanced transportation problem shown in Table 3. This transportation problem has two optimal solutions:

1 $z = 4 + 2 + 2 = 8, x_{12} = x_{25} = x_{56} = x_{33} = x_{44} = 1$ (all other variables equal 0). This solution corresponds to the path 1–2–5–6.

2 $z = 3 + 3 + 2 = 8, x_{13} = x_{35} = x_{56} = x_{22} = x_{44} = 1$ (all other variables equal 0). This solution corresponds to the path 1–3–5–6.

REMARK After formulating a shortest path problem as a transshipment problem, the problem may be solved easily by using LINGO or a spreadsheet optimizer.

TABLE 3
Transshipment Representation of Shortest Path Problem and Optimal Solution (1)

	Node					
Node	2	3	4	5	6	Supply
1	[4] 1	[3]	[M]	[M]	[M]	1
2	[0]	[M]	[3]	[2] 1	[M]	1
3	[M]	[0] 1	[M]	[3]	[M]	1
4	[M]	[M]	[0] 1	[M]	[2]	1
5	[M]	[M]	[M]	[0]	[2] 1	1
Demand	1	1	1	1	1	

Problems

Group A

1 Find the shortest path from node 1 to node 6 in Figure 3.

2 Find the shortest path from node 1 to node 5 in Figure 4.

3 Formulate Problem 2 as a transshipment problem.

4 Use Dijkstra's algorithm to find the shortest path from node 1 to node 4 in Figure 5. Why does Dijkstra's algorithm fail to obtain the correct answer?

5 Suppose it costs $10,000 to purchase a new car. The annual operating cost and resale value of a used car is shown in Table 4. Assuming that one has a new car at present, determine a replacement policy that minimizes the net costs of owning and operating a car for the next six years.

6 It costs $40 to buy a telephone from the department store. Assume that I can keep a telephone for at most five years and that the estimated maintenance cost each year of operation is as follows: year 1, $20; year 2, $30; year 3, $40; year 4, $60; year 5, $70. I have just purchased a new telephone. Assuming that a telephone has no salvage value, determine how to minimize the total cost of purchasing and operating a telephone for the next six years.

7 At the beginning of year 1 a new machine must be purchased. The cost of maintaining a machine i years old is given in Table 5.

The cost of purchasing a machine at the beginning of each year is given in Table 6.

There is no trade-in value when a machine is replaced. Your goal is to minimize the total cost (purchase plus maintenance) of having a machine for five years. Determine the years in which a new machine should be purchased.

Group B

8† A library must build shelving to shelve 200 4-inch high books, 100 8-inch high books, and 80 12-inch high books. Each book is 0.5 inch thick. The library has several ways to store the books. For example, an 8-inch high shelf may be built to store all books of height less than or equal to 8 inches, and a 12-inch high shelf may be built for the 12-inch books. Alternatively, a 12-inch high shelf might be built to store all books. The library believes it costs $2300 to build a shelf and that a cost of $5 per square inch is incurred for book storage. (Assume that the area required to store a book is given by height of storage area times book's thickness.)

Formulate and solve a shortest path problem that could be used to help the library determine how to shelve the books

FIGURE 4 Network for Problem 2

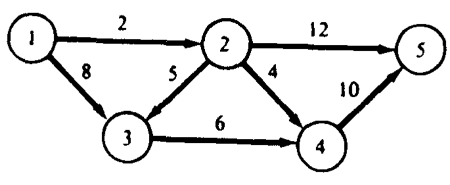


FIGURE 5 Network for Problem 4

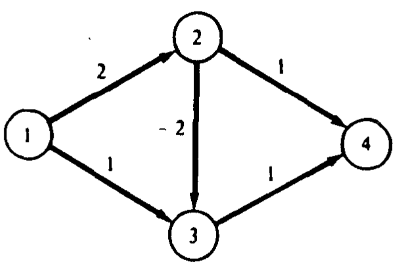


TABLE 4

Age of Car (years)	Resale Value	Operating Cost
1	$7000	$300 (year 1)
2	$6000	$500 (year 2)
3	$4000	$800 (year 3)
4	$3000	$1200 (year 4)
5	$2000	$1600 (year 5)
6	$1000	$2200 (year 6)

TABLE 5

Age at Beginning of Year	Maintenance Cost for Next Year
0	$38,000
1	$50,000
2	$97,000
3	$182,000
4	$304,000

TABLE 6

Year	Purchase Cost
1	$170,000
2	$190,000
3	$210,000
4	$250,000
5	$300,000

†Based on Ravindran (1971).

at minimum cost. (*Hint*: Have nodes 0, 4, 8, and 12, with c_{ij} being the total cost of shelving all books of height $> i$ and $\leq j$ on a single shelf.)

9 A company sells seven types of boxes, ranging in volume from 17 to 33 cubic feet. The demand and size of each box is given in Table 7. The variable cost (in dollars) of producing each box is equal to the box's volume: A fixed cost of $1000 is incurred to produce any of a particular box. If the company desires, demand for a box may be satisfied by a box of larger size. Formulate and solve a shortest path problem whose solution will minimize the cost of meeting the demand for boxes.

10 Explain how by solving a single transshipment problem you can find the shortest path from node 1 in a network to **each other node** in the network.

TABLE **7**

Box	1	2	3	4	5	6	7
Size	33	30	26	24	19	18	17
Demand	400	300	500	700	200	400	200

1.3 Maximum Flow Problems

Many situations can be modeled by a network in which the arcs may be thought of as having a capacity that limits the quantity of a product that may be shipped through the arc. In these situations, it is often desired to transport the maximum amount of flow from a starting point (called the **source**) to a terminal point (called the **sink**). Such problems are called **maximum flow problems**. Several specialized algorithms exist to solve maximum flow problems. In this section, we begin by showing how linear programming can be used to solve a maximum flow problem. Then we discuss the Ford–Fulkerson (1962) method for solving maximum flow problems.

LP Solution of Maximum Flow Problems

EXAMPLE **3** Sunco Oil wants to ship the maximum possible amount of oil (per hour) via pipeline from node *so* to node *si* in Figure 6. On its way from node *so* to node *si*, oil must pass through some or all of stations 1, 2, and 3. The various arcs represent pipelines of different diameters. The maximum number of barrels of oil (millions of barrels per hour) that can be pumped through each arc is shown in Table 8. Each of these numbers is called an **arc capacity**. Formulate an LP that can be used to determine the maximum number of barrels of oil per hour that can be sent from *so* to *si*.

Solution Node *so* is called the source node because oil flows out of it but no oil flows into it. Analogously, node *si* is called the sink node because oil flows into it and no oil flows out

FIGURE **6**
Network for Sunco Oil

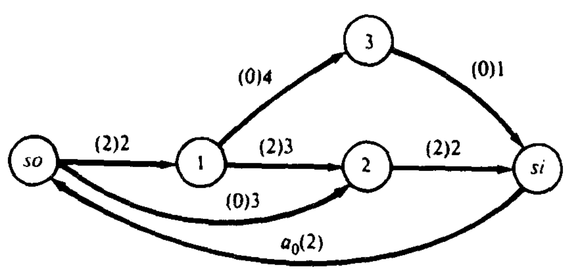


TABLE **8**
Arc Capacities for Sunco Oil

Arc	Capacity
$(so, 1)$	2
$(so, 2)$	3
$(1, 2)$	3
$(1, 3)$	4
$(3, si)$	1
$(2, si)$	2

of it. For reasons that will soon become clear, we have added an artificial arc a_0 from the sink to the source. The flow through a_0 is not actually oil; hence the term **artificial arc**.

To formulate an LP that will yield the maximum flow from node so to si, we observe that Sunco must determine how much oil (per hour) should be sent through arc (i, j). Thus, we define

$$x_{ij} = \text{millions of barrels of oil per hour that will pass through arc } (i, j) \text{ of pipeline}$$

As an example of a possible flow (termed a *feasible flow*), consider the flow indentified by the numbers in parentheses in Figure 6.

$$x_{so,1} = 2, \quad x_{13} = 0, \quad x_{12} = 2, \quad x_{3,si} = 0, \quad x_{2,si} = 2, \quad x_{si,so} = 2, \quad x_{so,2} = 0$$

For a flow to be feasible, it must have two characteristics:

$$0 \le \text{flow through each arc} \le \text{arc capacity} \tag{1}$$

and

$$\text{Flow into node } i = \text{flow out of node } i \tag{2}$$

We assume that no oil gets lost while being pumped through the network, so at each node, a feasible flow must satify (2), the *conservation-of-flow* constraint. The introduction of the artificial arc a_0 allows us to write the conservation-of-flow constraint for the source and sink.

If we let x_0 be the flow through the artificial arc, then conservation of flow implies that x_0 = total amount of oil entering the sink. Thus, Sunco's goal is to maximize x_0 subject to (1) and (2):

$$\begin{aligned}
\max z &= x_0 \\
\text{s.t.} \quad x_{so,1} &\le 2 \qquad \text{(Arc capacity constraints)} \\
x_{so,2} &\le 3 \\
x_{12} &\le 3 \\
x_{2,si} &\le 2 \\
x_{13} &\le 4 \\
x_{3,si} &\le 1
\end{aligned}$$

$$\begin{aligned}
x_0 &= x_{so,1} + x_{so,2} && \text{(Node } so \text{ flow constraint)}\\
x_{so,1} &= x_{12} + x_{13} && \text{(Node 1 flow constraint)}\\
x_{so,2} + x_{12} &= x_{2,si} && \text{(Node 2 flow constraint)}\\
x_{13} &= x_{3,si} && \text{(Node 3 flow constraint)}\\
x_{3,si} + x_{2,si} &= x_0 && \text{(Node } si \text{ flow constraint)}\\
x_{ij} &\geq 0
\end{aligned}$$

One optimal solution to this LP is $z = 3, x_{so,1} = 2, x_{13} = 1, x_{12} = 1, x_{so,2} = 1, x_{3,si} = 1, x_{2,si} = 2, x_0 = 3$. Thus, the maximum possible flow of oil from node so to si is 3 million barrels per hour, with 1 million barrels each sent via the following paths: so–1–2–si, so–1–3–si, and so–2–si.

The linear programming formulation of maximum flow problems is a special case of the minimum-cost network problem (MCNFP) discussed in Section 1.5. A generalization of the transportation simplex (known as the network simplex) can be used to solve MCNFPs.

Before discussing the Ford–Fulkerson method for solving maximum flow problems, we give two examples for situations in which a maximum flow problem might arise.

EXAMPLE 4 Fly-by-Night Airlines must determine how many connecting flights daily can be arranged between Juneau, Alaska and Dallas, Texas. Connecting flights must stop in Seattle and then stop in Los Angeles or Denver. Because of limited landing space, Fly-by-Night is limited to making the number of daily flights between pairs of cities shown in Table 9. Set up a maximum flow problem whose solution will tell the airline how to maximize the number of connecting flights daily from Juneau to Dallas.

Solution The appropriate network is given in Figure 7. Here the capacity of arc (i, j) is the maximum number of daily flights between city i and city j. The optimal solution to this maximum flow problem is $z = x_0 = 3, x_{J,S} = 3, x_{S,L} = 1, x_{S,De} = 2, x_{L,D} = 1, x_{De,D} = 2$. Thus, Fly-by-Night can send three flights daily connecting Juneau and Dallas. One flight connects via Juneau–Seattle–L.A.–Dallas and two flights connect via Juneau–Seattle–Denver–Dallas.

EXAMPLE 5 Five male and five female entertainers are at a dance. The goal of the matchmaker is to match each woman with a man in a way that maximizes the number of people who are

TABLE 9
Arc Capacities for Fly-by-Night Airlines

Cities	Maximum Number of Daily Flights
Juneau–Seattle (J, S)	3
Seattle–L.A. (S, L)	2
Seattle–Denver (S, De)	3
L.A.–Dallas (L, D)	1
Denver–Dallas (De, D)	2