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# Active Control of Noise and Vibration

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# Preface

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Active control of sound and vibration is a relatively new and fast growing field of research and application. The number of papers published on the subject have been more than doubling every year for the past ten years and each year more researchers are becoming involved in this fascinating subject. Because of this rapid growth and continuing new developments, it is difficult for any book to claim to cover the subject completely. However, we have attempted to include the most recent theoretical and practical developments, while at the same time devoting considerable space to fundamental principles which will not become outdated with time. We have also devoted space to explaining how active control systems may be designed and implemented in practice, and the practical pitfalls which must be avoided to ensure a reliable and stable system.

We have treated the active control of noise and the active control of vibration in a unified way, even though later on in the book some noise and vibration control topics are treated separately. The reason for the unified treatment is that it is becoming increasingly difficult to keep the two disciplines separate, as one depends so much on the other. For example, the treatment of the active control of sound radiated by vibrating structures would be incomplete if either active control of the radiated acoustic field or the active control of the structural vibration were omitted. Thus, in the first part of the book, which is concerned entirely with fundamental concepts of relevance to active noise and vibration control, an attempt has been made to combine the two subjects so that it can be seen how they are related and how they share many common concepts.

One interesting topic which we have omitted from this book is a discussion of patents. Not only is the subject a large one (with hundreds of patents already granted), it is rapidly growing and it is difficult to do justice to it in a book of this type. One gem of wisdom which we would like to share with our readers is that some of the patent holders are only too willing to sue for patent

infringement, even though it can be shown that prior knowledge existed before many of the patents. Thus, any company preparing to market any products containing active noise or vibration control should be prepared to fight a legal battle for their right to do so unless they have obtained a licence to use the technology from the patent owners. So far, it seems that lawyers and judges have made more money from active control than any engineering company, and they don't even own any patents! It is also of interest to ponder upon the number of patents which are granted which closely describe an aspect of active noise and vibration control which has been patented previously. All we can do here is try to appeal to some sense of reason as too much litigation of this type will stifle research, slow down new product development and enrich the legal profession – all of which we would be better off without.

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