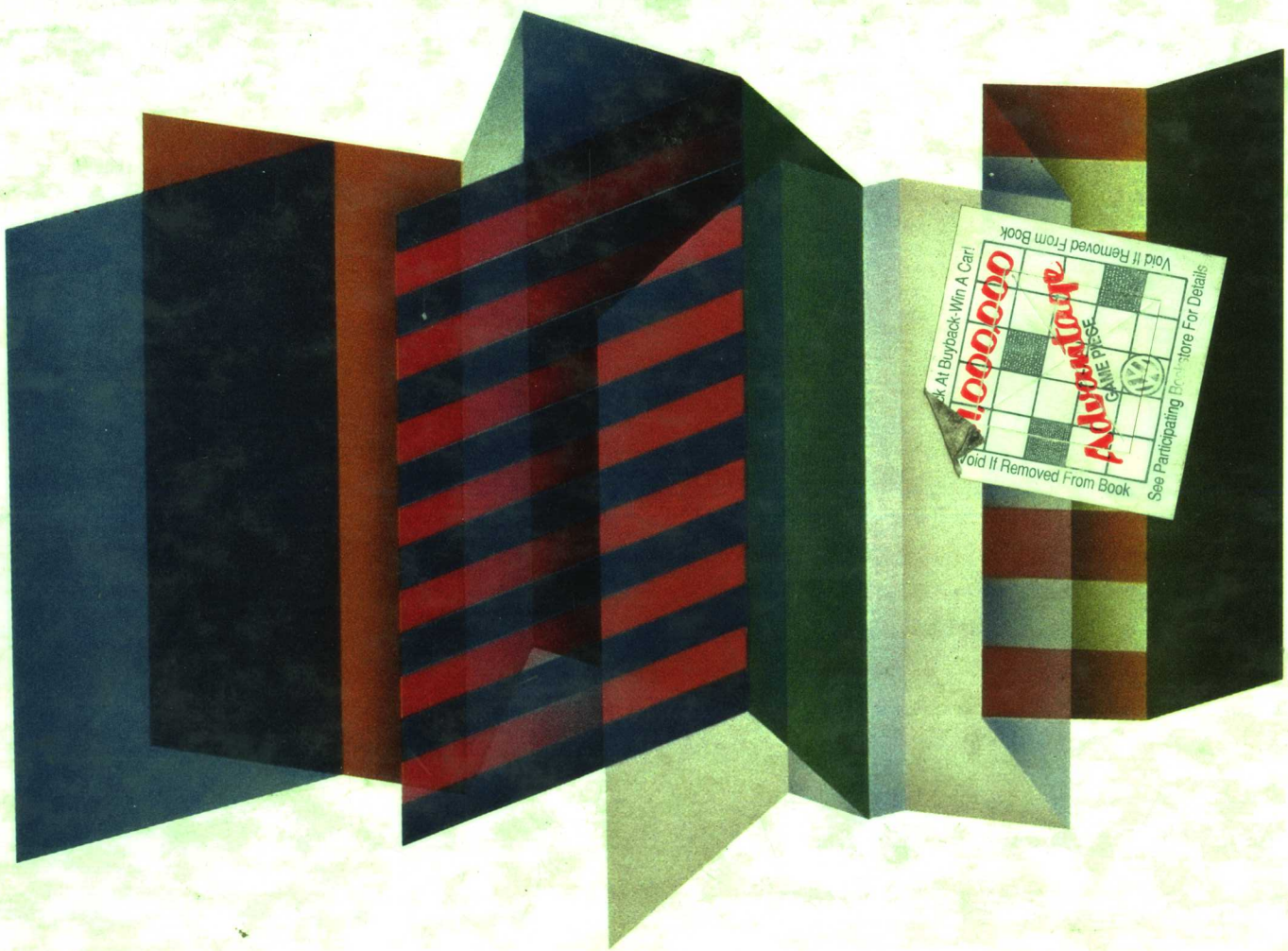
Lial/Miller

Beginning Algebra/4th



Beginning Algebra

Fourth Edition

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Glenview, Illinois

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To the Student

The *Study Guide: Beginning Algebra*, Fourth Edition, and the *Student Solutions Manual* are available from your local college bookstore. These books can help you study and review the course material.

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Preface

Ever since the first edition of *Beginning Algebra*, we have tried to produce a book that is both *useful* to the student and *helpful* for the instructor. For the student, we have written a book with explanations and examples that are clear, direct, and to the point. The exercise sets are carefully graded in difficulty, with examples corresponding to exercises. For the instructor, we have produced a complete instructional package. The text is carefully laid out to match standard courses, with the sections including only what an instructor would normally expect to find in each section. The supplemental package offers those items we have found useful in our own classes: tests, complete solutions to 75 percent of the exercises, additional exercises keyed to objectives, a *Study Guide*, audiotapes, and videotapes.

KEY FEATURES

Word Problems A problem-solving approach gives students early and repeated experience in solving applied problems. A list of steps for solving word problems is first presented in Chapter 2, yet even Chapter 1 has some simple word problems. Throughout the text students are given practice translating English words into algebraic symbols. In this way, students see word problems early and gradually improve their problem-solving skills.

Examples More than 350 worked-out examples clearly illustrate concepts and techniques. Second color is used to identify pertinent steps within examples, as well as to highlight explanatory side comments.

Keyed Objectives Each section opens with a list of skills that students should learn in that section. These objectives are keyed to the appropriate portions of each section with symbols such as **1** or **2**.

EXERCISES

Graded Exercises The range of difficulty in the exercise sets affords students ample practice with drill problems. Then they are eased gradually through problems of increasing difficulty to problems that will challenge outstanding students. More than 4000 drill exercises and 300 word problems, keyed to examples, are included.

Calculator Exercises Calculator exercises have been included throughout the book. These optional exercises are identified with the symbol  .

Review Exercises Beginning in Chapter 2, most exercise sets end with a few problems that help prepare students for the following section. For example, at the end of the section preceding the discussion of solving quadratic equations by factoring, students review the solution of equations such as $2x + 3 = 0$.

Chapter Review Exercises Extensive review exercises at the end of each chapter, nearly 650 in all, provide further opportunity for mastery of the material before students take an examination. These exercises are keyed to appropriate sections in the text.

Chapter Tests Sample tests, of a length comparable to that of actual classroom tests, have been made somewhat more difficult in this edition of the book. More than 250 test questions give additional practice to students.

SECOND COLOR

Second color is used pedagogically in the following ways.

- Screens set off key definitions, formulas, and procedures, helping students review.
- Color side comments within examples explain the structure of the problem.
- For clarity, the end of each example is indicated with a color symbol, ●.
- Warnings about common student errors are included in color when appropriate.

SUPPLEMENTS

Beginning Algebra, Fourth Edition, has an extensive supplemental package that includes testing materials, solutions, and electronic media.

The **Instructor's Guide** features five tests for each chapter. One version is a multiple-choice test. In addition to the chapter tests, two forms of a final examination are given, as well as a diagnostic pretest. Answers to all tests are provided in columns that can be lined up with the answer blanks of the student tests. Answers to even-numbered exercises in the text are given in this guide.

Additional Exercises, keyed to objectives, are also available. Ten to 20 exercises for each objective in the book help students review exactly where their difficulties are.

A **Solutions Manual,** featuring complete solutions to all even-numbered exercises, is available to adopters.

A **Student Solutions Manual** has solutions to one half of the odd-numbered exercises in the text. Some students use this volume as an additional source of examples.

A **Study Guide,** in a semiprogrammed format, provides additional practice and reinforcement for students.

Professional-quality **videotapes** are available at low cost to users of this text. The tapes amplify the ideas in the book that cause students the most difficulty.

Audiotapes that cover all topics in the text are available at no charge to users of the book. Students who need help with a particular topic or who have missed class find these tapes help them master the material.

We thank the many users of the previous editions of this book who were kind enough to share their experiences with us. This revision has benefited from their comments and suggestions. (See the list of acknowledgments that follows this Preface.)

We also thank the people who reviewed all or part of the revised manuscript and gave us many helpful suggestions: Ron Beeler, East Central College; Arthur Dull, Diablo Valley College; K. Elayn Gay, University of New Orleans; John H. Gray, University of North Alabama; George Grisham, Illinois Central College; Daniel A. Hogan, Hinds Junior College; Myrna L. Mitchell, Pima Community College; William C. Monnin, University of Southwestern Louisiana; John Monroe, University of Akron; David W. Sicks, Olympic College; Mary Jo Steig, Mesa Community College; George T. Wales, Ferris State College; Hettie M. Williams, Broward Community College; and Cynthia Yang, Miami University.

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Contents

1

Number Systems

These carpenters are using a ruler, which is based on the same idea as a number line.

1.1 Fractions

1. Write fractions in lowest terms.
2. Multiply fractions.
3. Divide fractions.
4. Write a fraction as an equal fraction having a given denominator.
5. Add fractions.
6. Subtract fractions.

To prepare for the study of algebra, we begin with a brief review of arithmetic. In everyday life, the numbers seen most often are the **whole numbers,**

$$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ \ldots,$$

and the **fractions,** such as

$$\frac{1}{2},\ \frac{2}{3},\ \frac{11}{12},\ \text{and so on.}$$

In a fraction, the top number is called the **numerator** and the bottom number is called the **denominator.**

1 A fraction is in **lowest terms** when the numerator and the denominator cannot both be divided by the same number other than 1 (without remainder). Use the following steps to write a fraction in lowest terms.

Writing a Fraction in Lowest Terms

Step 1 Find the largest number that will divide into both the numerator and the denominator. This number is called the **greatest common factor.**

Step 2 Divide both the numerator and the denominator by the greatest common factor.

EXAMPLE 1 Write each fraction in lowest terms.

(a) $\dfrac{10}{15} = \dfrac{10 \div 5}{15 \div 5} = \dfrac{2}{3}$

Since the largest number that can be divided without remainder into both 10 and 15 is 5, the number 5 is the greatest common factor of 10 and 15. Both numerator and denominator are divided by 5 to write the fraction in lowest terms.

(b) $\dfrac{15}{45} = \dfrac{15 \div 15}{45 \div 15} = \dfrac{1}{3}$

Both 15 and 45 can be divided by 3, 5, and 15. The largest of these three numbers, 15, is the greatest common factor, so use 15 to divide both the numerator and denominator. ●

2 The basic operations on whole numbers are addition, subtraction, multiplication, and division. These same operations apply to fractions. To *multiply* two fractions, multiply their numerators and then multiply their denominators. In symbols, if a/b and c/d are fractions, then

Multiplying Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

The dot indicates multiplication. For example,

$$3 \cdot 5 = 15 \quad \text{and} \quad 7 \cdot 8 = 56.$$

The answer to a multiplication problem is called the **product.**

EXAMPLE 2 Find the product of 3/8 and 4/9, and write it in lowest terms.

First, multiply 3/8 and 4/9.

$$\frac{3}{8} \cdot \frac{4}{9} = \frac{3 \cdot 4}{8 \cdot 9} = \frac{12}{72}$$

Write the product in lowest terms.

$$\frac{12}{72} = \frac{12 \div 12}{72 \div 12} = \frac{1}{6} \quad \bullet$$

3 Two fractions are **reciprocals** of each other if their product is 1. For example, 3/4 and 4/3 are reciprocals since

$$\frac{3}{4} \cdot \frac{4}{3} = 1.$$

Also, 7/11 and 11/7 are reciprocals of each other. The reciprocal is used to divide fractions. To *divide* two fractions, multiply the first and the reciprocal of the second, or

Dividing Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

The reason this method works will be explained in a later chapter. The answer to a division problem is called a **quotient.** For example, the quotient of 20 and 10 is 2, since $20 \div 10 = 2$.

EXAMPLE 3 Find the following quotients, and write them in lowest terms.

(a) $\frac{3}{4} \div \frac{8}{5} = \frac{3}{4} \cdot \frac{5}{8} = \frac{3 \cdot 5}{4 \cdot 8} = \frac{15}{32}$

(b) $\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \cdot \frac{8}{5} = \frac{3 \cdot 8}{4 \cdot 5} = \frac{24}{20} = \frac{6}{5}$ $\bullet$

4 All the fractions in an addition or subtraction problem must have the same denominator. If they do not, rewrite the fractions with a new common denominator. For example, to rewrite 3/4 as a fraction with a denominator of 32,

$$\frac{3}{4} = \frac{\quad}{32},$$

we need to find the number that can be multiplied by 4 to give 32. Since $4 \cdot 8 = 32$, use the number 8. We want the value of the original fraction, 3/4, to stay the same, so multiply 3/4 by the fraction 8/8, which equals 1.

$$\frac{3}{4} = \frac{3}{4} \cdot \frac{8}{8} = \frac{3 \cdot 8}{4 \cdot 8} = \frac{24}{32}$$

EXAMPLE 4 Write 5/8 as a fraction with a denominator of 72.

Since the quotient of 72 and 8 is 9, multiply 5/8 by 9/9.

$$\frac{5}{8} = \frac{5}{8} \cdot \frac{9}{9} = \frac{5 \cdot 9}{8 \cdot 9} = \frac{45}{72} \quad \bullet$$

5 The **sum** of two fractions having the same denominator is found by adding the numerators. If a/b and c/b are fractions with the same denominator, then

Adding Fractions

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}.$$

EXAMPLE 5 Add.

(a) $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

(b) $\frac{2}{10} + \frac{5}{10} = \frac{2+5}{10} = \frac{7}{10}$ •

If two fractions to be added do not have the same denominators, the rule above can still be used, but only after the fractions are rewritten with a common denominator.

EXAMPLE 6 Add.

(a) $\frac{1}{2} + \frac{1}{3}$

We cannot add until the fractions have the same denominator. Use 6 as a common denominator, since both 2 and 3 divide into 6. Write 1/2 and 1/3 as fractions with a denominator of 6.

$$\frac{1}{2} = \frac{1}{2} \cdot \frac{3}{3} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{and} \quad \frac{1}{3} = \frac{1}{3} \cdot \frac{2}{2} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$$

Now add.

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$$

(b) $3\frac{1}{2} + 2\frac{3}{4}$

Change both numbers to fractions, as follows.

$$3\frac{1}{2} = 3 + \frac{1}{2} = \frac{3}{1} + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2}$$

Also, $$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}.$$

Now add.

$$3\frac{1}{2} + 2\frac{3}{4} = \frac{7}{2} + \frac{11}{4} = \frac{14}{4} + \frac{11}{4} = \frac{25}{4} \text{ or } 6\frac{1}{4}$$ •

6 *Subtraction* of fractions is similar to addition. Just subtract the numerators instead of adding them, according to the following definition.

Subtracting Fractions

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

EXAMPLE 7 Subtract.

(a) $\frac{5}{8} - \frac{3}{8} = \frac{5-3}{8} = \frac{2}{8} = \frac{1}{4}$

(b) $\frac{3}{4} - \frac{1}{3}$

A common denominator is 12.

$$\frac{3}{4} - \frac{1}{3} = \frac{\mathbf{9}}{\mathbf{12}} - \frac{\mathbf{4}}{\mathbf{12}} = \frac{9-4}{12} = \frac{5}{12} \quad \bullet$$

1.1 EXERCISES

Write each fraction in lowest terms. See Example 1.

1. $\frac{7}{14}$ **2.** $\frac{3}{9}$ **3.** $\frac{10}{12}$ **4.** $\frac{8}{10}$

5. $\frac{16}{18}$ **6.** $\frac{14}{20}$ **7.** $\frac{50}{75}$ **8.** $\frac{32}{48}$

9. $\frac{72}{108}$ **10.** $\frac{96}{120}$ **11.** $\frac{120}{144}$ **12.** $\frac{77}{132}$

Find the products or quotients. Write answers in lowest terms. See Examples 2 and 3.

13. $\frac{3}{4} \cdot \frac{3}{5}$ **14.** $\frac{3}{8} \cdot \frac{5}{7}$ **15.** $\frac{1}{10} \cdot \frac{6}{5}$ **16.** $\frac{6}{7} \cdot \frac{1}{3}$

17. $\frac{9}{4} \cdot \frac{8}{15}$ **18.** $\frac{3}{5} \cdot \frac{20}{15}$ **19.** $\frac{3}{8} \div \frac{5}{4}$ **20.** $\frac{9}{16} \div \frac{3}{8}$

21. $\frac{5}{12} \div \frac{15}{4}$ **22.** $\frac{15}{16} \div \frac{30}{8}$ **23.** $\frac{15}{32} \div \frac{25}{8}$ **24.** $\frac{24}{25} \div \frac{3}{50}$

25. $\frac{5}{9} \cdot \frac{7}{10}$ **26.** $\frac{21}{30} \cdot \frac{5}{7}$ **27.** $\frac{13}{10} \cdot \frac{5}{3}$ **28.** $\frac{21}{16} \cdot \frac{8}{7}$

29. $\frac{28}{3} \cdot \frac{6}{7}$ **30.** $\frac{121}{9} \cdot \frac{18}{11}$ **31.** $\frac{28}{15} \div \frac{14}{5}$ **32.** $\frac{120}{7} \div \frac{45}{3}$