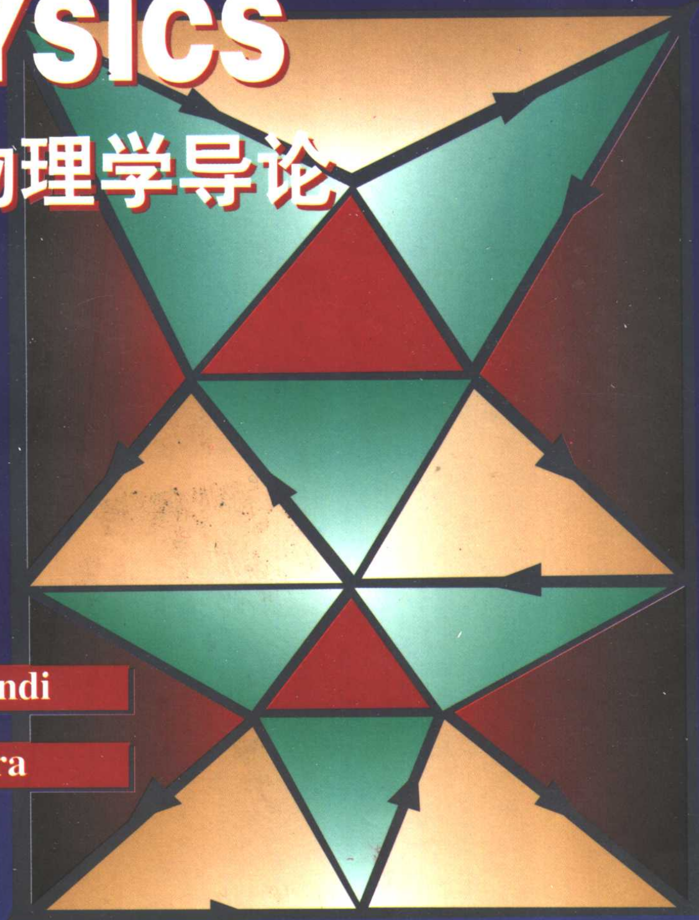
AN INTRODUCTION TO GEOMETRICAL PHYSICS

几何物理学导论



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书　　名：An Introduction to Geometrical Physics

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中 译 名：几何物理学导论

出 版 者：世界图书出版公司北京公司

印 刷 者：北京中西印刷厂

发　　行：世界图书出版公司北京公司（北京朝内大街 137 号　100010）

开　　本：1/32　850×1168　　印　张：22.5

出版年代：2000 年 6 月

书　　号：ISBN 7-5062-4717-8/ O・297

版权登记：图字 01-2000-0113

定　　价：80.00 元

Dedicated to

Nice, José ...

... Dina and Tito.

FOREWORD

This book grew out of courses given at the Instituto de Física Teórica for many years. As the title announces, it is intended as a first, elementary approach to "Geometrical Physics" — to be understood as a chapter of Mathematical Physics. Mathematical Physics is a moving subject, and has moved faster in recent times. From the study of differential equations and related special functions, it has migrated to the more qualitative realms of topology and algebra. The bridge has been the framework of geometry. The passage supposes an acquaintance with concepts and terms of a new kind, to which this text is a tentative introduction.

In its technical uses, the word "geometry" has since long lost its metric etymological meaning. It is the science of space, or better, of spaces. Thus, the name should be understood as a study of those spaces which are of interest in Physics. This emphasis on the notion of space has dominated the choice of topics – they will have in common the use of "spaces". Some may seem less geometric than others, but a space is always endowed with a few basic, irreducible properties enabling some kind of analysis, allowing a discussion of relations between its different parts.

Up until the sixties, General Relativity was seen as the prototype of geometrical intrusion in Physics. Gravitation stood apart as the sole example of geometrization amongst the fundamental interactions. Things changed a lot with the discovery of the renormalizability of gauge theories, and with their subsequent coming to age as *the* theories for all the other fundamental interactions. At least as a general principle, the gauge principle appears nowadays as a vast unifying idea. It came as an astonishing surprise that gauge theories are still more geometrical in character and structure than General Relativity.

Landau's criterium, so well proven for Physics as for good wines, has been loosely adopted: results less than ten years old in the field are given only limited attention. It is true that some acceleration in new developments is easily felt, but it is always graceful to present some seemingly objective alibi for the authors' limitations. Let it be so, and let us, like those real bourgogne connoisseurs, leave some recent vintages to rest in the shadows for some more time.

The text falls into five parts. Parts I, II and III constitute the main text. Part IV is made up of mathematical topics and Part V of physical topics. The main text is intended as an introductory, largely descriptive presentation of the basic notions of geometry from a physicist's point of view. Pedagogically, it would provide for a first one-semester course. But it is also a kind of guide to the topical chapters. It refers to Part IV for further details, and uses Part V as a source of examples. The main text is divided into chapters, these in sections and these in paragraphs. For instance, "§4.2.5" means chapter 4 of the main text, section 2, paragraph 5. Calls like "§n" refer to a numbered paragraph of the same section. Paragraphs with additional examples and complements, which can be skipped in a first lecture, are printed in smaller characters. The topical chapters are simply divided in sections. For instance, "Phys.7.5" means section 5 of the topical chapter Phys.7.

The topical sections oscillate between the "fascicule des résultats" and application-almost-exercise styles. The mathematical topics should not be mistaken as anything more than a guide for further study. They are a physicists' presentation of mathematical subjects in broad brushstrokes, intending to illustrate and complement the main text. The physical topics are similarly introductory, always in a physicists' view, trying never to say twain for two, and avoiding as long as possible the gobbledygook theoreticians became prone to in recent times. It tries to present well known subjects from a point of view which is more geometrical than the usual treatments. Some general references are given at the end of each topic, the footnotes being left for those of more immediate character.

Mathematics is, amongst many other things, the language of Physics. Repetitions are unavoidable and, as with all language apprenticeship, they may be even desirable. The pedagogical bias justifies some duplications, as that of indices at the beginning of each topical section, emphatically advocated by our students.

Like any text coming up from lecture notes, these pages owe a lot to many. To the financing agencies, like CNPq, FAPESP and CAPES, which supported the authors during the period. To our colleagues of São Paulo and Paris, in special C. E. Harle, B. M. Pimentel, D. Galetti, G. Francisco, R. A. Kraenkel, G. A. Matsas, L. A. Saeger, A. L. Barbosa, M. Dubois-Violette, R. Kerner and J. Madore. Of course, to the many consulted authors, hopefully all quoted in the reference list. And, last but not least, to our attentive and long-suffering editor, Tina Jeavons.

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0 SPACE AND GEOMETRY

What stuff 'tis made of, whereof it is born,
I am to learn.
Merchant of Venice

The simplest geometrical setting used — consciously or not — by physicists in their everyday work is the 3-dimensional euclidean space $\mathbf{E}^3$. It consists of the set $\mathbf{R}^3$ of ordered triples of real numbers such as $\mathbf{p} = (p^1, p^2, p^3)$, $\mathbf{q} = (q^1, q^2, q^3)$, etc, and is endowed with a very special characteristic, a metric defined by the distance function

$$d(\mathbf{p}, \mathbf{q}) = \left[\sum_{i=1}^{3} (p^i - q^i)^2\right]^{1/2} .$$

It is the space of ordinary human experience and the starting point of our geometric intuition. Studied for two and a half millenia, it has been the object of celebrated controversies, the most famous concerning the minimum number of properties necessary to define it completely.

From Aristotle to Newton, through Galileo and Descartes, the very word *space* has been reserved to $\mathbf{E}^3$. Only in the last century it has become clear that other different spaces could be thought of, and mathematicians have since greatly amused themselves by inventing all kinds of them. For physicists, the age-long debate shifted to another question: how can we recognize, amongst such innumerable possible spaces, that *real* space chosen by Nature as the stage-set of its processes? For example, suppose the space of our everyday experience consists of the same set of triples above, but with a different distance function, such as

$$d(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^{3} | p^i - q^i | .$$

This would define a different metric space, in principle as good as the one given above. Were it only a matter of principle, it would be as good as any other space given by any distance function with $\mathbf{R}^3$ as set point. It so happens, however, that Nature has chosen the former and not the latter space for us to live in. To know which one is *the* real space is not a simple question of principle — something else is needed. What else? The answer may seem rather trivial in the case of our home space, though less so in other spaces singled out by Nature in the many different situations which are objects of physical study. It was given by Riemann in his famous Inaugural Address[1]:

" ... those properties which distinguish Space from other conceivable triply extended quantities can only be deduced from experience."

Thus, *from experience*! It is experiment which tells us in which space we actually live in. When we measure distances we find them to be independent of the direction of the straight lines joining the points. And this isotropy property rules out the second proposed distance function while admitting the metric of the euclidean space.

In reality, Riemann's statement implies an epistemological limitation: it will never be possible to ascertain *exactly* which one is the real space. Other isotropic distance functions are, in principle, admissible and more experiments are necessary to decide between them. In Riemann's time already other geometries were known (those found by Lobachevsky and Boliyai) that could be as similar to the euclidean geometry as we might wish in the restricted regions experience is confined to. In honesty, all we can say is that $\mathbf{E}^3$, as a model for our ambient space, is strongly favored by present day experimental evidence in scales ranging from (say) human dimensions down to about 10^{-15} cm. Our knowledge on smaller scales is limited by our capacity to probe them. For larger scales, according to General Relativity, the validity of this model depends on the presence and strength of gravitational fields — $\mathbf{E}^3$ is good only as long as gravitational fields are very weak.

" These data are — like all data — not logically necessary, but only of empirical certainty . . . one can therefore investigate their likelihood, which is certainly very great within the bounds of observation, and afterwards decide upon the legitimacy of extending them beyond the bounds of observation, both in the direction of the immeasurably large and in the direction of the immeasurably small."

The only remark we could add to these words, pronounced in 1854, is that the "bounds of observation" have greatly receded with respect to the values of Riemann times.

" . . . geometry presupposes the concept of space, as well as assuming the

[1] A translation of Riemann's Address can be found in Spivak 1970, vol. II.