Heavy Quark Physics

ANEESH V. MANOHAR
AND
MARK B. WISE

CAMBRIDGE MONOGRAPHS
ON PARTICLE PHYSICS, NUCLEAR PHYSICS
AND COSMOLOGY

10

Heavy Quark Physics

ANEESH V. MANOHAR
University of California,
San Diego

MARK B. WISE
California Institute of Technology

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK http://www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA http://www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

First published 2000

Printed in the United States of America

Typeset in Times Roman 11/13 *System* LaTeX 2_ε [TB]

A catalog record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data

Manohar, Aneesh Vasant.
Heavy quark physics / Aneesh V. Manohar, Mark B. Wise.
p. cm. – (Cambridge monographs on particle physics, nuclear physics, and cosmology ; 10)
ISBN 0-521-64241-8 (hb)
1. Quarks. 2. Heavy particles (Nuclear physics) I. Wise, Mark B., 1953– . II. Title. III. Series.
QC793.5.Q252M36 2000 99-38167
539.7′2167 – dc21 CIP

ISBN 0 521 64241 8 hardback

Preface

We are entering an exciting era of B meson physics, with several new high luminosity facilities that are about to start taking data. The measurements will provide information on quark couplings and CP violation. To make full use of the experimental results, it is important to have reliable theoretical calculations of the hadronic decay amplitudes in terms of the fundamental parameters in the standard model Lagrangian. In recent years, many such calculations have been performed using heavy quark effective theory (HQET), which has emerged as an indispensible tool for analyzing the interactions of heavy hadrons. This formalism makes manifest heavy quark spin-flavor symmetry, which is exact in the infinite quark mass limit, and allows one to systematically compute the correction terms for finite quark mass.

This text is designed to introduce the reader to the concepts and methods of HQET, developing them to the stage where explicit calculations are performed. It is not intended to be a review of the field, but rather to serve as an introduction accessible to both theorists and experimentalists. We hope it will be useful not just to those working in the area of heavy quark physics but also to physicists who work in other areas of high energy physics but want a deeper appreciation of HQET methods. We felt that if the book is to serve this role, then it is important that it not be too long. An effort was made to keep the book at the 200-page level and this necessitated some difficult decisions on which subjects were to be covered.

The material presented here is not uniform in its difficulty. Section 1.8 on the operator product expansion, Section 4.6 on renormalons, and Chapter 6 on inclusive B decays are considerably more difficult than the other parts of the book. Although this material is very important, depending on the background of the reader, it may be useful to skip it on first reading. Chapter 3 involves some familiarity with radiative corrections in field theory as studied, for example, in a graduate course that discusses renormalization in quantum electrodynamics. Readers less comfortable with loop corrections can read through the chapter, accepting the results for the one-loop diagrams, without necessarily going through the detailed computations. A section on problems at the end of each chapter is intended to give the reader more experience with the concepts introduced in that chapter. The problems are of varying difficulty and most can be completed in a fairly short period of time. Three exceptions to this are Problem 2 of Chapter 3 and Problems 3 and 7 of Chapter 6, which are considerably more time-consuming.

This book could serve as a text for a one-semester graduate course on heavy quark physics. The background necessary for the book is quantum field theory and some familiarity with the standard model. The latter may be quite modest, since Chapter 1 is devoted to a review of the standard model.

The only references that are given in the text are to lattice QCD results or to experimental data that cannot be readily found by consulting the Particle Data Book (`http://pdg.lbl.gov`). However, at the end of each chapter a guide to some of the literature is given. The emphasis here is on the earlier papers, and even this list is far from complete.

We have benefited from the comments given by a large number of our colleagues who have read draft versions of this book. Particularly noteworthy among them are Martin Gremm, Elizabeth Jenkins, Adam Leibovich, and Zoltan Ligeti, who provided a substantial number of valuable suggestions.

Updates to the book can be found at the URL:

`http://einstein.ucsd.edu/hqbook`.

Contents

1
Review

The standard model of strong, weak, and electromagnetic interactions is a relativistic quantum field theory that describes all known interactions of quarks and leptons. This chapter provides a quick review of features of the standard model that are relevant for heavy quark systems, and of basic field theory techniques such as the operator product expansion. It will also serve the purpose of defining some of the normalization conventions and notation to be used in the rest of the book.

1.1 The standard model

The standard model is a gauge theory based on the gauge group $SU(3) \times SU(2) \times U(1)$. The $SU(3)$ gauge group describes the strong color interactions among quarks, and the $SU(2) \times U(1)$ gauge group describes the electroweak interactions. At the present time three generations of quarks and leptons have been observed. The measured width of the Z boson does not permit a fourth generation with a massless (or light) neutrino. Many extensions of the minimal standard model have been proposed, and there is evidence in the present data for neutrino masses, which requires new physics beyond that in the minimal standard model. Low-energy supersymmetry, dynamical weak symmetry breaking, or something totally unexpected may be discovered at the next generation of high-energy particle accelerators.

The focus of this book is on understanding the physics of hadrons containing a bottom or charm quark. The technically difficult problem is understanding the role strong interactions play in determining the properties of these hadrons. For example, weak decays can be computed by using a low-energy effective weak Hamiltonian. Any new physics beyond the standard model can also be treated by using a local low-energy effective interaction, and the theoretical difficulties associated with evaluating hadronic matrix elements of this interaction are virtually identical to those for the weak interactions. For this reason, most of the discussion in this book will focus on the properties of heavy quark hadrons as computed in the standard model.

The matter fields in the minimal standard model are three families of spin-1/2 quarks and leptons, and a spin-zero Higgs boson, shown in Table 1.1. The index i on the Fermion fields is a family or generation index $i = 1, 2, 3$, and the subscripts L and R denote left- and right-handed fields, respectively,

$$\psi_L = P_L \psi, \qquad \psi_R = P_R \psi, \tag{1.1}$$

where P_L and P_R are the projection operators

$$P_L = \frac{1}{2}(1 - \gamma_5), \qquad P_R = \frac{1}{2}(1 + \gamma_5). \tag{1.2}$$

Q_L^i, u_R^i, d_R^i are the quark fields and L_L^i, e_R^i are the lepton fields. All the particles associated with the fields in Table 1.1 have been observed experimentally, except for the Higgs boson. The $SU(2) \times U(1)$ symmetry of the electroweak sector is not manifest at low energies. In the standard model, the $SU(2) \times U(1)$ symmetry is spontaneously broken by the vacuum expectation value of the Higgs doublet

Table 1.1. *Matter fields in the standard model*[a]

Field	$SU(3)$	$SU(2)$	$U(1)$	Lorentz
$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	**3**	**2**	1/6	(1/2, 0)
u_R^i	**3**	**1**	2/3	(0, 1/2)
d_R^i	**3**	**1**	−1/3	(0, 1/2)
$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$	**1**	**2**	−1/2	(1/2, 0)
e_R^i	**1**	**1**	−1	(0, 1/2)
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	**1**	**2**	1/2	(0, 0)

[a] The index i labels the quark and lepton family. The dimensions of the $SU(3)$ and $SU(2)$ representations and their $U(1)$ charge are listed in the second, third, and fourth columns, respectively. The transformation properties of the fermion fields under the Lorentz group $SO(3, 1)$ are listed in the last column.

H. The spontaneous breakdown of $SU(2) \times U(1)$ gives mass to the $W^{\pm}$ and Z^0 gauge bosons. A single Higgs doublet is the simplest way to achieve the observed pattern of spontaneous symmetry breaking, but a more complicated scalar sector, such as two doublets, is possible.

The terms in the standard model Lagrangian density that involve only the Higgs doublet

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{1.3}$$

are

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) - V(H), \tag{1.4}$$

where D_μ is the covariant derivative and $V(H)$ is the Higgs potential

$$V(H) = \frac{\lambda}{4}(H^\dagger H - v^2/2)^2. \tag{1.5}$$

The Higgs potential is minimized when $H^\dagger H = v^2/2$. The $SU(2) \times U(1)$ symmetry can be used to rotate a general vacuum expectation value into the standard form

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \tag{1.6}$$

where v is real and positive.

The generators of the $SU(2)$ gauge symmetry acting on the Higgs (i.e., fundamental) representation are

$$T^a = \sigma^a/2, \quad a = 1, 2, 3, \tag{1.7}$$

where the Pauli spin matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1.8}$$

and the generators are normalized to $\operatorname{Tr} T^a T^b = \delta^{ab}/2$. The $U(1)$ generator Y is called hypercharge and is equal to $1/2$ acting on the Higgs doublet (see Table 1.1). One linear combination of $SU(2) \times U(1)$ generators is left unbroken by the vacuum expectation value of the Higgs field H given in Eq. (1.6). This linear combination is the electric charge generator $Q = T^3 + Y$, where

$$Q = T^3 + Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{1.9}$$

when acting on the Higgs representation. It is obvious from Eqs. (1.6) and (1.9) that

$$Q\langle H \rangle = 0, \tag{1.10}$$

so that electric charge is left unbroken. The $SU(3) \times SU(2) \times U(1)$ symmetry of the standard model is broken to $SU(3) \times U(1)_Q$ by the vacuum expectation value of H, where the unbroken electromagnetic $U(1)_Q$ is the linear combination of the original $U(1)$ hypercharge generator, Y, and the $SU(2)$ generator, T^3, given in Eq. (1.9).

Expanding H about its expectation value

$$H(x) = \begin{pmatrix} h^+(x) \\ v/\sqrt{2} + h^0(x) \end{pmatrix} \tag{1.11}$$

and substituting in Eq. (1.5) gives the Higgs potential

$$V(H) = \frac{\lambda}{4}(|h^+|^2 + |h^0|^2 + \sqrt{2}v\,\mathrm{Re}\,h^0)^2. \tag{1.12}$$

The fields h^+ and $\mathrm{Im}\,h^0$ are massless. This is an example of Goldstone's theorem. The potential has a continuous three-parameter family of degenerate vacua that are obtained from the reference vacuum in Eq. (1.6) by global $SU(2) \times U(1)$ transformations. [Of the four $SU(2) \times U(1)$ generators, one linear combination Q leaves the vacuum expectation value invariant, and so does not give a massless mode.] Field excitations along these degenerate directions cost no potential energy and so the fields h^+ and $\mathrm{Im}\,h^0$ are massless. There is one massive scalar that is destroyed by the (normalized) real scalar field $\sqrt{2}\,\mathrm{Re}\,h^0$. At tree level, its mass is

$$m_{\mathrm{Re}\,h^0} = \sqrt{\frac{\lambda}{2}}\,v. \tag{1.13}$$

Global $SU(2) \times U(1)$ transformations allow the space–time independent vacuum expectation value of H to be put into the form given in Eq. (1.6). Local $SU(2) \times U(1)$ transformations can be used to eliminate $h^+(x)$ and $\mathrm{Im}\,h^0(x)$ completely from the theory, and to write

$$H(x) = \begin{pmatrix} 0 \\ v/\sqrt{2} + \mathrm{Re}\,h^0(x) \end{pmatrix}. \tag{1.14}$$

This is the standard model in unitary gauge, in which the $W^\pm$ and Z bosons have explicit mass terms in the Lagrangian, as is shown below. In this gauge, the massless fields h^+ and $\mathrm{Im}\,h^0$ are eliminated, and so do not correspond to states in the spectrum of the theory.

The gauge covariant derivative acting on any field ψ is

$$D_\mu = \partial_\mu + igA_\mu^A T^A + ig_2 W_\mu^a T^a + ig_1 B_\mu Y, \tag{1.15}$$

where T^A, $A = 1, \ldots, 8$, are the eight color $SU(3)$ generators T^a, $a = 1, 2, 3$ are the weak $SU(2)$ generators, and Y is the $U(1)$ hypercharge generator. The generators are chosen to be in the representation of the field ψ on which the covariant derivative acts. The gauge bosons and coupling constants associated with these gauge groups are denoted A_μ^A, W_μ^a, and B_μ and g, g_2, and g_1, respectively. The kinetic term for the Higgs field contains a piece quadratic in the gauge fields when expanded about the Higgs vacuum expectation value using Eq. (1.11). The quadratic terms that produce a gauge-boson mass are

$$\mathcal{L}_{\substack{\text{gauge-boson}\\ \text{mass}}} = \frac{g_2^2 v^2}{8}(W^1 W^1 + W^2 W^2) + \frac{v^2}{8}(g_2 W^3 - g_1 B)^2, \tag{1.16}$$

where for simplicity of notation Lorentz indices are suppressed. The charged W-boson fields

$$W^\pm = \frac{W^1 \mp i W^2}{\sqrt{2}} \tag{1.17}$$

have mass

$$M_W = \frac{g_2 v}{2}. \tag{1.18}$$

It is convenient to introduce the weak mixing angle θ_W defined by

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \qquad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}. \tag{1.19}$$

The Z-boson field and photon field $\mathcal{A}$ are defined as linear combinations of the neutral gauge-boson fields W^3 and B,

$$\begin{aligned} Z &= \cos\theta_W W^3 - \sin\theta_W B, \\ \mathcal{A} &= \sin\theta_W W^3 + \cos\theta_W B. \end{aligned} \tag{1.20}$$

The Z boson has a mass at tree level

$$M_Z = \frac{\sqrt{g_1^2 + g_2^2}}{2} v = \frac{M_W}{\cos\theta_W}, \tag{1.21}$$

and the photon is massless.

The covariant derivative in Eq. (1.15) can be reexpressed in terms of the mass-eigenstate fields as

$$\begin{aligned} D_\mu = \partial_\mu &+ i g A_\mu^A T^A + i\frac{g_2}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) \\ &+ i\sqrt{g_1^2 + g_2^2}(T_3 - \sin^2\theta_W Q)Z_\mu + i g_2 \sin\theta_W Q \mathcal{A}_\mu, \end{aligned} \tag{1.22}$$

where $T^\pm = T^1 \pm i T^2$. The photon coupling constant in Eq. (1.22) leads to the relation between the electric charge e and the couplings $g_{1,2}$,

$$e = g_2 \sin\theta_W = \frac{g_2 g_1}{\sqrt{g_1^2 + g_2^2}}, \tag{1.23}$$

so the Z coupling constant $\sqrt{g_1^2+g_2^2}$ in Eq. (1.22) is conventionally written as $e/(\sin\theta_W\cos\theta_W)$.

Outside of unitary gauge the H kinetic term also has a piece quadratic in the fields where the Goldstone bosons h^+, $\mathrm{Im}\, h^0$ mix with the longitudinal parts of the massive gauge bosons. This mixing piece can be removed by adding to the Lagrange density the 't Hooft gauge fixing term

$$\mathcal{L}_{\substack{\text{gauge}\\ \text{fix}}} = -\frac{1}{2\xi}\sum_a\left[\partial^\mu W_\mu^a + ig_2\xi(\langle H\rangle^\dagger T^a H - H^\dagger T^a\langle H\rangle)\right]^2 - \frac{1}{2\xi}\left[\partial^\mu B_\mu + ig_1\xi(\langle H\rangle^\dagger Y H - H^\dagger Y\langle H\rangle)\right]^2, \tag{1.24}$$

which gives the Lagrangian in R_ξ gauge, where ξ is an arbitrary parameter. The fields $h^\pm$ and $\mathrm{Im}\, h^0$ have mass terms proportional to the gauge fixing constant ξ. In Feynman gauge $\xi = 1$ (the easiest for doing calculations), these masses are the same as those of the $W^\pm$ and Z. $\mathrm{Im}\, h^0$ and $h^\pm$ are not physical degrees of freedom since in unitary gauge $\xi\to\infty$ their masses are infinite and they decouple from the theory.

$SU(3)\times SU(2)\times U(1)$ gauge invariance prevents bare mass terms for the quarks and leptons from appearing in the Lagrange density. The quarks and leptons get mass because of their Yukawa couplings to the Higgs doublet,

$$\mathcal{L}_{\text{Yukawa}} = g_u^{ij}\,\bar u_R^i H^T\epsilon Q_L^j - g_d^{ij}\,\bar d_R^i H^\dagger Q_L^j - g_e^{ij}\,\bar e_R^i H^\dagger L_L^j + \text{h.c.} \tag{1.25}$$

where h.c. denotes Hermitian conjugate. Here repeated indices i, j are summed and the antisymmetric matrix ϵ is given by

$$\epsilon = \begin{pmatrix} 0 & 1\\ -1 & 0\end{pmatrix}. \tag{1.26}$$

Color indices and spinor indices are suppressed in Eq. (1.25). Since H has a vacuum expectation value, the Yukawa couplings in Eq. (1.25) give rise to the 3×3 quark and lepton mass matrices

$$\mathcal{M}_u = vg_u/\sqrt{2},\quad \mathcal{M}_d = vg_d/\sqrt{2},\quad \text{and } \mathcal{M}_e = vg_e/\sqrt{2}. \tag{1.27}$$

Neutrinos do not get mass from the Yukawa interactions in Eq. (1.25), since there is no right-handed neutrino field.

Any matrix M can be brought into diagonal form by separate unitary transformations on the left and right, $M\to LDR^\dagger$, where L and R are unitary, and D is real, diagonal and nonnegative. One can make separate unitary transformations on the left- and right-handed quark and lepton fields, while leaving the kinetic energy terms for the quarks, $\bar Q_L^i i\partial\!\!\!/\, Q_L^i$, $\bar u_R^i i\partial\!\!\!/\, u_R^i$, and $\bar d_R^i i\partial\!\!\!/\, d_R^i$, and also those for the leptons, invariant. The unitary transformations are

$$\begin{aligned} u_L &= \mathcal{U}(u,L)\,u'_L, & u_R &= \mathcal{U}(u,R)\,u'_R, \\ d_L &= \mathcal{U}(d,L)\,d'_L, & d_R &= \mathcal{U}(d,R)\,d'_R, \\ e_L &= \mathcal{U}(e,L)\,e'_L, & e_R &= \mathcal{U}(e,R)\,e'_R. \end{aligned} \tag{1.28}$$

Here u, d, and e are three-component column vectors (in flavor space) for the quarks and leptons, and the primed fields represent the corresponding mass eigenstates. The transformation matrices $\mathcal{U}$ are 3×3 unitary matrices, which are chosen to diagonalize the mass matrices

$$\mathcal{U}(u,R)^\dagger \mathcal{M}_u \mathcal{U}(u,L) = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \tag{1.29}$$

$$\mathcal{U}(d,R)^\dagger \mathcal{M}_d \mathcal{U}(d,L) = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \tag{1.30}$$

and

$$\mathcal{U}(e,R)^\dagger \mathcal{M}_e \mathcal{U}(e,L) = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}. \tag{1.31}$$

Diagonalizing the quark mass matrices in Eqs. (1.29) and (1.30) requires different transformations of the u_L and d_L fields, which are part of the same $SU(2)$ doublet Q_L. The original quark doublet can be rewritten as

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} = \begin{pmatrix} \mathcal{U}(u,L)\,u'_L \\ \mathcal{U}(d,L)\,d'_L \end{pmatrix} = \mathcal{U}(u,L) \begin{pmatrix} u'_L \\ V d'_L \end{pmatrix}, \tag{1.32}$$

where the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix V is defined by

$$V = \mathcal{U}(u,L)^\dagger\, \mathcal{U}(d,L). \tag{1.33}$$

It is convenient to reexpress the standard model Lagrangian in terms of the primed mass-eigenstate fields. The unitary matrices in Eq. (1.32) leave the quark kinetic terms unchanged. The Z and $\mathcal{A}$ couplings are also unaffected, so there are no flavor-changing neutral currents in the Lagrangian at tree level. The W couplings are left unchanged by $\mathcal{U}(u, L)$, but not by V, so that

$$\frac{g_2}{\sqrt{2}}\, W^+ \bar{u}_L \gamma^\mu d_L = \frac{g_2}{\sqrt{2}}\, W^+ \bar{u}'_L \gamma^\mu V d'_L. \tag{1.34}$$

As a result there are flavor-changing charged currents at tree level.

The CKM matrix V is a 3×3 unitary matrix, and so is completely specified by nine real parameters. Some of these can be eliminated by making phase redefinitions of the quark fields. The u and d quark mass matrices are unchanged if one makes independent phase rotations on the six quarks, provided the same phase is used for the left- and right-handed quarks of a given flavor. An overall equal phase rotation on all the quarks leaves the CKM matrix unchanged, but the remaining five rotations can be used to eliminate five parameters, so that V is written in terms of four parameters. The original Kobayashi-Maskawa parameterization of V is

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}, \tag{1.35}$$

where $c_i \equiv \cos\theta_i$, and $s_i \equiv \sin\theta_i$ for $i = 1, 2, 3$. The angles θ_1, θ_2, and θ_3 can be chosen to lie in the first quadrant, where their sines and cosines are positive. Experimentally it is known that these angles are quite small. The CKM matrix is real if $\delta = 0$, so that $\delta \neq 0$ is a signal of CP violation in the weak interactions. It describes the unitary transformation between the mass-eigenstate basis $d^{i\prime}$, and the weak interaction eigenstate basis d^i. The standard notation for the mass-eigenstate fields is $u'^1 = u$, $u'^2 = c$, $u'^3 = t$, $d'^1 = d$, $d'^2 = s$, $d'^3 = b$.

So far we have only considered the left-handed quark couplings to the gauge bosons. For the right-handed quarks there are no W-boson interactions in the standard model, and in the primed mass-eigenstate basis the couplings of the Z, photon, and color gauge bosons are flavor diagonal. The analysis for leptons is similar to that for quarks, with one notable difference – because the neutrinos are massless, one can choose to make the same unitary transformation on the left-handed charged leptons and neutrinos. The analog of the CKM matrix in the lepton sector can be chosen to be the unit matrix, and the leptons can be chosen to be simultaneously mass and weak eigenstates. We adopt the notation $\nu'^1 = \nu_e$, $\nu'^2 = \nu_\mu$, $\nu'^3 = \nu_\tau$, $e'^1 = e$, $e'^2 = \mu$, $e'^3 = \tau$. From now on, we will use the mass-eigenstate basis for labeling the quark and lepton fields.

1.2 Loops

Loop diagrams in the standard model have divergences from the high-momentum (ultraviolet) region of the momentum integrals. These divergences are interpreted by a renormalization procedure; the theory is regulated in some way and terms that diverge as the regulator is removed are absorbed into the definitions of the couplings and masses. Theories in which all divergences in physical quantities (e.g., S-matrix elements) can be removed in this way using a finite number of counterterms are called renormalizable. In the unitary gauge, $\xi \to \infty$, the standard model is manifestly unitary (i.e., only physical degrees of freedom propagate because the "ghost" Higgs associated with $h^\pm$ and $\mathrm{Im}\, h^0$ have infinite mass). The vector-boson propagator

$$-i\,\frac{g_{\mu\nu} - k_\mu k_\nu/M_{W,Z}^2}{k^2 - M_{W,Z}^2} \tag{1.36}$$

is finite as $k \to \infty$, and naive power counting suggests that the standard model is not renormalizable. In the Feynman gauge, $\xi = 1$, the vector-boson propagator is

$$-i\,\frac{g_{\mu\nu}}{k^2 - M_{W,Z}^2}, \tag{1.37}$$

which falls off as $1/k^2$, and naive power counting shows that the standard model is renormalizable. The potentially disastrous divergences that occur in the unitary gauge must cancel. However, unitarity is not manifest in the Feynman gauge because the unphysical degrees of freedom associated with $h^\pm$ and $\mathrm{Im}\,h^0$ are included as intermediate states in Feynman diagrams. The standard model is manifestly unitary in one gauge and manifestly renormalizable in another. Gauge invariance assures us that the theory is both unitary and renormalizable.

In this book we will regularize Feynman diagrams by using dimensional regularization. Diagrams are calculated in $n = 4 - \epsilon$ dimensions, and the ultraviolet divergences that occur in four dimensions appear as factors of $1/\epsilon$, as $\epsilon \to 0$.

To review how dimensional regularization works, consider the quantum electrodynamics (QED) Lagrangian

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + i\bar{\psi}^{(0)}\gamma^\mu\left(\partial_\mu - ie^{(0)}\mathcal{A}_\mu^{(0)}\right)\psi^{(0)} - m_e^{(0)}\bar{\psi}^{(0)}\psi^{(0)}, \tag{1.38}$$

which is part of the standard model Lagrangian. The superscript (0) is used to denote a bare quantity. Here

$$F_{\mu\nu}^{(0)} = \partial_\mu \mathcal{A}_\nu^{(0)} - \partial_\nu \mathcal{A}_\mu^{(0)} \tag{1.39}$$

is the bare electromagnetic field strength tensor. In n dimensions, the action

$$S_{\text{QED}} = \int d^n x\; \mathcal{L}_{\text{QED}} \tag{1.40}$$

is dimensionless, since $e^{iS_{\text{QED}}}$ is the measure in the Feynman path integral (we use units where $\hbar = c = 1$). It follows that the dimensions of the fields, the coupling constant $e^{(0)}$, and the electron mass, $m_e^{(0)}$, are

$$\begin{aligned}
\left[\mathcal{A}^{(0)}\right] &= (n-2)/2 = 1 - \epsilon/2, \\
\left[\psi^{(0)}\right] &= (n-1)/2 = 3/2 - \epsilon/2, \\
\left[e^{(0)}\right] &= (4-n)/2 = \epsilon/2, \\
\left[m_e^{(0)}\right] &= 1.
\end{aligned} \tag{1.41}$$

The bare fields are related to the renormalized fields by

$$\begin{aligned}
\mathcal{A}_\mu &= \frac{1}{\sqrt{Z_A}}\mathcal{A}_\mu^{(0)},\\
\psi &= \frac{1}{\sqrt{Z_\psi}}\psi^{(0)},\\
e &= \frac{1}{Z_e}\mu^{-\epsilon/2}e^{(0)},\\
m_e &= \frac{1}{Z_m}m_e^{(0)}.
\end{aligned} \tag{1.42}$$

The factor of $\mu^{-\epsilon/2}$ is included in the relation between the bare and renormalized electric couplings so that the renormalized coupling is dimensionless. Here μ is a parameter with dimensions of mass and is called the subtraction point or renormalization scale of dimensional regularization. In terms of these renormalized quantities the Lagrange density is

$$\begin{aligned}
\mathcal{L}_{\text{QED}} &= -\frac{1}{4}Z_A F_{\mu\nu}F^{\mu\nu} + iZ_\psi\bar{\psi}\gamma^\mu\left(\partial_\mu - i\mu^{\epsilon/2}Z_e\sqrt{Z_A}e\mathcal{A}_\mu\right)\psi\\
&\quad -Z_m Z_\psi m_e\bar{\psi}\psi,\\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\left(\partial_\mu - i\mu^{\epsilon/2}e\mathcal{A}_\mu\right)\psi - m_e\bar{\psi}\psi + \text{ counterterms.}
\end{aligned} \tag{1.43}$$

It is straightforward to compute the renormalization constants $Z_{A,\psi,e,m}$ by using the formula for one-loop integrals in dimensional regularization,

$$\begin{aligned}
&\int \frac{d^n q}{(2\pi)^n}\frac{(q^2)^\alpha}{(q^2-M^2)^\beta}\\
&\quad = \frac{i}{2^n\pi^{n/2}}(-1)^{\alpha+\beta}(M^2)^{\alpha-\beta+n/2}\frac{\Gamma(\alpha+n/2)\Gamma(\beta-\alpha-n/2)}{\Gamma(n/2)\Gamma(\beta)},
\end{aligned} \tag{1.44}$$

and the Feynman trick for combining denominators,

$$\begin{aligned}
\frac{1}{a_1^{m_1}\cdots a_n^{m_n}} &= \frac{\Gamma(M)}{\Gamma(m_1)\cdots\Gamma(m_n)}\\
&\quad\times\int_0^1 dx_1 x_1^{m_1-1}\cdots\int_0^1 dx_n x_n^{m_n-1}\frac{\delta\left(1-\sum_{i=1}^n x_i\right)}{(x_1 a_1+\cdots+x_n a_n)^M},
\end{aligned} \tag{1.45}$$

where

$$M = \sum_{i=1}^n m_i.$$

The Z's are determined by the condition that time-ordered products of renormalized fields (i.e., Green's functions) be finite when expressed in terms of the