OPTIMAL CONTROL OF CREDIT RISK

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Chapter 1

INTRODUCTION

Guarantees, whether implicit or explicit, are a widespread component of financial contracts and provide the conceptual framework for credit risk analysis. Foremost among explicit private guarantees are guarantees of the debt obligations of subsidiaries from parent corporations; letter-of-credit guarantees provide by commercial banks; swap guarantees; mortgage guarantees; and insurance contracts of all sorts. Public guarantees are ubiquitous. Government guarantees of loans made to private corporations have made headlines on each side of the Atlantic. Indeed, government-issued small business guarantees as well as export-oriented and industry-targeted guarantees represent current government practices for financing economic activity. Even more important may be the role of guarantees of deposits through the Federal Deposit Insurance Corporation (FDIC), and less extensive, but similar, guarantees of pension benefits, student loans, residential mortgages, etc.

But implicit guarantees are also widely used. As a matter of fact. any risky loan can be considered a combination of a riskless contract and a guarantee, as shown by Merton and Bodie (1992).

The use of guarantees is likely to become even more widespread in the future. The enormous amount of recent financial investment products highlights the importance of contract fault or credit sensitivity, as it appears for example in swap trading.

In general, any form of default or signals the existence of an implicit guarantee. Starting with Merton (1974) up to a flow of recent papers with different approaches, the academic literature has considered credit risk mostly as a pricing issue. The challenge has been to find the right model to price credit risk and many have added to the theory from this side, either by following and extending Merton's approach (Shimko and alii (1993), Longstaff and Schwartz (1995), Anderson and Sundaresan (1996), Leland and Toft (1996), Mella-Barral and Perraudin (1997) amongst others) or by choosing an alternative path (Duffie and Singleton (1996), Duffie and Huang (1996), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997) amongst others). See Cossin and Pirotte (2000) for a survey of credit risk models.

Credit risk pricing, following Merton's terminology, corresponds to guarantee pricing as any long position in a credit risky contract can be considered identical to a portfolio of a long position in a riskless contract and a short position in a guarantee (Merton and Bodie, 1992). As such, the pricing research on credit risk is valid, with slight modification to fit the institutional context, for pricing the guarantees that pervade financial contracts, such as letter-of-credit guarantees, mortgage guarantees, swap guarantees, public guarantees (such as for deposit insurance, pension benefits, etc.), and all the implicit embedded guarantees that appear in risky contract (see Hirtle, 1987, and the Appendix of this monograph for a survey of guarantees). Many applications using traditional financial engineering on the Merton approach to pricing credit risk and guarantees have been developed for each of this situations (e.g., Jones and Mason, 1980, on classical debt, Cooper and Mello on swaps, 1991, Hsieh and alii, 1994, on PBGC insurance premiums, Marcus and Shaked, 1984, and Crouhy and Galai, 1991, on deposit insurance) .

Banks and other financial intermediaries, on another hand, have for a long time mitigated their credit risk exposures (or other short positions in guarantees, to use the Merton terminology) by using collaterals rather than pricing them. This is particularly evident in the OTC markets, e.g. the swap market, where counterparties of different quality main not obtain different rates but will often be asked for a collateral. It is well known that collaterization substitutes (at least in part, see Cossin and Pirotte, (1997) on swap pricing and Cossin and Hricko (2000) for theoretical results in the field) for credit risk pricing. In our knowledge, very little academic research bears on what an optimal policy of collaterization should be (some research analyzes the role of collaterals in game-theoretical models of debt renegotiation -see Bester, 1994).

Given that credit risk and guarantees are pervasive financial instruments, their optimal control can be an important competitive advantage for companies, while in the case of public finance, it may result in significant savings for taxpayers. In many cases, the option-based valuation approach to guarantees could not be directly implemented or sufficient to manage the risks involved (as in deposit insurance). Therefore, the guarantor has somehow to devise policies to keep under control the value of the assets underlying the guarantee. Knowing that the actions allowed on the part of the guarantor are essentially of two types, namely a) to monitor these assets, possibly with cost, and b) to control them with cost (for example, seize them under some circumstances or restrict their use, or just ask for additional collateral), the programs implementing these policies should provide the guarantor with the optimal timing and strength of its controls. Current general practice relies on standard rules of thumb to monitor the assets, tracking them through annual or quarterly audits(or even daily,as for example in margin accounts). Alternatively, seizure or control policies of assets securing guarantees have been based on static capital ratios, with some discretionary power often used by the guarantor to skirt these rules (see for example the 1978 guidelines set by the Controller of the Currency, the Federal Reserve and the FDIC, so-called CAMEL ratings). But none of these procedures are enough to ensure profit maximization. The same can be said about premiums based on the historic value of the assets (even those calculated using refined theories such as contingent claim analysis). Indeed, fair premiums are valid in so far as neither the guaranteed party nor the guarantor can influence the dynamic path of the assets or liabilities backing the guarantee without the immediate knowledge of the counterparty. In a realistic agency setting, however, a moral hazard problem arises since the guaranteed party has strong incentives to alter the stochastic process of the asset's prices, either by hiding its true value or by increasing its variance so as to inflate the value of the corresponding put option. Thus unless the guarantor can monitor the assets continuously, risk-based premiums will be insufficient to ensure that the requested premium is in line with the guarantor's cost. Moreover, for regulatory or marketing reasons, it is not that easy to implement risk-based premiums. There are many cases in which only a global approximation of risk, or a discrete approximation such as the risk categories in insurance, is used in calculating the premium. But in addition, the available data history could be affected by different sources of errors or inaccuracies.

It follows from the previous considerations that the efficiency of offering guarantees depends on optimal monitoring and optimal control. Here, we propose a set of optimization programs that solve the control problem by first characterizing the minimum cost of guarantees and then the optimal behavior leading to that cost. Even though they represent a first attempt at modeling what should impact the decision to ask for more or less collateral during the life time of a contract or guarantee, our programs will remain valid for the control of any financial contract at risk of default (swaps, CMOs, etc.), since as we said before, any risky loan can be decomposed in a safe contract and a guarantee. We treat our optimal control problem following the methodology of *impulse control* proposed by Bensoussan and Lions (1973). Impulse control techniques are not frequently analyzed in the available financial literature and should prove useful to banks and financial intermediaries that want to optimize their collaterization policy (notably when it bears on large amounts, for example for AAA set-ups). In this respect, our work proposes a new framework that should be thought of as a basis to be refined for practical use. The formulation of the original singular stochastic control problem lead us to a set of quasi-variational inequalities (as developed in Bensoussan and Lions, 1982; Glowinski, Lions and Trémolières, 1981; and Hlavácek and Alii, 1988) which we try to solve numerically in a simple set-up. We finally discuss the type of results provided by our analysis.

The monograph is organized as follows. In Chapter 1, we start by reviewing the main avenues of literature related to our problem. Chapter 2 provides a brief overview of the main optimal control principles which we use later, while Chapter 3 presents the models and their setting. Here we define the relevant variables, the model parameters, and the solution approaches to be considered. In the remaining chapters, we propose two sets of programs. One set of programs will apply in cases where the information on the assets' value is readily available (*full observation* case), while the other applies when costly audits are needed in order to assess this value (*partial observation* case). Chapter 4 deals with the full observation case, first as an impulse control problem and then as an optimal stopping problem. Chapter 5 proceeds similarly for the partial observation case. In either case, the modeling stage lead us to a set of *quasi-variational inequalities* which we attempt to solve numerically in the simpler case of full observations. This is done in Chapter 6. Finally, a simulation analysis is carried out in Chapter 7, in which we study the influence on the control process of changes in the different model parameters. This precedes a discussion on possible extensions in Chapter 8 and some concluding remarks in Section 9. An Appendix provides for a survey of different situations involving financial guarantees and the need for optimal control.

Chapter 2

LITERATURE REVIEW

Various strands of financial and mathematical literature relate to the research on the control of guarantees. On guarantees, the main corpus of academic research focuses on valuation, with special attention given to deposit insurance valuation . Research on the control of guarantees is more limited, often focused on optimal seizure rules for regulators, with the exception of a paper studying general management of guarantees (Merton and Bodie, 1992) and of another one studying the impact of collateral on the pricing of guarantees (Cossin and Hricko, 2000).

The study here does not rely exclusively on the guarantee literature; because of the technology involved, it also uses the literature of portfolio selection with transactions costs as well as the literature of machine quality control. The following section describes these different strands of literature as they relate to this study.

1. GUARANTEE VALUATION

The financial, political, and institutional stakes of accurately valuing guarantees have long been recognized. Indeed, an exact theory of guarantee valuation is now well developed. Merton (1977) initiated the use of contingent claim analysis to value guarantees, with an application to deposit insurance. He noted that guarantees are isomorphic to put options, with particular maturities and exercise prices, as well as underlying processes. For example, Merton presented a model in which the price of deposit insurance (as a specific example of guarantee) is equal to the value of a put option written on a bank's assets with a strike price equal to the amount of insured deposits. He assumes in this model that the guarantor examines a bank at specified time intervals, hence obtaining from these time intervals the maturity of the claim. In another study, Merton (1978) derives what would be a fair one-time payment by banks (i.e., a chartering fee) for deposit insurance, given that there is a cost in auditing banks at random time intervals. Merton's two original papers (1977 and 1978) influenced many subsequent studies, of which a few are surveyed here (see in particular Jones and Mason (1980); Sosin (1980); Langetieg et alii (1982); Bulow (1981 and 1983); Harrison and Sharpe (1982); Baldwin, Lessard, and Mason (1983); Bulow and Scholes (1983); Marcus and Shaked (1984); Flannery and James (1984); Ronn and Verma (1986); Pennacchi (1987a and b); Marcus (1987); Thomson (1987); Selby, Franks, and Karki (1988); Cummins (1988); Bodie (1991); Crouhy and Galai (1991). Among early uses of option pricing in insurance valuation, note also Sharpe (1976); Mayers and Smith (1977); Treynor (1977).).

Specific analyses to a wide range of guarantees followed. Among these, Jones and Mason (1980) use numerical solutions (obtained by the method of Markov chains) to value a range of guarantees: full guarantee of non-callable coupon debt; partially guaranteed issue of non-callable coupon debt; junior and senior non-callable debt with guarantees; callable coupon debt. Sosin (1980) extended this approach by recognizing potential wealth transfers to the original stockholders from the guarantee. Regarding as irrational the tendency for equity holders to redistribute wealth away from themselves, he assumes a no-loss, no-gain condition for existing bondholders (for his analysis, Sosin considers a politically driven, non-profitable project, that decreases the value of the firm (p.1212). He then assumes that protective covenants of senior debt holders prevent redistributions away from them in order to allocate the value of the claims on the firm value between new equity holders, old equity holders, subordinated debt holders and senior debt holders (p.1213). Because "it is irrational for equity holders to knowingly redistribute wealth away from themselves" (p.1213), the firm recapitalizes at the time of investment.)

Baldwin, Lessard and Mason (1983) warn against the "budgetary time bombs" constituting uncontrolled guarantees offered by the government (in that case, the Canadian Government) and urge governments to account for the fair value of guarantees as measured through contingent claim analysis. Such accounting is now mandated by law (1990) in the United States through the Office of Management and Budget and in Canada, but not, in our knowledge, in Europe.

Selby, Franks and Karki (1988) value loan guarantees and the wealth transfers that may arise as a result of the changes in capital structure introduced by the new guaranteed loan. They assume a no-loss, no-gain condition for shareholders more fitting to the United Kingdom context than Sosin's no-loss, no-gain condition for bondholders. This condition arises from different bankruptcy rules. In the United Kingdom, when a firm is in financial distress, a trustee (known as the "receiver") can be appointed by the lenders to manage the firm. As a result, wealth transfers against shareholders may be difficult to reverse. In the United States, however, such a firm can seek protection from the courts under Chapter 11 of the 1978 Bankruptcy Act; only rarely is existing management removed. In the specific case of ICL, the UK government did not require renegotiation of any of the outstanding debt (while lenders to Chrysler redeemed a third of their debt at a price substantially below par). The authors also use compound options to deal with an outstanding hierarchy of loans with more than one maturity. They then apply their valuation theories to the valuation of the loan guarantee provided in 1981 by the United Kingdom government to International Computers Limited (ICL). Cummins (1988) develops risk-based premium formulas for insurance guarantee funds, both for ongoing insurance with or without jumps in assets behavior and in policy cohorts (where liabilities eventually run off to zero as claims are paid). He shows how the use of risk-based premiums can relieve the adverse incentives of firms arising from the use of non-risk-based or "flat" premiums.

Cooper and Mello (1991) develop a partial equilibrium model for swap default that can provide banks with a measure of their swap transactions net of default risk, and regulators with a consistent way of measuring the potential default risk to effectively control banks. They characterize the transfers arising among shareholders, debtholders, and swap counterparties and obtain closed-form solutions for the value of the default risk in the swap. They do not, however, consider collaterization of swaps. Bodie (1991) examines the guarantee provided by the sponsor of a defined benefit pension plan by referring to its equivalent put option and discusses the optimal hedging strategies that the plan sponsor should follow.

Cossin and Hricko (2000) address the issue of pricing credit risk with the specific guarantee of a collateral in a structural form framework. Notice that the issue of pricing an instrument that is collateralized with another risky instrument is not trivial and becomes complex when marking-to-market or margin calls are considered. Margrabe(1978) has mentioned the analogy between an exchange option and a margin account and provides the pricing for a very simple framework with no marking-to-market. Stulz and Johnson (1985) have priced secured debt using contingent claim analysis and study the use of collaterization in a corporate finance framework, analyzing the impact of collaterization on the value of the firm. The rest of the economic literature has addressed the rationale behind the use of collateral in debt contracts and is an extension of the questions arising in the theory of debt (see Benjamin, 1978; Plaut, 1985; Bester, 1994) but has not been concerned with pricing the credit risk with collateral or with evaluating the impact of haircut levels on the credit risk value.

2. DEPOSIT INSURANCE VALUATION

The issue of deposit insurance pricing dominates the literature on guarantee valuation. As already noted, the original Merton papers (1977 and 1978) focus on that particular application.

Marcus and Shaked (1984) use Merton's (1977) model to estimate from bank-stock market data the fair value of deposit insurance for a sample of large commercial banks. They find that the vast majority of larger banks (publicly traded as they use market data) are overcharged for deposit insurance. Marcus and Shaked assume, however, that regulators could control the banks (i.e., implement new capital requirements) at no cost as they valued the put option (the security equivalent of the deposit insurance guarantee) from one audit to another. In other words, they assume that just after the next audit, the value of the FDIC's liability is 0 because, at that time, regulators can implement full control.

Pennacchi (1987b) generalizes Merton's model and considers alternative policy assumptions concerning an insuring agency's pricing of insurance (i.e., fixed rates over different risk classes versus risk-sensitive rates) and methods for handling bank closings by the agency (direct payment to depositors versus arrangement of a merger). Pennacchi also describes incentives for risk-taking by the banks under the different policies.

Ronn and Verma (1986) present an empirical valuation methodology also based on Merton (1977), using only market data on equity to determine asset value and volatility (by inverting the valuation of equity as a call option on assets), rather than any data provided by bank management or by FDIC audits. Kane (1985, 1986) explores the consequences of valuing deposit insurance incorrectly and the policy implications of using contingent claim analysis. He also discusses the constraints faced by FDIC that may prevent it from closing insolvent banks, allowing "zombie banks" to persist.

Flannery and James (1984) use stock market data to obtain effective maturities for different bank liabilities such as demand deposits, regular savings accounts, small denomination time deposits. Use of their methodology may help determine values of puts on these liabilities. Thomson (1987) uses market information to value deposit guarantees. He decomposes the FDIC guarantee into the guarantee of insured deposits, a conditional guarantee of the bank's uninsured deposits, and a guarantee of stockholders' residual on the future earnings of the bank (since the FDIC may fail to close out the stockholders' position in an insolvent bank).

Crouhy and Galai (1991) use contingent claim analysis to develop a model of a financial intermediary and obtain the valuation of equity and demand deposits for the intermediary. They also obtain valuation for fair deposit insurance premium and further analyze the impact of different types of regulation for control of the assets (e.g. reserve requirements, capital requirements, interest rate ceilings, etc.) on the banking industry.

3. CONTROL OF GUARANTEES

The formal literature is more scarce on the control of ongoing guarantees. A special branch of the literature focuses on the optimal seizure of banks by regulators.

Campbell and Glenn (1984) focus on the determinants of an optimal price for deposit insurance and optimal policy for closing insured institutions, analyzing the practicality of a private deposit insurance system. In no way, however, do they provide the means of obtaining this optimal closure policy. Their study focuses on measurement problems and the impact of either early or late closure, and the control problems arising from the closure rule itself.

Pennacchi (1987a) was the first to formally analyze the link between pricing of deposit insurance and optimal control. He studies the impact of regulatory control in insuring deposits over the valuation of the insuring's agency liability. He examines two cases of full control (following any audit, regulators can compel a capital deficient bank to add more capital) and of no control (regulatorscan close a bank only when it has negative net worth and cannot influence its capital structure before that). The two cases can also be understood as being limited-term insurance for the first case and unlimited term for the second. Audits occur in his model at Poisson distributed random time intervals (as in Merton 1978) and in a non-endogenous manner. He shows that for a large sample of banks, the FDIC should be seen as overcharging the banks if actual control is full control and as undercharging them if actual control is closer to no control.

Acharya and Dreyfus (1989) derive optimal regulatory policies for closing or seizure of a bank by FDIC as functions of rate of flow of bank deposits, interest rate on deposits, the economy's risk-free interest rate, and the regulator's audit/administration costs. Baldwin (1991) analyzes the phenomenon of "asset stripping" and its impact on the cost of deposit insurance when there is information asymmetry between management and deposit insurance agencies. She discusses regulatory control of banks and the source of strength doctrine, all elements that would have to be considered in a direct application of our model to deposit insurance. Additionally, she shows how new equity infusions or secured interests in specific assets (collaterals) ought to be the modern regulatory response to bank diversification and the rise of secondary markets. Both solutions can be modeled following our program, resulting in optimal control.

Gennotte and Pyle (1991) provide an analysis of effects of deposit guarantees on banks' loan portfolios when there is imperfect regulatory control of the banks' assets from the guarantor. They show how deposit guarantees lead banks to engage in inefficient investment and how tightening control can actually lead to an increase in per-unit asset risk (and thus possibly to an increase in the probability of default).

Merton and Bodie (1992) treat the management of guarantees as a general issue, clarifying in the process the context in which the guarantee control problem arises. Further, their discussion specifies when and how the optimal behavior derived in our paper can be implemented. They show why guarantees, whether they are explicit or not, need to be managed. The institutional structure in which guarantees originate (government versus bank, for example) certainly affects the management issue but does not preclude the issue from arising in any way. Stressing the different dimensions of guarantee management, Merton and Bodie show that profitability of guarantee management arises by combining the right mix of adequate premiums with control of operating costs as well as the frequency and severity of shortfall losses. They categorize the three tools of guarantee management as (1) restriction of types of assets, (2) the monitoring and seizure of assets, and (3) risk-based premiums. Any form of guarantee management is a mix of these three tools. Depending on regulation, taxes, the institutional structure, or the structure of information, some of these tools will be preferred. In some cases, information is so cheap to obtain, for example, that monitoring almost exclusively is an excellent solution. (See the margin account example.) In others, risk-based premiums will not be allowed because of regulation, so the other two forms of management must dominate. (See the deposit insurance example.) Valuation theory addresses only the premium calculation part. Still needed is a formal theory of the monitoring and control timing. When there are costs involved in monitoring the guaranteed assets, continuous monitoring is not feasible at finite cost. A trade-off occurs between the cost of frequent monitoring and the possibility of the collateral assets falling substantially between audits, thus jeopardizing the guarantor's position.

Our perspective here is to build programs determining (1) when a guarantor should audit a company for which it provides guarantees, and

(2) when it should control the assets' level to minimize its costs over time. This approach differs from previous studies (such as Pennacchi,1987a; Acharya and Dreyfus,1989; Gennotte and Pyle, 1991) because it presents a general rather than specific framework and because it proposes a mathematical technology still rarely used in finance (and never used for that particular problem).

Note that unexpected changes in assets level or variance can be linked to problems arising from asymmetric information between the guarantor and the guaranteed party. Moral hazard, adverse selection, or agency problems can all arise in different situations (e.g., moral hazard is a problem is the company buys a put from the guarantor and then can increase the value of the put by increasing the variance of the assets). Our programs partly help to overcome these problems. This monograph allows both for situations where there is full information (no cost to audit) and partial observation(costly audits). It allows for multiple controls applied at various times during the guarantee's lifetime (impulse control) as well as for a onetime application of control, such as seizure of a bank by a regulator (stopping time). The programs presented here can be implemented directly, although applications to real-world situations will increase both conceptual and mathematical complexities already met here. No unusual data are necessary. Modern computers are required to handle the program through numerical analysis. Guarantors who today apply rules of thumb for timing their (often very costly) audits may gain by similar programs to optimize their behavior.

4. OTHER APPLICATIONS

The problem of guarantee control as analyzed here is similar to both the problem of portfolio selection and the problem of machine-quality control, as explained in the following sections. When there are fixed costs to transacting or to controlling (depending on the case), the three problems are known in mathematics as "impulse control" problems. We show the link between the different approaches to "impulse control" and the origin of the "quasi-variational inequalities" technology, as developed in Bensoussan and Lions (1982) and used in the analysis to follow.

The problem of optimal guarantee control is homologous in many ways to the problem of portfolio selection when there are fixed transactions costs. The portfolio selection program of an investor can be written as

$MaxE\left\{\int e^{-\delta t}.u(c(t)).dt\right\}$ with t the time between 0 and maturity T, $u(.)$ the utility function of consumption $c(t)$ and δ the discount rate.

This program can be easily compared to the objective function of the complete observation program developed later. When considering only one risky asset with Brownian motion dynamics and one riskless asset,