DEREK I. BLOOMFIELD

# INTERMEDIATE ALGEBRA

# Intermediate Algebra

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# Preface

## Intended Level

This book is the third in a series of basic mathematics texts which include:

**From Arithmetic to Algebra**

**Introductory Algebra**

**Intermediate Algebra**

It is intended for students who have either completed a first course in algebra in high school or the equivalent college introductory algebra course.

This book is complete insofar as it starts at the beginning of algebra and then proceeds through the usual intermediate algebra topics. However, the topics normally considered in an introductory algebra course are presented here at a faster pace and with more of the unusual cases pointed out.

It is intended to prepare a student who plans to take as the next course finite mathematics, brief calculus, business mathematics, or college algebra.

The book can be used in a variety of instructional modes:

1. As a conventional lecture-type class.
2. As a self-study program in which the student works at his or her own pace.
3. As a mathematics laboratory in which video tapes or computer-based drill-type materials are used.

The book consists of ten chapters, with each chapter presented in the following manner: An explanation of the concepts for a particular topic is given; step-by-step examples illustrating the fundamental concepts are given; and more than a sufficient number of similar problems are given in the exercise sets. The exercises are designed to progress from simple to more difficult in order to help the student work his or her way up to the more difficult problems. Answers to odd-numbered exercises are given at the end of the text. The goal of this method of presentation is to gain a correct understanding of the topics and the skills to solve many different types of problems.

## Outstanding Features

- Rules highlighted in boxes.
- Over 700 worked-out examples with step-by-step explanations are given.
- Over 2700 exercises have been carefully chosen to clarify explanations and provide drill.
- STOP warnings are given to students about common errors.
- Word problems covering a wide variety of applications are found throughout the text. These shown the power of algebra in real-world situations.
- Chapter summaries are provided with definitions and rules restated, along with an example of each concept for quick review.
- Achievement tests at the end of each chapter examine the student's mastery of the materials in that chapter.
- Answers to odd-numbered exercises and all achievement test questions are given in the answer section. Page numbers of the answers are given after each set of exercises for easy reference.

## An Instructor's Manual is Available Containing:

- Alternate forms of achievement tests for each chapter.
- A comprehensive final exam that may be tailored to individual instructor's needs.
- Answers to even-numbered exercises.

## ACKNOWLEDGMENTS

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My whole family gave up many hours of their precious time. I thank them for their inspiration, their understanding, and their love.

## A Note from the Author to the Student

I have written this book with the belief that every student can learn algebra if he or she wants to. If you have decided that you would like to learn algebra, here are some suggestions to help you attain success.

1. Attend all of your classes. You can't hope to learn what goes on in class unless you are there. Even if you don't understand everything that's being taught, you'll pick up a lot.
2. Work lots of problems. Even if you don't always have a thorough understanding of what you're doing, if you do enough problems, the concepts eventually filter through.
3. Use your book. The worked-out examples are there to help you do the exercises. Go over the examples several times until you can repeat the procedures. Understanding will eventually follow.
4. Work on a regular basis. Try not to get behind. Solving today's problems usually depends on yesterday's results, so it's very hard to catch up once you're behind.
5. Try to remain confident. Even if you've never had any real success at algebra before, I'm convinced that you can succeed if you keep trying.

This book is written for you, the student, so please write to me if you have any suggestions for improving it.

*Derek I. Bloomfield*

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# 1 Fundamental Definitions and Concepts

## 1.1 SETS

The concept of sets is basic to all areas of mathematics and serves to unify the various branches. A knowledge of the following definitions and operations will prove useful in any further study that you undertake in mathematics.

We use the word **set** to mean any group or collection of things. Sets are usually named by using a capital letter, such as $A$, $B$, or $C$ and the individual items in the set are referred to as **elements** or **members** of the set. The elements of a set are enclosed in braces and the symbol $\in$ is used to indicate that an element belongs to the set.

***Example 1:***

(a) The set $B$ containing the first five counting numbers is written as $B = \{1, 2, 3, 4, 5\}$.

(b) Since 4 is an element of set $B$, we can write $4 \in B$.

(c) We indicate that 6 is not an element of set $B$ by writing $6 \notin B$.

(d) The set of letters in the name of the capital of Nebraska is written $C = \{L, I, N, C, O\}$. The last two letters $L$ and $N$ in Lincoln are not written since elements in sets are not repeated.

(e) $H = \{8, 9, 10, 11, \ldots\}$ is the set of counting numbers greater than 7. The three dots after the 11 indicate that the numbers keep going in the same pattern. This notation is used when it is not practical (or not possible) to list all the elements of the set.

## Cardinal Number of a Set

The cardinal number of a set is the number of elements in the set. The cardinal number of the set $A = \{a, b, c, d\}$ is 4 and we write $n(A) = 4$.

***Example 2:***

(a) If $B = \{2, 4, 6\}$, then $n(B) = 3$.
(b) If $T = \{4\}$, then $n(T) = 1$.
(c) If $Q = \{0\}$, then $n(Q) = 1$.

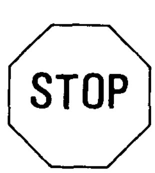
**Finite set.** Any set that has a cardinal number equal to a whole number (zero or a counting number) is called a **finite set.** Keep in mind that finite sets can be very large. For example, the set $T = \{1, 2, 3, 4, \ldots 1{,}000{,}000{,}000\}$ has 1 billion elements in it, but it has a cardinal number equal to 1,000,000,000 which is a whole number. Therefore, $T$ is a finite set.

**Infinite set.** Any set that is not a finite set is an infinite set. An example of an infinite set is $R = \{2, 4, 6, 8, \ldots\}$. The cardinal number of an infinite set is not a whole number. (Why not?)

**Empty set or null set.** The empty set is a set that contains no elements and is denoted by the symbol $\varnothing$. It has a cardinal number equal to 0: $n(\varnothing) = 0$.

***Example 3:*** Label the following as finite or infinite.

| | |
|---|---|
| (a) $R = \{6\}$ | Finite |
| (b) Set of odd counting numbers | Infinite |
| (c) $M = \{a, e, i, o, u\}$ | Finite |
| (d) $\varnothing$ | Finite |




A common error is made by trying to write the empty set as $\{\varnothing\}$. The set $\{\varnothing\}$ cannot be the empty set since it contains *one* element, $\varnothing$. It has a cardinal number equal to 1. The empty set has *no* elements. It has a cardinal number equal to 0.

## Subsets

Set $B$ is a subset of set $A$ if every element of set $B$ is also an element of set $A$. This is written $B \subseteq A$.

***Example 4:***

(a) If $A = \{1, 2, 3, 4\}$ and $B = \{1, 3\}$, then $B \subseteq A$.
(b) If $P = \{1, 2, 3, 4\}$ and $Q = \{4, 5\}$, then $Q$ is not a subset of $P$ and we write $Q \not\subseteq P$.
(c) Given $W = \{0, 1, 2, 3, 4, \ldots\}$ and $E = \{2, 4, 6, 8, \ldots\}$, $E \subseteq W$ since every even whole number (set $E$) is a whole number (set $W$). As you can see, subsets are not restricted to finite sets.
(d) If $A = \{3, 4, 5\}$, then $A \subseteq A$ because every element of $A$ is an element of $A$. Every set is a subset of itself.
(e) For consistency, the empty set is thought of as being a subset of every set. For any set $A$, $\varnothing \subseteq A$.

## Operations on Sets

There are two basic set operations, union and intersection.

The **union** of two sets $A$ and $B$, denoted $A \cup B$, is the set of all elements in $A$ or in $B$ or in both.

The **intersection** of two sets $A$ and $B$, denoted $A \cap B$, is the set of all elements common to both $A$ and $B$.

***Example 5:*** Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$, and $C = \{1, 3, 5\}$.

(a) $A \cup B = \{1, 2, 3, 4, 6\}$.
(b) $A \cap B = \{2, 4\}$.
(c) $A \cup \varnothing = \{1, 2, 3, 4\}$.
(d) $B \cup C = \{1, 2, 3, 4, 5, 6\}$.
(e) $B \cap C = \varnothing$. Sets $B$ and $C$ have no elements in common and are said to be **disjoint**.
(f) $B \cap \varnothing = \varnothing$.

## EXERCISE 1.1

Write exercises 1–10 using set notation.

1. The set of counting numbers between 5 and 10.
2. The set of letters in the word *genius*.
3. The set of states beginning with the word *New*.

4. The set of the names of the days of the week.

5. The set of even counting numbers larger than 5.

6. The set of odd counting numbers less than 10.

7. The set of months of the year beginning with the letter $H$.

8. The set of three-sided rectangles.

9. The set of letters in the name of the first president of the United States.

10. The set of counting numbers that are evenly divisible by 3.

For exercises 11–16, indicate which sets are finite and which are infinite and in the case of finite sets write their cardinal numbers.

11. $T = \{10, 20, 30, \ldots, 700\}$

12. The set of people currently living in Atlanta, Georgia, who are over 12 feet tall.

13. The set of even counting numbers between 9 and 17.

14. $Z = \{1000, 2000, 3000, \ldots\}$

15. The letters of the alphabet.

16. The set of counting numbers that can be evenly divided by 5.

For exercises 17–28, if set $A = \{2, 4, 6, \text{July}, *\}$, indicate whether the statements are true or false.

17. $4 \in A$

18. $\text{June} \in A$

19. $\{*\} \subseteq A$

20. $A \subseteq A$

21. $\varnothing \subseteq A$

22. $\{2, \text{July}\} \subseteq A$

23. $\varnothing \in A$

24. $\{2, 4, 6\} \subseteq A$

25. $\{4, 6\} \in A$

26. $A \in A$

27. $\{\varnothing\} \subseteq A$

28. $\{\varnothing\} \in A$

For exercises 29–37, let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7, 9\}$, and $D = \{6, 7\}$. Find the following:

29. $A \cup C$

30. $A \cap B$

31. $A \cup D$

32. $B \cap A$ 33. $A \cap D$ 34. $C \cap A$

35. $D \cap D$ 36. $D \cup D$ 37. $\varnothing \cap A$

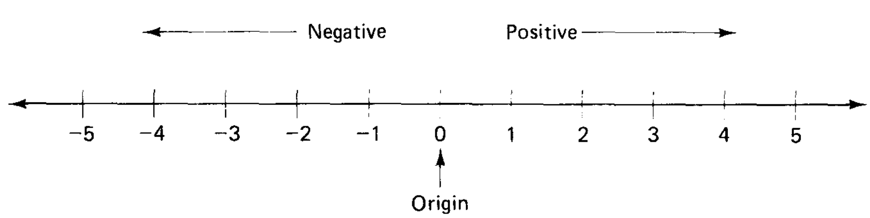
*Answers to odd-numbered exercises on page 465.*

## 1.2 THE REAL NUMBERS

The real numbers are the language of algebra, and in this section we will describe the set of real numbers in terms of its subsets and look at some of its properties.

### The Integers

The positive and negative counting numbers along with zero make up the signed numbers or integers. A convenient way of illustrating the integers is on a number line.




If we represent the set of integers by $I$, we have

$$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

### The Rational Numbers

A **rational number** is any number that can be written as the quotient $a/b$, where $a$ and $b$ are integers and $b$ does not equal 0.

Examples of rational numbers are

$$\frac{-3}{4} \quad \frac{1}{2} \quad \frac{8}{5} \quad \frac{7}{-4} \quad \frac{6}{1}$$

Rational numbers written in the form $\frac{a}{b}$ are called fractions, where $a$ is the numerator and $b$ is the denominator.

An expression like $\frac{7}{0}$ is not defined since 0 is not allowed as a denominator. A complete discussion involving division and 0 is given in the next section.

All integers are rational numbers since any integer can be written as a fraction with the given integer as the numerator and 1 as the denominator; for example, $-6 = \frac{-6}{1}$

## Terminating and Repeating Decimals

Since a fraction is a quotient, the numerator may be divided by the denominator. For example, $\frac{5}{8}$ can be written as the result obtained by dividing 5 by 8.

$$8\overline{)5.0^{2}0^{4}0}\quad = .6\,2\,5$$

The decimal equivalent for $\frac{5}{8}$, .625, is called a **terminating** decimal since the sequence of digits comes to an end.

Similarly, $\frac{4}{11}$ can be expressed as a decimal:

$$11\overline{)4.0000\ldots}\quad = .3636\ldots$$

If we continue the division, the 36 pattern continues to repeat. The decimal .363636... is called a **repeating** decimal. This is also written as $.\overline{36}$ where the bar is placed over the digits that repeat.

***Example 1:*** The following are rational numbers.

| *Terminating decimals* | *Repeating decimals* |
|---|---|
| (a) $\frac{1}{4} = .25$ | (a) $\frac{1}{3} = .333\ldots$ or $.\overline{3}$ |
| (b) $\frac{3}{8} = .375$ | (b) $\frac{3}{11} = .2727\ldots$ or $.\overline{27}$ |
| (c) $\frac{2}{5} = .4$ | (c) $\frac{4}{7} = .571428571428\ldots$ or $.\overline{571428}$ |

To summarize:

> Every rational number may be written as either a terminating or repeating decimal.

## Irrational Numbers

Consider the decimal

$$.010010001000010000001\ldots$$

The three dots indicate that the sequence of digits does not end, so it is **nonterminating**. If you examine it carefully, you will see that the pattern does not repeat so it is also a **nonrepeating** decimal. Such numbers are called irrational numbers. It can be shown that irrational numbers *cannot* be written as the quotient of two integers.

> An **irrational number** is a nonterminating, nonrepeating decimal. An irrational number cannot be written as the quotient of two integers.

***Example 2:*** The following are irrational numbers.

(a) 5.121121112...
(b) $\pi = 3.14159\ldots$
(c) $\sqrt{2} = 1.4142\ldots$
(d) Square roots of nonperfect squares like

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \text{ and } \sqrt{7}$$

are all irrational numbers.

STOP

A common misconception is that $\pi = \frac{22}{7}$. Since it is the quotient of two integers, $\frac{22}{7}$ is a rational number. Dividing 22 by 7 gives $3.\overline{142857}$, a repeating decimal; so $\frac{22}{7}$ is a **rational** approximation to $\pi$, and $\pi$ is an **irrational** number.

## The Real Numbers

> The **real numbers** consist of all the rational numbers plus all the irrational numbers. Each real number is represented by a point on the *real number line.*

***Example 3:*** The following are real numbers.

| | |
|---|---|
| (a) $\frac{9}{16}$ | fraction (rational) |
| (b) $\sqrt{3}$ | irrational |
| (c) $-\sqrt{5}$ | irrational |
| (d) $.\overline{231}$ | repeating decimal (rational) |
| (e) $-.343343334\ldots$ | nonrepeating decimal (irrational) |
| (f) 1.341 | terminating decimal (rational) |

The position of these numbers on the real number line is illustrated below.