MECHANICAL VIBRATION ANALYSIS

P SRINIVASAN

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P SRINIVASAN
Indian Institute of Science
Bangalore

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Sponsoring Editor : RAJIV BERI
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To My Wife
Srimathi

कर्मणो ह्यपि बोद्धव्यं बोद्धव्यं च विकर्मणः ।
अकर्मणश्च बोद्धव्यं गहना कर्मणो गतिः ॥१७॥

The truth about action must be known;
And the truth of prohibited action must also be known;
Even so, the truth about inaction must be known.
For mysterious are the ways of action.
GITA: Chapter 4, Verse 17

Preface

To consider things in themselves without losing track of them in the course of reasoning

POINSOT
(Translated from French)

Vibrations are an uniquitous, universal and multifaceted phenomenon where physical principles, mathematical theorems and engineering practice intermingle. It is an interdisciplinary field where the physicist, mathematician and engineer interact in a closed loop. This fact is often forgotten and the subject of vibration is compartmentalised into mutually exclusive disciplines. While books on mathematics dealing with vibrations are concerned with the nature of solution, viz. the existence uniqueness and stability of the differential equations, books on classical physics which treat vibration as "small motion" are mostly concerned with the phenomenological aspects of vibration. Engineering books, on the other hand, are hardware and gadget-oriented.

The necessity to write yet another book on vibration has arisen out of the author's desire to weld these diverse aspects into one homogeneous, coherent and integrated text which will not appear amateurish to the physicist-mathematician or excessively academic to the engineer-technologist.

With this objective in view the physical principles are introduced and explained clearly. The mathematical models are set up, equations of motion derived, solutions obtained and their limitations stated precisely. A unique feature of the mathematical presentation is the way it is constructed piece by piece, in such a way as to emphasise its physical significance and conform to Poinsot's counsel of perfection cited above. The application of these principles to engineering problems is given concisely and solved examples are provided to improve comprehension.

The first five chapters of the book are devoted to the exposition of the single degree of freedom system. The emphasis on the vibrations of the single degree of freedom system not only brings out the basic nature of the book but also provides the foundation to the various applications of vibration theory, to vibration isolation, vibration instrumentation and vibration testing, which are essential elements in the effective control of vibration.

Chapter 6 is devoted to the study of coupled vibrations or the vibrations of multi-degree of freedom systems. The study is restricted to systems with two degrees of freedom in the first instance, because it is felt that the student can easily grasp the essential ideas of the more general theory of a multi-degree of freedom system without getting lost in complex algebra in the course of his studies. Also, the study of a two-degree-of-freedom system helps the student understand the control of vibration by the use of dynamic vibration absorbers which are designed on the basis of the theory of vibrations of two-degree-of-freedom systems.

Lagrange's equation, the most elegant equation in analytical dynamics forms the subject matter of Chapter 7. The basic structure of the Lagrangian which is pivotal in the formulation of the equations of motion is discussed in detail and its application to vibration problems is illustrated by a number of examples.

Chapter 8 is devoted to frequency analysis and computation and mode-shape calculation. Computing techniques using matrix interaction, transfer matrix, Myklestad-Prohl method, Rayleigh-Ritz and Stodola-Vianello approaches are discussed in detail, which will help the student to write programmes and solve problems with the help of modern high-speed digital and hybrid computers. Rayleigh's minimum principle and the convergence of Stodola's process are demonstrated to provide a sound theoretical foundation in the use of these widely-applied techniques of frequency analysis.

Chapter 9 deals with one-and two-dimensional wave equations which are exemplified by a vibrating string and membrane. The wave nature of the motion, transcendental character of the frequency equation, and significance of the state propagation velocity are discussed. Hamilton's principle of least action is stated and the method of setting up equations of motion using the technique of the calculus of variation demonstrated.

In Chapter 10 the vibration of important load-carrying elements, such as beams, plates, rings and shells is discussed. Equations of motion of these elements are solved by both exact as well as approximate procedures. The importance of these elements in engineering design and construction is highlighted.

From the above it is clear that this book fulfils not only the needs of mechanical engineering undergraduates, but also of first year post-graduate students. It also caters to the needs of aeronautical, civil, chemical, electrical and electronic engineers. To the practising engineer, who is not a specialist in vibration, it serves as a reference manual.

While the theoretically-oriented engineer-scientist will welcome the practical slant given to the book, the practically-oriented engineer-technologist will appreciate the theoretical bias. This book is thus a happy blend of theory and practice and opens the window to the panorama of vibration.

In dealing with a subject which is so vast, it is natural that some areas might not have been highlighted sufficiently. But whatever areas have been covered have been explored in depth and even the more advanced concepts, such as nonlinear resonance, parametric resonance, self-resonance, etc. have been defined and clarified using the simple theory that has been presented.

It is hoped that this book will serve as a bridge between the elementary texts on the one hand and advanced treatises on the other. It is a book at an intermediate level of difficulty, striking a balance between classical and modern areas and paving the way to a deeper understanding of the phenomena from more advanced sources.

During the course of writing this book, the author had the benefit of discussing with his students, research fellows and colleagues at the Indian Institute of Science. The task of preparing this manuscript was lightened by the efficient help rendered by M/s A. S. Rao, C. Shankaranarayana Rao and T. M. Mahadeviah. To all of them the author wishes to extend his sincere thanks. The author's thanks are also due to the Ministry of Education and Social Welfare for the financial assistance extended in the preparation of this manuscript.

If this book helps even in a small measure to improve the quality of machines built for the benefit of mankind, the author's aspirations would have been more than fulfilled because he is aware of the danger that in trying to please many he may please none.

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Notation

The notation adopted in this book is the internationally accepted one with the following simplifications.

The English letter p is used throughout the book to denote the natural frequency in preference to ω_n. The greek letter ω is used throughout to denote the forcing frequency. The nondimensional ratios are denoted by a bar, namely, $\bar{p}$ which is $\frac{\omega}{p}$.

Differentiation with respect to time is denoted by a dot and differentiation with respect to the argument by a prime. For continuous systems differentiation with respect to time is denoted by u_t which is $\frac{\delta u}{\delta t}$. Sometimes, the following notations are also adopted, viz.,

$$u_{t^2} = \frac{\delta^2 u}{\delta t^2} \; ; \; u_{x^2} = \frac{\delta^2 u}{\delta x^2} \; ; \; u_{tx} = \frac{\delta^2 u}{\delta x \delta t}$$

to save effort in writing.

For example, the Timoshenko beam equation is written as

$$y_{t^4} + a y_{t^2} + b y_{t^2x^2} + c y_{x^4} = 0$$

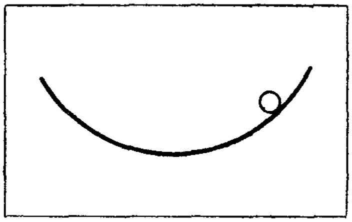
A unique teaching aid provided in the textbook is the portrayal of the vibratory responses on a triangle called the response triangle which not only facilitates remembering the lengthy formulae easily, but also serves as a ready reckoner.

The diagram provided at the beginning of each chapter not only adds zest to the topics discussed but also conveys succinctly the essential content of the chapter. A brief summary highlights the important concepts that are covered in the chapter.

Contents

1

Introduction



Vibration is ubiquitous

Vibration is a universal phenomenon. It manifests itself in many forms and permeates the entire universe. The universe would not be in its present form but for the motion of celestial bodies in their orbits and their perturbations. It is no wonder, therefore, that astronomers, such as Copernicus and Kepler, who were engaged in the study of planetary motion, were the pioneers in the field of vibration studies. As knowledge progressed, the astronomers were joined in this quest by a band of physicists—the contributions of Newton and Rayleigh in the field of vibration dynamics were enormous and remain unsurpassed even today. Simultaneously, mathematicians, such as Lagrange and Hamilton were busy laying the foundations of analytical dynamics to which branch of science the subject of "small motions" or vibration belongs. With the ushering in of the Industrial Revolution the engineer emerged on the scene with vibration problems which began to bedevil the design and construction of machines and structures. Vibration problems facing the industry were grappled by engineers, such as Timoshenko and Den Hartog, who drew copiously from the stockpile of knowledge built up by astronomers, physicists and mathematicians over the centuries.

As technology advanced, became diversified and sophisticated, the engineer was faced with problems for which either solutions did not exist or the existing solutions were inadequate. At this juncture the digital and hybrid computers came of age. With the developments in integrated circuits, electronic component and microprocessor industries, devices, such as spectral analysers, variable head shakers, controllable balancing machines, and piezoelectric accelerometers became available. Making use of the revolution in software and hardware technology, engineers began to tackle complicated vibration problems to keep the wheels of industry moving. The subject of vibration is, therefore, a highly interdisciplinary area where engineering practice, physical principles and mathematical theory are combined and the engineer, physicist and mathematician meet and interact in a closed loop. In the loop, we take the engineering view of vibration. But the scope and field of engineering vibrations is so vast, embracing as it does such diverse fields as entertainment electronics at one end to biomechanics at the other, that it is not possible to study this subject in one single volume. We, therefore, restrict our study to a small portion of the subject, viz., "mechanical vibrations".

"Mechanical vibrations", as the name implies, are vibrations executed by mechanical systems which are made up of bodies interconnected by elastic elements and constrained to move relative to one another in a predetermined manner from the first input stage to the last output stage. For purposes of analysis we treat a body as a finite collection of material particles. A particle in turn is idealised as a mass point whose dimensions are ignored in considering its motion. The motion of a lumped entity such as a particle in space is specified by its coordinates, velocities and accelerations. The number of independent coordinates required to completely specify the motion of a particle are called the "degrees of freedom" of the particle.*

Having thus defined a mechanical system and quantified it, we shall now proceed to define the term "vibration". Consider a simple system consisting of a cylinder P constrained to roll without slipping on the inside of a plate bent into a circular arc as shown in Fig. 1.1(a). We discretise or treat the cylinder as a mass point and denote its position at any instant of time by the angle ϕ it makes with the vertical. When the cylinder is at rest at the bottom of the plate, its angular displacement ϕ, angular velocity $\dot{\phi}$ and angular acceleration $\ddot{\phi}$ are zero. This implies that the net torque acting on the cylinder about O is zero. The bottommost position or the position of rest is, therefore, synonymous with the position of zero torque on the cylinder. We identify this position as the position of stable equilibrium of the system. From mechanics we know that the position of stable equilibrium is also the position of minimum potential energy.

When the cylinder is perturbed from its position of rest, it executes to and fro motion about the equilibrium position. This motion is possible because a restoring torque (due to gravity) tends to return the cylinder to its position of rest. If we ignore the friction between the cylinder and the plate, the energy possessed by the system in passing through the equilibrium position takes it to the other extreme end of the plate and the gravitational force once again tends to pull it to its original position and the motion persists. From this example it may be noted that a vibration is nothing but a "slightly perturbed**" motion of a body about the position of stable equilibrium. We shall see later on that it may also be a small perturbation about a position of steady rotation.

*A more precise definition of "degrees of freedom" is given in Sec. 7.1.

**The reason for restricting the motion to small perturbation is given in Sec. 2.5.

If the bent plate is flattened out and the cylinder P perturbed from its position of rest, as shown in Fig. 1.1 (b), it will never return to its original position as there is no torque tending to return it to its equilibrium position and the cylinder will simply roll, executing a rigid body motion.

If the plate of Fig 1.1(a) is inverted and the cylinder P placed at the top as shown in Fig. 1.1(c), it may be noted that upon perturbing the cylinder from its position of rest it will fly away and never return to its original position as there is no restoring torque tending to bring it back to its equilibrium position. Hence no to and fro motion about the position of rest is possible. This position of P is identified as the position of unstable equilibrium and the motion executed by the cylinder may be termed as a "flyaway" motion.

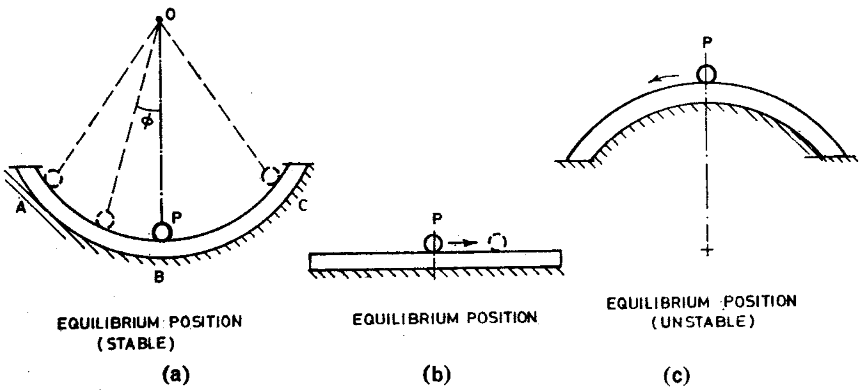


Fig. 1.1 (a) A cylinder rolling on a plate bent into the form of a circular arc
(b) A cylinder in rigid translation on a flat plate
(c) A cylinder at the top of the bent plate inverted

Upon smearing the inside of the plate (Fig 1.1(a)) with a highly viscous fluid and perturbing the cylinder P from its position of rest, it may be noted that the body will return to the equilibrium position after an infinitely long time without executing any to and fro motion on account of the frictional resistance, which ultimately reduces the restoring torque on the cylinder to zero. Such a creeping motion is called a "die away motion" and does not constitute a vibration.

From these examples it may be noted that the important elements that constitute or make up a vibratory system are: an inertia element in a position of stable equilibrium, a restoring element and the initial condition inputs of displacement and velocity. Once set in motion, in the absence of any friction, the oscillations persist for all time.

An important characteristic of such a motion is its repeatability or periodicity. A motion $\phi(t)$ is said to be periodic or reproducible if there is a nonzero number τ such that

$$\phi(t+\tau) = \phi(t) \tag{1.1}$$

for every value of the argument t. The lowest of such τ, called the period of motion, represents the time interval in which the motion repeats itself in all its aspects. For instance, the simple harmonic motion $y = a \sin (pt - \alpha)$ which reproduces itself in all its entirety after a time $\tau = \frac{2\pi}{p}$ is the simplest example of periodic motion. The total number of vibrations performed in a given time is called the frequency. The frequency and periodic time are related to one another by the equation

$$\tau = \frac{2\pi}{p} \tag{1.2}$$

where p represents the number of vibrations in 2π seconds. It is defined as the angular frequency or circular frequency and is expressed in radians per second. From the above definition it is clear that

$$p = 2\pi f \tag{1.3}$$

where f is the frequency in cycles per second, abbreviated as cps. This unit of frequency is sometimes called "hertz" after Hermann Hertz, who discovered radiowaves. One hertz represents a frequency of one cycle per second. From Eqs. (1.2) and (1.2) it is obvious that

$$\tau = \frac{1}{f} \tag{1.4}$$

The periodic time in seconds is the reciprocal of frequency in hertz.

The French mathematician Fourier, in a prize-winning paper presented to the French Academy of Science in Paris in 1811, has shown that any complex periodic motion $\phi(pt)$ can be represented by a linear combination of several simple harmonic motions, such as, $a_1 \sin (p_1 t - \alpha_1)$, ..., $a_n \sin (p_n t - \alpha_n)$, i.e.,

$$\phi(pt) = \tfrac{1}{2} a_0 + a_1 \sin (p_1 t - \alpha_1) + a_2 \sin (p_2 t - \alpha_2) + \dots + a_n \sin (p_n t - \alpha_n) \tag{1.5}$$

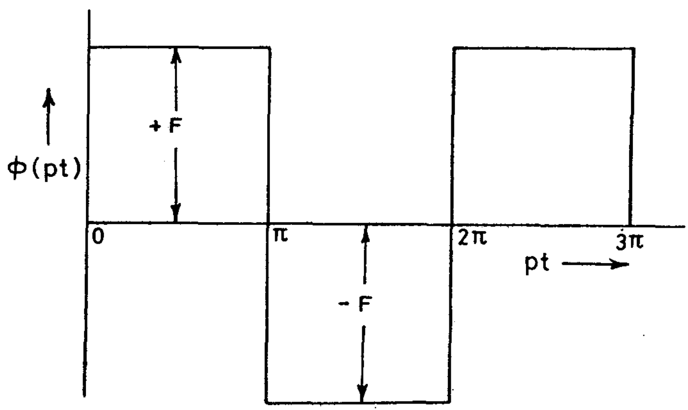


Fig. 1.2 A square wave or meander function

where a_0, a_1, ..., a_n and α_1, ..., α_n are constants to be evaluated. This theorem shows that a simple harmonic motion is the basic building block from which any general periodic motion may be built up by superposition.

Figure 1.2 shows a square wave or meander function. Upon Fourier analysing the wave it can be shown that it is a superposition of an infinite number of simple harmonic motions of amplitudes $\frac{4F}{\pi}$, $-\frac{4F}{3\pi}$, $+\frac{4F}{5\pi}$, etc. with frequencies, p, $3p$, $5p$ etc. respectively. Hence

$$\phi(pt) = \frac{4F}{\pi}\left\{\sin pt - \frac{1}{3}\sin 3pt + \frac{1}{5}\sin 5pt - \dots\right\} \tag{1.6}$$

A simple harmonic motion can be represented by the projection of a radius vector of magnitude a revolving with an angular velocity pt in the counterclockwise direction as shown in Fig. 1.3.

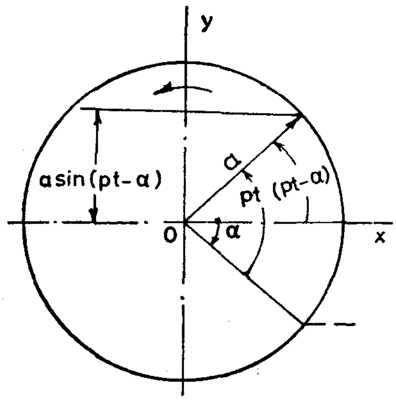


Fig. 1.3 Vector representation of simple harmonic motion

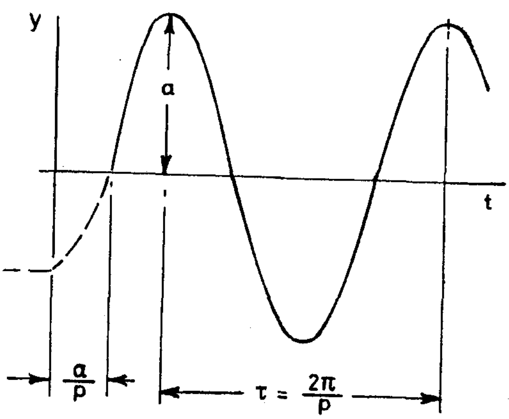


Fig. 1.4 Periodic motion of amplitude *a*, phase angle α and period τ