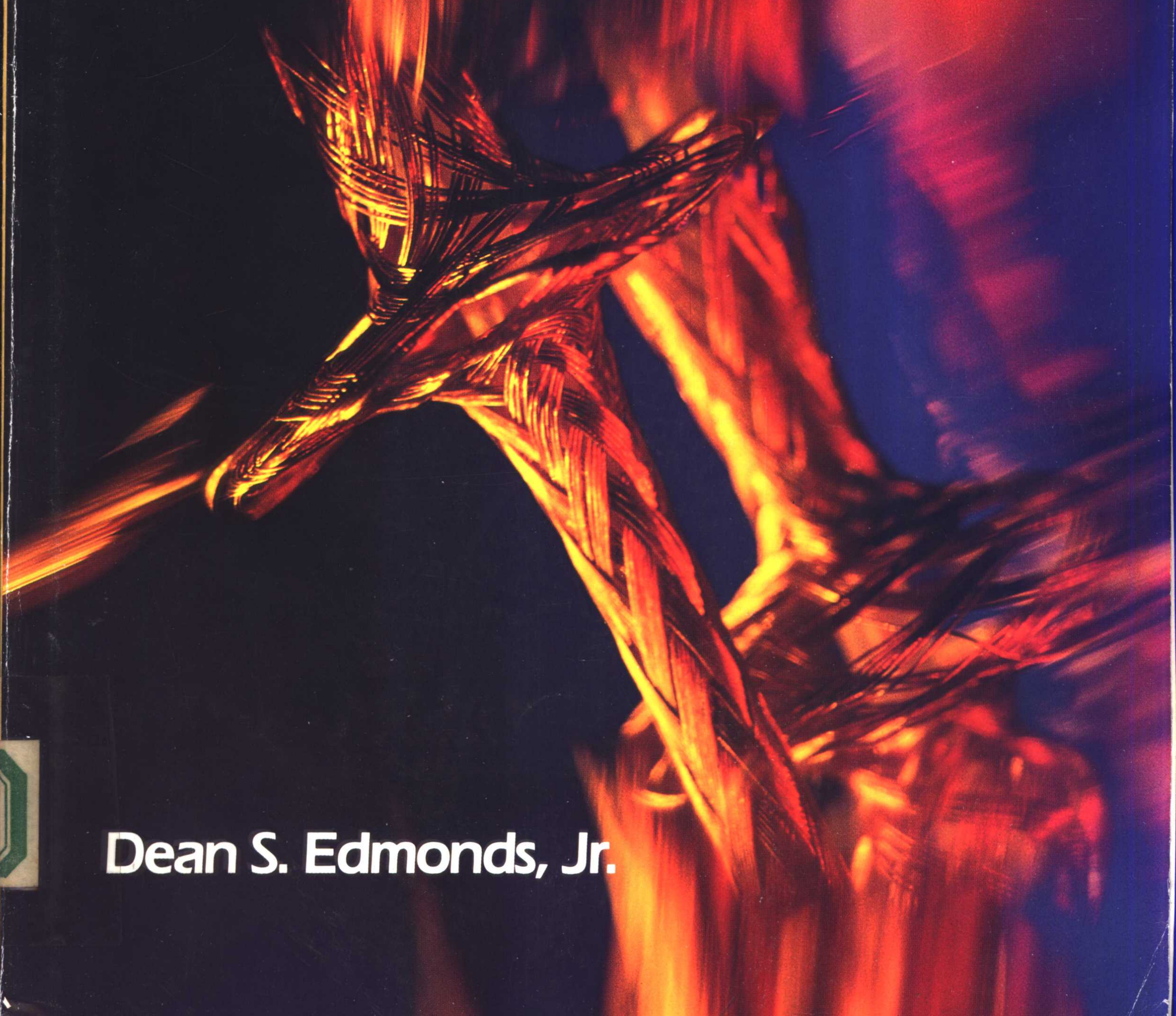
Cioffari's Experiments in College Physics

Eighth Edition



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Cioffari's Experiments in College Physics 8TH EDITION

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Preface

The eighth edition of *Cioffari's Experiments in College Physics* is, like its predecessors, intended for use in the laboratory segment of either a calculus- or algebra-based freshman physics course. The present revision features a rewriting of experiments in order to improve procedures and simplify data-taking. Thus an audio signal generator has been specified for Experiments 38 and 39 on vacuum-tube and transistor amplification; a special thermionic diode has been introduced in Experiment 36 on diode characteristics; and this latter experiment has been separated from the one on rectifier circuits (Experiment 37) so that either or both can be done, but neither is so long that it is difficult to finish in a normal laboratory period. This division has allowed the zener diode and some of its applications to be included, and in fact, Experiments 35–40 constitute a brief introduction to electronics that can relieve the instructor of the necessity of fitting some mention of this subject into the very tight schedule of the lecture part of the usual first-year physics course.

Two new experiments have been added—one on Coulomb's law (Experiment 23) and the other on field plotting (Experiment 24)—to cover the electrostatics gap of earlier editions. Quantitative experiments in electrostatics are always difficult because of the problem of measuring quantity of charge; the Coulomb's law experiment is really a measurement of the force between the plates of a parallel-plate capacitor, but at least the theory of this force is a direct consequence of the inverse-square force law between point charges. Moreover, the measurement is made using a simple attachment to the current-balance apparatus specified in Experiment 25 for the fundamental law of the magnetic force between parallel currents. A new procedure allowing a much more accurate determination of the force between the plates or the wires at a given spacing is presented in both these experiments.

Users of previous editions have felt a need for greater emphasis on the use of SI units and on the distinction between mass and weight. These comments have been taken very much to heart in the preparation of the present edition, and a special section of the Introduction is devoted to these topics. The fact that mass and weight are two very different things is described in some detail and is further pointed out whenever it comes up in the experimental work. Note is also taken of such practical considerations as the fact that balances (which really measure weight) are usually calibrated in grams, and the advantage of using centimeters, millimeters, or Angstrom units in cases where the lengths in question make it inconvenient to quote them in meters. Even the gauss gets introduced, since magnetic field strengths are so often given in this unit, but its relation to the tesla and the tesla's being the SI unit that must be used in all the equations of electromagnetism are carefully pointed out. Hopefully the new edition will succeed in helping the student avoid units problems and the all-too-prevalent confusion about mass and weight.

Finally, the popularity of the Apparatus Notes in the seventh edition has brought about their expansion into a separate *Instructor's Guide*, which features more details on the apparatus, including construction notes for certain items that are better built than purchased, some sample data, and answers to the problems in the Questions sections. I hope that the new manual will be a great help both to instructors charged with setting up a laboratory from scratch and to veterans of many years of experience with first-year physics courses.

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Dean S. Edmonds, Jr.

Introduction

The purpose of the physics laboratory is to supply the practical knowledge necessary for a well-rounded understanding of physics and the physicist's way of looking at the universe. A further aim is to develop familiarity with the experimental method of scientific investigation and to give you experience in the actual handling of laboratory apparatus. It is one thing to study a certain model of some physical phenomenon and deduce that certain results should be observed. It is quite another to set up an experiment in which these observations can be made and thus produce data on the basis of which the model's validity may be tested. In particular, obtaining experimental results depends on your ability to make accurate measurements of physical quantities in the real world. A major purpose of every experiment in this book is to provide practice in doing so.

INSTRUCTIONS

The instructions for each experiment include some basic theory on the phenomenon to be investigated and a description of the procedure to be used. You should study these carefully before coming to the laboratory to avoid waste of valuable laboratory time figuring out what should be done. You will be told well in advance which experiments are to be performed and the date for which each is scheduled so that there will be time for proper preparation. The necessary equipment will be laid out at each assigned place in the laboratory. Missing or defective apparatus should be reported to the laboratory instructor immediately. The instructor should also be consulted if you have any questions about the experiment.

Record all observations and data in the blank tables provided for this purpose in each experiment. Columns in these tables are already suitably labeled, but be careful to note the units in which each of the observed quantities is measured. Instruments should be read to the limit of their possibilities by estimating the last figure of the reading, that is, the fraction of the smallest scale division. Record each measurement directly on the data sheet exactly in the form in which it is made without any mental calculation. Do not use "scratch" data sheets from which data are to be transcribed onto the blank ones provided in this book. Very neat data sheets can be made out this way, but mistakes can also creep in. The instructor is interested in an *original* data sheet and is willing to put up with a certain amount of sloppy penmanship in order to see the direct recording of the actual data taken in the laboratory.

Whenever feasible, make calculations in the spaces provided in the manual. If the required computations are too long, complete them on a separate sheet of paper, and include it in the report. Each set of calculations should be headed by the pertinent equation so that anyone reading the report can see what mathematical operations are being performed and why. An electronic calculator of your own may and indeed should be used, but be careful. Erroneous entries produce erroneous results, and all numbers should be looked at to be sure no careless mistake has crept in. A quick order-of-magnitude check by hand is sometimes useful, but bear in mind that practice in arithmetic is *not* an important goal of the physics laboratory.

The questions at the end of each experiment are to be answered in the spaces provided for this purpose. Use proper English so that communication between you and a reader will not be impeded. Complete all calculations and as much of the remainder of the report as possible during the laboratory period. However, unfinished reports may be completed outside the laboratory and should be handed in at the time and place specified by the instructor.

CARE OF APPARATUS

The apparatus provided with each experiment has been set up to work properly in the arrangement described for that experiment and is in some cases very delicate. Use extreme care in handling it. The instructions for each experiment include a list of the required equipment, and you should check this list against the items on the work bench to make sure everything necessary is there and in good condition. Anything missing or broken should be reported to the instructor. At the end of the period, check the apparatus again and leave it neatly arranged.

Whenever an experimental setup has been assembled, it should be checked before being placed in operation so that any mistakes that might keep it from working properly or that might cause actual damage can be found and corrected. In particular, electrical circuits should be examined carefully for proper wiring. Application of power to a circuit containing wiring errors can cause serious damage. The source of power (battery or power supply) should always be connected last, and the circuit should be checked and approved by the instructor before this final connection is made. Special care should be exercised in setting meters to the proper range, as these items are expensive and easily destroyed if excessive current is allowed to pass through the movement. Whenever the range of a meter or any wiring in a circuit is to be changed, the source of power should always be disconnected first to eliminate the possibility of electrical shock or damage due to a temporary wrong connection.

THE REPORT

You must prepare a report of the work done in each experiment and hand it in at the beginning of the next laboratory period or at some other time designated by the instructor. The report will be graded and returned as soon as possible, after which it may be kept in a folder or binder for future reference. The report should include:

1. A title page. This should carry your name, the date, and the name and number of the experiment.
2. The instruction sheets. These are the pages describing the purpose of the experiment, the theory, the apparatus, and the procedure. Perforations allow these pages to be easily torn out of this book for inclusion in the report.
3. All original data and observations. As already noted, these are entered in the blank data tables provided in each experiment. The data table sheets are also perforated so that they can be easily removed from the book.
4. All the required calculations. Make these in the spaces provided. The calculation sheets are then detached along their perforations for inclusion in the report. If extra calculation pages are used, include them in the report in the proper order.
5. Graphs and diagrams, whenever they are required. Graph paper pages are provided as needed in this manual and are also perforated so that they can be easily detached and inserted in the report.
6. A summary and discussion of the results. The summary is included in tabular form under the data. It usually involves a comparison of the computed results with the accepted values together with the percent errors involved. You are encouraged to add a brief discussion of the sources of these errors and any other comments you would like to make about the working of the experiment.
7. Answers to the questions at the end of the experiment. The answers are written in the space provided after each question. The question sheets are then torn out along the perforations and added to the report. Take care to use complete sentences and in general to make the answers as clear and readable as possible. Use extra sheets if needed and then include them in the proper order as in the case of the calculations.

UNITS

If the properties of our physical world are to be investigated quantitatively, units must be introduced in terms of which the quantities we wish to measure can be stated. We are all familiar with units of length such as feet, inches, and centimeters and units of time such as hours and minutes, but because physics is a precise science, we have to look at such things more carefully and be very precise in our definitions.

Although the choice of units is quite arbitrary, two paramount considerations must be observed when such choices are made: (a) The chosen unit must be of a size that is convenient to use for the proposed measurements, and (b) everyone must agree on its definition. This latter requirement is accomplished by international agreement. The most recent conference for this purpose (the fourteenth) was held in 1971. The so-called fundamental units of length, mass, time, and electric current were agreed upon as a result of these conferences. In the case of length, the meter was established as the basic unit and defined in terms of the wavelength of the light emitted by the krypton 86 atom when undergoing a transition between a particular pair of its allowed levels of energy. Note that such an agreement involves a definition that is accessible to everyone—krypton atoms are all identical and anyone, anywhere in the world, can get some krypton and follow the prescribed procedure for determining the specified wavelength. An identical situation arose in the case of the second, the basic unit of time. The standard second has been defined in terms of the period of a certain frequency observed in the cesium atom. The basic unit of mass, on the other hand, is the kilogram, which everybody has agreed is the mass of a platinum-iridium cylinder kept in the vaults of the International Bureau of Weights and Measures near Paris. These units are called *fundamental*, not because physics gives them some fundamentally special status but because we can express all other units in terms of them. Thus velocity is measured in meters per second (m/s), acceleration in meters per second per second (m/s^2), and force in kilogram-meters per second per second ($kg\text{-}m/s^2$). In the case of a unit like force, which is an unwieldy combination of fundamental units, a special name is given to represent that combination. It has become customary to honor famous scientists by using their names for this purpose; thus our unit of force is called the newton (after Isaac Newton) and is defined as a kilogram-meter per second per second.

The meter, the kilogram, and the second suffice to give us all the units we need for measurements in mechanics. But electricity introduces a new physical quantity, electric charge, for which we need an additional fundamental unit. Actually, because a given amount of charge is hard to determine precisely, the International Conference agreed instead on the size of the ampere, the unit of electric current. Clearly, since by definition of current an ampere is a unit of charge going by in a second, and since a second has already been defined, a precise definition of the unit of electric charge is immediately obtainable. It is called the coulomb after the French physicist Charles Augustin Coulomb, just as the name of the unit of current honors André Marie Ampere.

Although the meter, the kilogram, and the second are convenient in size for measurement of the lengths, masses, and times encountered in everyday life, both smaller and larger units are needed in many areas of physics. Accordingly, a system of prefixes indicating multiples by powers of ten is used to express such quantities. These prefixes are listed in Table III at the end of the book. Some common examples are the centimeter (10^{-2} m), the millimeter (10^{-3} m), the kilometer (10^3 m); and for time, the millisecond (10^{-3} s), the microsecond (10^{-6} s) and, in this day of electronic circuitry, the nanosecond (10^{-9} s). Note that the standard unit of mass is the kilogram, not the gram, although this presents no particular problem, a standard gram being a mass of 10^{-3} kg. Conversion to other units in common use but not in the agreed-upon system, such as the foot, the mile, the minute, and the hour are given in Table II at the end of the book, which points out that in the so-called English system the unit of length is now defined by making the inch *exactly* 2.54 cm.

The system agreed upon at the 1971 International Conference and described earlier is called the SI system, an abbreviation for the full French title "Système Internationale d'Unites." Because the meter, the kilogram, the second, and the ampere (and hence the coulomb) are its fundamental units, the SI system is essentially identical with what was previously called the MKS system (for meter-kilogram-second). The only difference, in fact, lies in the precise definitions of the meter and second described above. These definitions superceded earlier, less precise ones, but the number of wavelengths of the krypton light making up a meter and the number of cycles of the cesium frequency making up a second were chosen to agree with the earlier definitions of these units within the precision already established. Thus existing secondary standards did not have to be altered following the 1971 conference. For our purposes, the terms MKS system and SI system are synonymous, but we shall use the presently preferred designation SI. We will also follow current practice in using the SI system in this book; however, there will be some exceptions. For example, meter sticks, vernier calipers, and micrometer calipers are calibrated in centimeters, and recording the reading of such instruments in centimeters should certainly be permissible. But conversion to meters will usually be advisable if not essential when the data is used in calculations.

One other difficulty that needs to be overcome right from the start is the confusion between mass and weight. *It is imperative to remember that weight is a force, namely, for our purposes in a laboratory on the earth's surface, the force with which objects are attracted to the earth by gravity.* Mass, on the other hand, is an intrinsic property of all objects. Since the force of gravity goes inversely with the square of the distance separating the attracting objects, an object can be made weightless by taking it far away from the earth and all other heavenly bodies, but it never becomes massless. Confusion arises because an object's mass is apparently responsible for its gravitational attraction to other objects, and in fact the gravitational force is proportional to the masses involved. Thus a mass m will be attracted to the earth with a force W (its weight) that is proportional to m. As the proportionality constant is a known number (the acceleration of gravity g), a measurement of the weight of an object also determines its mass. As a result, scales and balances, which actually measure weight, are often calibrated in grams or kilograms, which are units of mass. Thus it is all too easy to say that a certain object "weighs so many grams." Great care must be taken not to do this but to say rather that the object's mass is so many grams, even though the instrument making the measurement is actually measuring the force of gravity on the object. The distinction between mass and weight will be carefully observed throughout this book, although sometimes a seeming contradiction may appear. Thus it is common parlance to speak of "a set of weights," meaning objects of calibrated mass to be used in balances to produce known forces of gravity. We shall adopt such usual terms rather than the somewhat forced "set of calibrated masses," but the distinction between mass and weight must be kept in mind at all times, even when confusing situations such as those just mentioned arise.

PLOTTING OF CURVES

Graphs are of particular importance in physics because they display the relationship between pairs of interdependent quantities in a readily visualized form. Thus if two quantities x and y have the linear relation

$$y = ax + b \tag{1}$$

a graph of y against x will be a straight line whose slope is a and whose y intercept (the value of y at which the line crosses the y axis, that is, at $x = 0$) is b. Conversely, if a given theory predicts that a certain physical quantity y depends linearly on another physical quantity x, this conclusion can be tested experimentally by measuring corresponding values of x and y and plotting these results. The plotted points will readily show whether a straight line can be drawn through them, even if various errors cause them to have a "scatter" instead of all lying right on a line. If there is a scatter, the straight line that represents the best average should be drawn as shown in Fig. 1. There are numerous rules for obtaining the line that is a true "best fit" to a given set of experimental points, but a simple determination by visual inspection using a transparent (plastic) straight edge is usually good enough and is all that will be required in these experiments. If the scatter of points is so large that a good decision as to where to draw a straight line through them cannot be made, the conclusion that y depends linearly on x should be seriously questioned. If, on the other hand, the points readily define a straight line, not only does the resulting graph supply evidence that x and y are indeed linearly related, but the value of a in Equation 1 may be obtained by finding the line's slope. Note that this is a convenient method of getting the average value of a. Moreover, a value for b representing an average result of all the plotted data can be read directly off the graph. This simple procedure is equivalent to the much more tedious one of determining the best values of a and b by a "best fit" calculation.

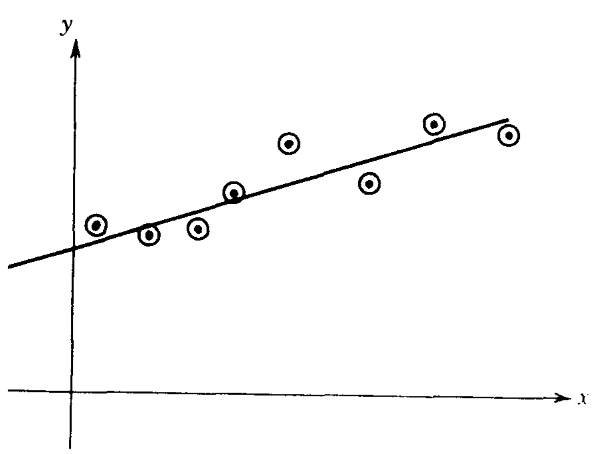

Figure 1 *A "best fit" straight line drawn through a set of experimental points*

A linear relation between x and y is easily recognized when the points are plotted, but other relationships are not so obvious. Thus, suppose the relationship to be investigated were

$$y = ax^2 + b \tag{2}$$

Plotting y against x would give a curve, but it would be very difficult to distinguish this curve from the curve resulting from, say, $y = ax^3 + b$. In fact, the straight line is the only graph that is really obvious. However, if the validity of Equation 2 is to be tested graphically, a new variable $u \equiv x^2$ can be introduced so that Equation 2 becomes

$$y = au + b \tag{3}$$

Then, if Equation 2 is valid, a plot of y against u (that is, against x^2) will yield a readily recognizable straight line. This procedure may be used in many cases where a new variable u may be substituted for a function of x to produce a linear relation such as Equation 3.

A special case of this procedure arises when y depends exponentially on x, so that

$$y = Ae^{ax} \tag{4}$$

where e is the base of natural logarithms and is approximately equal to 2.718. This situation occurs often enough to merit special treatment. The first step is to take the natural logarithm of both sides of Equation 4:

$$\ln y = \ln Ae^{ax} = \ln A + \ln e^{ax} = \ln A + ax \tag{5}$$

Equation 5 is just like Equation 1 except that $\ln y$ rather than y is to be plotted against x. This plot will thus be linear if Equation 4 is valid, and the y intercept (b in Equation 1) will be $\ln A$. Hence A can be found by taking the antilog of the intercept.

Because this situation arises often, special graph paper is printed on which the graduations along the ordinate (y axis) are logarithmically rather than linearly spaced. This means that if a value of y is plotted on the given ordinate scale, the actual position of the point along the y axis will be proportional to the logarithm of y. In other words, plotting a value of y on this special graph paper automatically takes the logarithm, making a separate calculation of $\ln y$ for each value of y unnecessary. However, note that commercial logarithmic graph paper is set up for common (base 10) logarithms rather than natural (base e) logarithms. Taking the common log of both sides of Equation 4 yields

$$\begin{aligned}\log y = \log Ae^{ax} &= \log A + \log e^{ax}\\ &= \log A + ax \log e\\ &= \log A + a(0.4343)x \end{aligned}\tag{6}$$

The slope of the resulting straight line is now $0.4343a$ rather than just a. Notice also that because the y axis is graduated logarithmically, the value of A may be read off from the intercept directly, the paper having also automatically done the job of taking the antilog.

Another popular special case arises when a functional relation of the form

$$y = ax^n \qquad (7)$$

is to be investigated. Although this can be handled by introducing $u \equiv x^n$ as already discussed, such a procedure requires calculating x^n for each value of x. Since n may be any number, positive or negative, such calculations can get tedious unless a reasonably sophisticated calculator is available. Another very convenient method is again to take the logarithm of both sides of Equation 7. Then,

$$\log y = \log a + n \log x \qquad (8)$$

and a plot of log y against log x will produce a straight line with slope n.* You may object that since you must look up the logarithms of all the values of both x and y, things haven't been simplified much, but again special graph paper is available that makes this calculation unnecessary. Since log y is now to be plotted against log x rather than x, this graph paper has both the ordinate and the abscissa graduated logarithmically. It is therefore called full log or log-log paper, whereas paper with the ordinate graduated logarithmically and the abscissa linearly is called semilog. The logarithmic scales are called 1-cycle, 2-cycle, etc., depending on the number of powers of ten covered on the axis in question. Thus, an axis graduated logarithmically from 1 to 10 is called 1-cycle; from 1 to 100, 2-cycle; etc. Scales of up to 5 cycles are available commercially, and in the case of full log paper there are various standard combinations of numbers of cycles along the ordinate and the abscissa. Appropriate graph paper pages are included in this book as needed; refer to Experiment 33 for an example of semilog paper and to Experiment 36 for the full log type.

In drawing graphs, scales for the coordinate axes should be chosen so that the curve extends over most of the graph sheet and so that decimal parts of units are easily determined. This can be done if each small division is made equal to one, two, five, or ten units. The same scale need not be used for both axes. The independent variable should be plotted along the x axis and the dependent variable along the y axis. Each axis should be labeled with the name of the quantity being plotted and the scale divisions used. The numbers should increase from left to right and from bottom to top. Each graph should have a title indicating what the curve is intended to show.

Each point should be plotted as a dot surrounded by a small circle, which shows where the point is located even if the dot is obscured by the curve drawn through it. A straight line (or smooth curve if a straight-line plot is not being sought) should then be drawn through the dots. The curve need not pass through all the dots but should be drawn so as to fit them as closely as possible, as already mentioned. In general, as many points will lie on one side of the curve as on the other. The extent to which the plotted points coincide with the curve is a measure of the accuracy of the results.

SIGNIFICANT FIGURES

The numbers dealt with in mathematics are exact numbers. That is, when a mathematician writes 2 it means 2.00000 . . . , and all subsequent calculations assume that the 2 means exactly two, not the tiniest fraction more or less. In physics the situation is very different. Many of the numbers dealt with come from measurements of physical quantities, and these can never be exact. For example, suppose that a distance is measured with an ordinary centimeter rule and found to be 5.23 cm. In this measurement, the 3 is an estimate, for the smallest divisions on a centimeter rule are millimeters (tenths of a centimeter). The 3 represents a guess as to where between the 5.2 and 5.3 cm divisions the end of the measured distance lies. The statement that the distance was found to be 5.23 cm does not mean that it is exactly 5.23 cm but merely that it is probably not less than 5.22 cm or more than 5.24 cm. If a high-quality micrometer had been used, the distance might have been found to be 5.2347 cm, where the 7 represents a guess as to where the micrometer's index line fell between the .234 and the .235 divisions. Thus the micrometer yields a much more precise value of the measured length than does the centimeter rule, but it too is not exact. More precise measurement methods might give further decimal places that cannot be determined with the micrometer any more than the 4 and the 7 could be found with the centimeter rule. Thus, when the result of the centimeter rule measurement has been written as 5.23 cm, it doesn't mean that the distance is exactly 5.23 cm or that zeros can be written after the 3. Nothing can be written after the 3 because the instrument being used gives no information as to what to write there.

The 5, the 2, and the 3 in the centimeter rule measurement are called *significant figures* because they each give trustworthy information about the size of the physical quantity being measured. The centimeter rule is quite good enough as a length-measuring instrument to determine that the length in question lies between 5.2 and 5.3 cm, and the 3 in the next place represents a significant guess as to where between 5.2

*Notice that in this case there is no intercept in the usual sense. Log y = log a and $y = a$ when $x = 1$.

and 5.3 cm the actual length lies. The centimeter rule measurement is thus *good to three significant figures*, whereas the micrometer measurement gave five significant figures, the micrometer being a much more precise length-measuring instrument than the centimeter rule. The 3 in the centimeter rule measurement and the 7 in the micrometer measurement are less significant than the other figures but are still considered significant because they give some real information about the desired length even though there is some doubt about their actual values. Clearly, however, if there is some doubt about them, any figures that might get written to the right of them in the respective cases would be meaningless. In particular, one should be careful not to write zeros there. If the length measurement made with the centimeter rule were recorded as 5.230 cm, the zero would be a significant figure and would mean that somehow someone was able to interpolate between the 5.2 and 5.3 divisions to 1/100 rather than just 1/10 of the space between them. Indeed, the micrometer measurement shows that the figure to the right of the 3 should be 4, so that putting a zero there says something that isn't true. Always take care to distinguish between zeros that are significant and those that are not. In general, zeros that merely serve to place the decimal point are not significant. Thus, if the length measurement were to be stated in meters, the two zeros in 0.0523 m would not be significant. They merely place the decimal point appropriately in the three-significant-figure measurement. However, if in measuring the distance with the centimeter rule the end of this distance appeared to fall right opposite the .2 cm division following the 5 cm mark, it would be recorded as 5.20 cm and the zero would be significant. In general, zeros appearing to the right of figures that are already to the right of the decimal point must be regarded as significant, for if they weren't they wouldn't be there. Zeros between other figures and the decimal point should usually be regarded as serving only to place the decimal point. The example of the length of 0.0523 m is typical, there being no doubt that the zeros are not significant. There are some ambiguous cases, however. Suppose that a certain race course is found to be 1.2 km long. As written, this is a two-significant-figure measurement. The same result may be given as 1200 m. Here again the zeros are not significant but must be present in order to properly locate the decimal point. Without the knowledge that the original measurement of 1.2 km contained only two significant figures, however, there is no way to tell whether these zeros are significant or not. In such cases, the experimenter must refer to the measuring instrument to determine how many significant figures are justified.

There is usually no problem in deciding how many significant figures a given measurement should contain, but difficulties arise when these numbers are used in calculations. This is because mathematics assumes that all numbers are exact and thus automatically fills all places to the right of the last significant figure with zeros even though this is physically wrong. The calculations then often produce a great many figures that look as if they were significant but really are not, for clearly no mathematical manipulation can give a result whose precision is greater than that of the quantities put into it. Some examples may serve to show how this problem should be handled.

1. Addition and subtraction: When carrying out addition or subtraction by hand, do not carry the result beyond the first column that contains a doubtful figure. This means that all figures lying to the right of the last column in which all figures are significant should be dropped. Thus in obtaining the sum of these numbers

806.5		806.5
32.03	they should be written as	32.0
0.0652		0.1
125.0		125.0
		963.6

Note that, in dropping nonsignificant figures, the last figure retained should be unchanged if the first figure dropped is less than 5 and should be increased by 1 if the first figure dropped is 5 or greater. This is a normal convention to which this book will adhere.

If an electronic calculator is used, the numbers to be added may be entered without the bother of determining which figures to drop, in which case all figures will appear in the sum. This is like adding the numbers as given on the left in the example above. The result will be 963.5952. You must then look at the data and observe that in two of the numbers being added there is no indication of what the figure in the second decimal place should be. The result must therefore be rounded off to one decimal place by dropping the 952. Since the 9 is equal to or greater than 5, the figure in the first decimal place is raised by one to give 963.6 as before.

2. Multiplication and division: The operations of multiplication and division usually produce many more figures than can be justified as significant, so that results must be properly rounded off. The rule is to retain in the result only as many figures as the number of significant figures in the least precise quantity in the data. Suppose the area of a plate is to be measured. A centimeter rule is used to find that the plate has a length of 7.62 cm and a width of 3.81 cm. As in the earlier example with the centimeter rule, these measurements each contain three significant figures, of which the third is doubtful. If the area is now found by multiplying 3.81×7.62 either by hand or with a calculator, 29.0322 cm^2 will be obtained. This number appears to have six significant figures, but the two original quantities have only three each. Therefore, only three significant figures should be retained in the

result, which should be written as 29.0 cm^2. With some exceptions, the measurements to be made in the experimental work covered in this book will contain three or four significant figures, so that when an electronic calculator is used, the final result must be rounded off to the number of figures that can be justified by the data as being significant.

THEORY OF ERRORS

All measurements are affected by errors; this means that measurements are always subject to some uncertainty. There are many different types of errors, such as personal, systematic, instrumental, and accidental errors. Personal errors include blunders, such as mistakes in arithmetic, in recording an observation, or in reading scale divisions. Another important kind of personal error is known as personal bias, such as trying to fit the measurements to some preconceived idea, or being prejudiced in favor of the first observation. Systematic errors are characterized by their tendency to be in one direction only, either positive or negative. For example, if a meter stick is slightly worn at one end, and measurements are taken from this end, then a constant error will occur in all these measurements. Instrumental errors are those introduced by slight imperfections in the manufacture or calibration of the instrument. The worn meter stick just mentioned or an electrical meter that has not been properly set to zero with no input are examples of instrumental errors. Note that such errors are usually also systematic. Accidental errors are deviations beyond the control of the observer. These errors are due to jarring, noise, fluctuations in temperature, variations in atmospheric pressure, and the like. Included in this category are variations in observed data due to inherently random processes such as the intersurface actions that produce the force we call friction and the radioactive decay of atomic nuclei. Since the causes just listed for accidental errors are essentially random in nature, all these causes of data variation are subject to treatment by statistical methods, as will be discussed here.

It will be assumed in these experiments that instrumental errors due to improper calibration, zeroing, etc., have been prevented by proper inspection and adjustment of the equipment and that care has been taken to eliminate systematic errors, personal errors, and personal bias. There remain accidental errors, which make themselves known by causing a spread in the values obtained when a given measurement is repeated several times. Two examples may serve to illustrate how this comes about.

Consider first the distance measurement with the centimeter rule discussed in connection with significant figures. In the measurement of 5.23 cm, the 3 was doubtful, being an interpolation between the 5.2 and 5.3 cm divisions, which are the smallest divisions on the centimeter rule. If a two-significant-figure result were adequate, the distance could have been quoted as 5.2 cm. In this case, if the measurement were repeated many times, even by different experimenters, the likelihood is that 5.2 cm would be obtained each time. No accidental error is revealed because the measuring instrument is not being pushed to the limit of its precision and random processes in the experiment (such as small variations in the length of the rule and/or the distance being measured due to temperature fluctuations) are negligible compared to the smallest scale unit in the measuring instrument (the millimeter divisions on the centimeter rule in this case). The precision here is said to be limited by the scale of the instrument.

However, if the distance measurement is repeated with an estimated interpolation made each time between the 5.2 and 5.3 cm divisions, the same estimate may not always be made. This would be especially true if each measurement were made by a different experimenter who had no knowledge of the others' results. Thus, one might guess 5.22 cm, another 5.21, another 5.24, etc. To handle this situation, a mean or average of the various measured values is calculated. As will be discussed shortly, this average is more accurate than any one of the measurements alone and can in fact be shown to improve in accuracy as the square root of the number of individual measurements made. Clearly, a way of improving the accuracy of experimental data is to measure each quantity many times, and an important matter of judgment in experimental work is to decide on how many times a given quantity is to be measured. In this regard, it must be remembered that to measure something N times takes N times as long as measuring it once, but the accuracy obtained by doing so is only $\sqrt{N}$ times as great. Thus, if a certain measurement takes one minute, making it ten times will take only ten minutes but will yield over three times the accuracy. However, making it a hundred times will take an hour and forty minutes, but this investment in time will only yield another threefold increase in accuracy. Clearly, a compromise based on the accuracy required, the time needed for a particular measurement, and the time available must be reached in each case.

A second example, one dominated by random processes inherent in the experiment itself, is that of the range of a spring-operated gun. The experimental setup is shown in Figure 2. The spring gun consists of a tube containing a spring-loaded plunger. A small steel ball is placed in the tube against the plunger. The plunger is pulled back a given distance, compressing the spring by a known amount, and is then released sharply, propelling the ball out of the tube. The ball

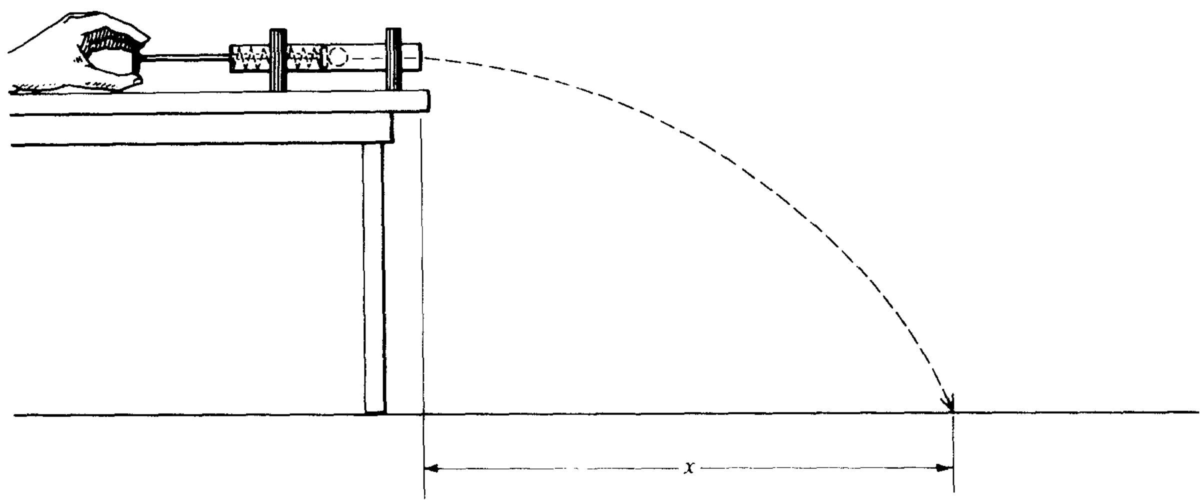

Figure 2 *The range of a spring gun*

strikes the floor at a horizontal distance x from the end of the tube (the gun's muzzle), this distance being the range in question.

If this experiment is repeated under conditions made as identical as possible to those in effect on the first try (the ball is carefully put back in the tube, the plunger is pulled back by a distance made as closely equal as possible to that used the first time, and care is taken to release the plunger in the same way), will the ball strike the floor at exactly the same point? Simple theory predicts that it will, but small variations in the distance the plunger was pulled back, in the state of the spring, and in the condition of the surfaces of the ball and the inside of the tube—all random, uncontrollable effects—will cause the measured range to vary somewhat on subsequent shots. Indeed, no one would really expect successive shots from a gun to all land in precisely the same spot even though the gun was clamped in a fixed position and given the same charge each time. Instead, a spread of impact points would be expected, as shown in the plan view of the spring-gun experiment in Fig. 3. The extent of the spread may be reduced by using great care in the experimental technique (wiping off the ball after each shot, handling it with plastic gloves to prevent getting fingerprints on it, taking care in the measurement of how far the plunger is retracted, and releasing the plunger smartly each time), but the spread can never be reduced to zero. The size of the spread is a measure of the precision of the experiment. An estimate of this precision is very desirable in all experimental work, and the following discussion will show how the extent of the spread can be used to express such an estimate quantitatively.

According to statistical theory, the arithmetic mean or average of a number of observations will give the most probable result. This is clear from the results

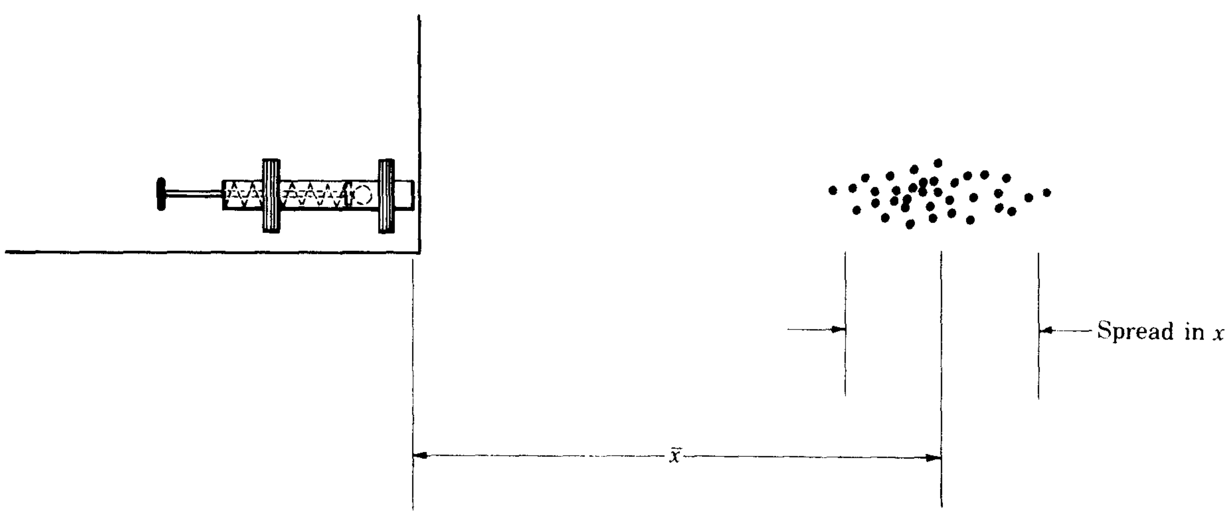

Figure 3 *The range of a spring gun. Plan view*

of the range experiment illustrated in Fig. 3. If a single number is to be quoted as the range of the spring gun, it should be the distance from the gun muzzle to the center of the distribution of impact points. In the absence of any peculiar experimental effects, we expect the distribution of points to be densest near the center, to thin out as we go away from the center, and to be symmetrical (to show as many impact points beyond the center as short of it). Hence in this normal case the *average* range $\bar{x}$ is also the *median* (the midpoint of the distribution with as many points with bigger x as with smaller x) and the *most probable* value (the point near which there is the greatest density of points). Thus the first step in data analysis is to find the average of the distances from the gun muzzle to all the individual impact points. This is shown on the left in Table I. Note that these measurements are made with a meter stick and could therefore be given to one more significant figure by interpolation between the millimeter divisions on the stick. If this order of precision were wanted, each value of x would be measured several times by different investigators, each of whom would make an interpolation, and an average value obtained for each x. Then every entry in the left-hand column of Table I could be quoted to two decimal places (five significant figures). However, this would take a great deal of time and effort, all of which would be wasted because the spread in the data is several centimeters, making the fifth significant figure in each measurement nonsignificant in the final result. In other words, the random effects in the experiment dominate the picture and limit the useable precision of the measuring instrument.

One obvious way of expressing the extent of the spread in a set of experimental data is to note the deviation of each measurement from the average or arithmetic mean just found. In the example of the spring-gun range experiment, these deviations (differences between each measurement and the average) are tabulated in the middle column of Table I and their average is then computed. Note that, in computing this average, no account is taken of the algebraic signs of the deviations. A deviation represents an error—a difference between a particular measurement and the average of all the measurements, this average being the closest available approximation to the true value of the quantity being measured. Which way the deviation lies makes no difference; it is still an error. The average error is a measure of the scatter of the observed values about their average. The average deviation thus found is therefore often called the average error, and for the purposes of the elementary laboratory, it may be taken as the possible error in the mean value. Consequently the result of the range measurement should be written as 133.9 ± 3.1 cm to show that the true value of the range has a high probability of lying between 130.8 cm (133.9 − 3.1 cm) and 137.0 cm (133.9 + 3.1 cm). Actually, a statistical analysis shows that if a very large number of range measurements were made, 57.5% of them would lie inside this interval. That is, 57.5% of the impact points would be between 130.8 and 137.0 cm from the gun muzzle.

Table I *The Range of a Spring Gun*

Range, cm	Deviations, cm	Deviations Squared, cm²
134.2	+0.3	0.09
139.5	+5.6	31.36
133.0	−0.9	0.81
136.6	+2.7	7.29
129.4	−4.5	20.25
127.8	−6.1	37.21
130.6	−3.3	10.89
136.5	+2.6	6.76
135.3	+1.4	1.96
131.9	−2.0	4.00
138.1	+4.2	17.64
11⌊1472.9	11⌊33.6	11⌊138.26
$\bar{x}$ = 133.9 cm	a.d. = 3.1 cm	12.57
		$\sigma = \sqrt{12.57} = 3.5$ cm

Statistical theory also presents some other useful ways of stating the accuracy of an experimental result. For example, the fact that the average of a set of measurements gets more and more accurate in proportion to the square root of the number of measurements made can be reflected in the stated error by dividing the average error by $\sqrt{N}$, where N is the number of measurements. The result is called the *average deviation of the mean* (A.D.). Thus,

$$\text{A.D.} = \frac{\text{a.d.}}{\sqrt{N}} \tag{9}$$

where a.d. stands for the average deviation *from* the mean, that is, the average error already discussed. The A.D. is a measure of the deviation of the arithmetical mean from the true value and is in this context generally known as the probable error. The significance of the A.D., from probability theory, is that the chances are 50% that the true value of the quantity being observed will lie within ±A.D. of the mean. Thus, in the example of the spring gun, the mean of the measured ranges is 133.9 cm and the average deviation from the mean (the a.d.) is 3.1 cm, which says that on the average the readings differ from the mean (133.9 cm) by ±3.1 cm. The average deviation *of* the mean (the A.D.) is $3.1/\sqrt{11} = 0.9$ cm, which says that the chances that the true value of the range will lie in the interval 133.9 ± 0.9 cm are 50%, while the chances that it will lie outside this interval are also 50%.

Another (and, from the standpoint of statistical theory, most important) measure of the *dispersion* (scatter of experimental points) is the *standard deviation*. This is defined as the square root of the average of the squares of the individual deviations, or, mathematically, by

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_N - \bar{x})^2}{N}} \tag{10}$$

where σ is the standard deviation, $x_1, x_2, \ldots, x_N$ are the N individual measurements, and $\bar{x}$ is their average. Note that the signs of the various deviations make no difference in calculating σ since each is squared. An example of a standard deviation is given in the right-hand column in Table I. Like other measures of dispersion, the standard deviation gives information about how closely the distribution is grouped about the mean. Statistical analysis shows that for a large number of normally distributed measurements, 68.3% of them will fall within the interval $x \pm \sigma$. In the results of our hypothetical range experiment, this is 133.9 ± 3.5 cm, and after a large number of firings we would expect to find that about 68% of the impact points lay between 130.4 and 137.4 cm from the gun muzzle.

When a very large number of measurements of a given quantity are made and variations between the different values obtained are due to truly random effects, a *normal distribution* of these values will be found. The word *distribution* as used here means an expression of the relative frequency with which the different observed values occur. Such an expression often takes the form of a graph in which the number of observed values in a small interval centered on a particular value of x is plotted against x. Thus, suppose in the experiment with the spring gun a very large number of range observations were made. We could, for example, count the number of such observations falling in the interval 127.0 ± 0.5 cm and plot this number as the ordinate of a point whose abscissa was 127 cm. Another point would be the number of observations falling between 127.5 and 128.5 cm plotted with an abscissa of 128 cm, and this process could be continued until we found somewhere beyond 140 cm that there were no more observed points to plot. A smooth curve could then be drawn through the plotted points. According to our earlier discussion, this curve should show a maximum at the mean value $\bar{x} = 133.9$ cm and should fall off symmetrically on either side. Such a curve, called the *normal curve*, was first discovered by a famous French mathematician, De Moivre, while working on certain problems in games of chance. It was also derived independently by Laplace and Gauss, who made statistical use of it and found that it accurately represents the errors of observation in scientific measurements. The curve is also known as the *normal probability* curve because of its use in the theory of probability, as the *normal curve of error*, and as the *Gaussian curve*. Here error is used to mean a deviation from the true value. Whenever any measurements are made in which there are random fluctuations, the results predicted by the normal curve are found to be valid. Thus it has been found that this curve describes very well many distributions that arise in the fields of the physical sciences, biology, education, and the social sciences.

The mathematical representation of the normal curve is given by the equation

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-(x-\bar{x})^2/2\sigma^2} \tag{11}$$

where y represents the distribution function. In accordance with the foregoing discussion of how the normal curve is obtained in a physical case, the number $y\Delta x$ is the number of measurements of x (out of a very large total number N) that fall within the very small interval Δx centered on the value of x for which y was computed. Thus suppose we want to know how many measurements of x will fall in the vicinity of a certain value x_0. Putting x_0 into Equation 11 for x, we calculate the corresponding value y_0 for y. Then $y_0\Delta x$ is the number of measurements of x expected to be found in the interval $x_0 - \frac{1}{2}\Delta x$ to $x_0 + \frac{1}{2}\Delta x$. Thus y_0 is to be interpreted as the number of measurements *per unit interval in x* falling in the neighborhood of x_0. Note that $y_0\Delta x$ is the area of a tall, thin rectangle Δx wide located at x_0 on the x axis of the coordinate system in which the normal curve is plotted and extending up to the curve (a height y_0). Because this area represents the number of measurements of x falling within Δx, the area under the complete curve should be equal to N, the total number of measurements made. The factor $1/\sigma\sqrt{2\pi}$ in Equation 11, called a normalizing factor, is chosen to bring this about. It follows that the area under the curve between ordinates erected at some pair of values $x = a$ and $x = b$ is the number of measurements falling between a and b.

The curve is bell-shaped and symmetrical about the line $x = \bar{x}$. It has a maximum for this value of x (bearing out the idea that the mean value of x is also the most probable value) and falls quite rapidly toward the x axis on both sides. For different normal distributions, the curve has the same general shape, but its steepness, height, and location along the x axis will depend on the values of N, $\bar{x}$, and σ. The characteristic properties of the normal curve can be studied very readily by representing it in a new set of variables by means of a mathematical transformation. The first step is to divide Equation 11 through by N. The quantity $(y/N)\Delta x$ is then the fraction of the total number of measurements or the probability of obtaining a measurement in the narrow interval Δx centered on the value of x for which y is computed. Put another way, y/N is the probability *per unit interval in x* of getting a value of x lying in that interval whenever a measurement of x is made. Clearly, the area under the curve of y/N is unity, corresponding to the fact that the probability of getting *some* value of x is 1 or 100%. Next, we take the origin of our transformed coordinate system to be at the arithmetic mean and use the standard deviation as the unit of measurement along our new horizontal axis. This is done by choosing a new variable t related to x by the equation $t = (x - \bar{x})/\sigma$. When this is substituted into Equation 11 with N divided out and the normalizing factor appropriately modified, we obtain in place of $y(x)$ a new function $\phi(t)$ given by

$$\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \tag{12}$$

This is the *standard form* of the normal curve, where $\phi(t)$ is the distribution and t is the variable. For this distribution, the average value of t is zero and the standard deviation is equal to one. The advantage of representing the normal curve in the standard form is that the area under the curve between any two values of t may be calculated once and for all. These values are tabulated and can be obtained from tables of probability integrals. Any normal distribution may then be expressed in the standard form and the required calculations of the characteristics of the distribution can easily be made.

Some interesting properties of this curve, which is shown in Fig. 4, deserve mention. Note that theoretically it extends from $-\infty$ to $+\infty$, but practically it is so close to the axis beyond $t = \pm 3$ that the area under the curve beyond these points is negligible. The variable t is measured in units of σ along the horizontal axis, and the mean, the median, and the most probable value all coincide at the origin ($t = 0$). The total area under the curve is equal to 1. Hence the area under any portion of the curve represents the relative frequency (expressed as a fraction of unity or as a percent) with which measurements in that interval occur. Numerical values of such areas may be obtained from tables and can be changed into the actual frequencies of occurrence by multiplying by N.

The *percentage distribution* of area under the normal curve is given in Fig. 4, where σ is the unit of measurement. The significance of the values given in the figure is that if the values of x are normally distributed, the probability that a value chosen at random will fall within the range $\bar{x} \pm \sigma$ is 0.683. The probability that it will lie within the range $\bar{x} \pm 2\sigma$ is 0.9545. The probability that it will lie within the range $x \pm 3\sigma$ is 0.9973. Thus the probability that it will lie outside of this range is only 0.0027 or 0.27%. Hence a deviation

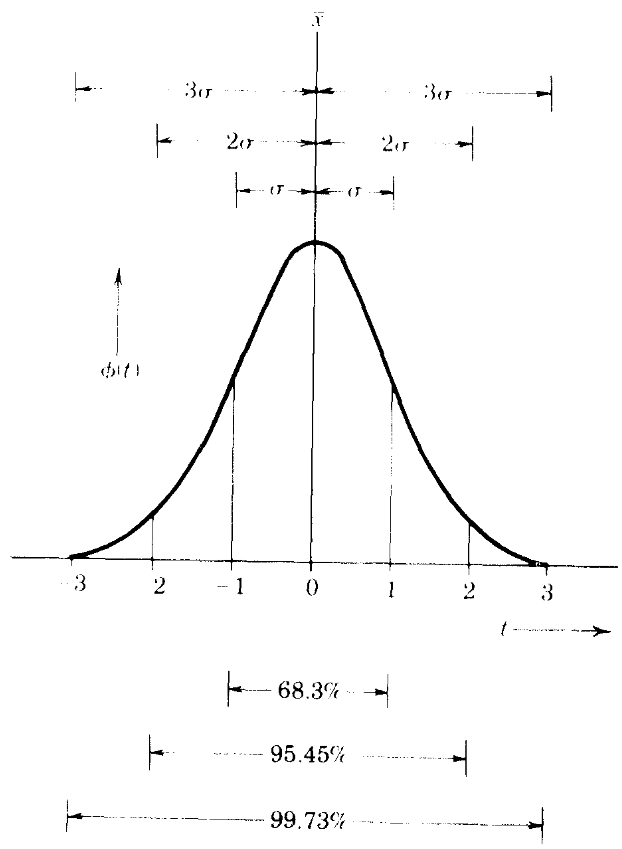

Figure 4 *The percentage distribution of area under the normal curve*

of 3σ on both sides of the arithmetic mean includes practically the whole of a normal distribution. The probability that any particular value of x will lie in the range $\bar{x} \pm$ a.d. is also easily found from Equation 12 and Fig. 4. The average deviation from the mean (a.d.) is simply the average $\bar{t}$ of t expressed in units of σ and can be calculated from Equation 12. The result is

$$\bar{t} = \sqrt{\frac{2}{\pi}} = 0.798$$

$$\text{a.d.} = \sigma\bar{t} = \sigma\sqrt{\frac{2}{\pi}} = 0.798\sigma \tag{13}$$

The area under the curve of Fig. 4 between −0.798 and +0.798 is then found to be 0.575 or 57.5% of the total area. This means that, as noted earlier, 57.5% of a large number of measurements will fall within the interval $\bar{x} \pm$ a.d. Similarly, the area under the curve between −1 and +1 is found to be 0.683 of the total area so that, also as noted earlier, 68.3% of a large number of measurements will fall within the interval $\bar{x} \pm \sigma$.

PERCENT ERROR

The error in a measured quantity is often conveniently expressed as a percent of the quantity itself. Since the true value is usually not known, one of the errors previously discussed must be used in calculating this percent. For this purpose, the probable error is usually chosen because it is the same as the A.D. and therefore the easiest to calculate from the tabulated data. This percent error, also called the percent deviation of the mean, or the percent A.D., is equal to the A.D. divided by the arithmetic mean of the measured values and multiplied by 100 to give the result in terms of a percent as desired. That is

$$\%\ \text{A.D.} = \frac{\text{A.D.}}{M} \times 100\%$$

where M is the arithmetic mean. This is the quantity usually considered in judging the accuracy of a series of measurements. In the example of the range of the spring gun, the percent A.D. is

$$\frac{0.9}{133.9} \times 100\% = 0.67\%$$

If the true or accepted value of a quantity is known, the actual error can be calculated as the difference between the result obtained from the experiment (the mean value M of the measurements) and the true value M_t. The relative error is then the ratio of the error to the true value, and the percent error is this ratio times 100%. Thus,

$$\%\ \text{error} = \frac{M - M_t}{M_t} \times 100\%$$

For example, suppose the velocity of sound in dry air at 0°C is measured and found to be 333.1 m/s, while the accepted value is 331.4 m/s. The error is 1.7 m/s. The relative error is 1.7/331.4 or 0.005. The percent error is

$$\frac{1.7}{331.4} \times 100\% = 0.5\%$$

There is no definite value for the allowable percent error to be expected in the following experiments. In many cases it is reasonable to expect results within 1%, while in others the error may be 5% or more, depending on the apparatus used. However, all measurements should be made with the greatest care, so as to reduce the error as much as possible.

CALCULATING WITH ERRORS

Whenever an experimental result is used in a calculation, account must be taken of the fact that an error is associated with it. Suppose we have two results x_1 and x_2 with respective standard deviations σ_1 and σ_2. We quote these results as $x_1 \pm \sigma_1$ and $x_2 \pm \sigma_2$ on the basis that the 68% chance that the true values lie within these ranges is good enough.* If the theory of the experiment requires these two quantities to be added, the sum is $x_1 + x_2 \pm \sigma_1 \pm \sigma_2$. It would, of course, be very nice if the error in one measurement was in the opposite direction from and therefore cancelled the error in the other, but this happy event can hardly be assured. It is much safer to assume the worst, that is, that the errors are in the same direction so that the sum becomes $x_1 + x_2 \pm (\sigma_1 + \sigma_2)$. We conclude that to be on the safe side we should add the errors in the individual quantities to obtain the error in the sum. Statistics show, however, that this approach is unduly pessimistic and that, in fact, when standard deviations are being used, the standard deviation σ_s in the sum is the square root of the sum of the squares of the individual standard deviations. Thus,

$$\sigma_s = \sqrt{\sigma_1^2 + \sigma_2^2}$$

The procedure is identical in the case of a subtraction, but care should be taken to note that the percent error can increase tremendously when two quantities of about the same value are to be subtracted one from the other. In such a case, we get $x_1 - x_2 \pm \sqrt{\sigma_1^2 + \sigma_2^2}$, and if x_1 and x_2 are nearly equal, their difference may be smaller than the error. This means that the errors associated with x_1 and x_2 are large enough so that we cannot tell whether the quantities are equal or slightly unequal and hence whether or not a difference actually exists. Consider, for example, two automobiles driving down the highway with one slowly passing the other. The problem is to measure the passing speed, that is, the difference between the speeds of the individual automobiles. The best way to do this (since we are not interested in the individual speeds) is to measure the relative speed directly, but suppose experimental difficulties made this impossible so that the only data obtainable were the speedometer readings in each car. We find that one reads 61 miles per hour and the other 62 miles per hour. Can we conclude that one car is passing the other at the rate of 1 mph? Not really, for automobile speedometers are good to only two significant figures with the last one in doubt. There is therefore an error of about 1 mph in each speedometer, so that their readings should be reported as 61 ± 1 and 62 ± 1 mph respectively. This being the case, the difference must be given as $1 \pm \sqrt{1^2 + 1^2} = 1 \pm 1.4$ mph. From this result alone we cannot tell whether the cars are proceeding side by side or even, if they are not, which is passing the other. Certainly there seems to be a bias in favor of one car's doing the passing, but a reasonably certain conclusion simply cannot be drawn on the basis of the

* For the experiments covered in this manual, the a.d., which is easier to find than σ, will usually be good enough.