QUANTUM MECHANICS

VOLUME I

ALBERT MESSIAH

BY

G. M. TEMMER

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Saclay, France

TRANSLATED FROM THE FRENCH

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OUTLINE

VOLUME I

THE FORMALISM AND ITS INTERPRETATION

SIMPLE SYSTEMS

VOLUME II

SYMMETRIES AND INVARIANCE

METHODS OF APPROXIMATION

ELEMENTS OF RELATIVISTIC QUANTUM MECHANICS

PREFACE

Nowadays, there hardly exists a branch of physics which one can seriously approach without a thorough knowledge of Quantum Mechanics. Its presentation, which is given in this work is, I hope, simple enough to be accessible to the student, and yet sufficiently complete to serve as a reference book for the working physicist.

This book resulted from a course given at the Center of Nuclear Studies at Saclay since 1953. Numerous discussions with students as well as with my colleagues, have helped me considerably in clarifying its presentation. Several people to whom I had transmitted certain parts of the manuscript, have kindly given me their criticism; among them I should like to mention Messrs. Edmond Bauer and Jean Ullmo, to whom I am indebted for interesting remarks concerning the presentation of principles. I am more particularly grateful to Mr. Roger Balian for having critically examined a large portion of the manuscript, and for having suggested to me a large number of improvements. Finally, I wish to thank those of my students who were kind enough to check over the text and the calculations of the various chapters, and to help me with the correction of the proofs.

The problems which occur at the end of each chapter were chosen not only for their educational value, but also to point out certain properties worthy of interest; this may explain the relative difficulty of certain ones among them.

The several works or articles cited as references have the purpose of aiding the reader to complete or round out certain passages. It was out of the question to give a complete bibliography of the various subjects treated here. An entire volume would not have sufficed for that.

October, 1958 ALBERT MESSIAH

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…"Il lui proposa de faire le voyage de Copenhague, et lui en facilita les moyens" (*Candide*)

PART ONE

THE FORMALISM AND ITS INTERPRETATION

CHAPTER I

THE ORIGINS OF QUANTUM THEORY

1. Introduction

According to the *classical doctrine* – generally adopted by physicists until the beginning of the 20th century – one associates with physical systems whose evolution one wishes to describe, a certain number of quantities or dynamical variables; each of these variables possesses at each instant a *well-defined value*, and the specification of this set of values defines the dynamical state of the system at that instant. One further postulates that the evolution in time of the physical system is entirely determined if one knows its state at a given initial instant. Mathematically this fundamental axiom is expressed more precisely by the fact that the dynamical variables satisfy a system of differential equations of the first order, as a function of time. The program of Classical Theoretical Physics thus consists in enumerating the dynamical variables of the system under study, and then in discovering the equations of motion which predict its evolution in accord with experimental observation.

From the formulation of Rational Mechanics by Newton until the end of the 19th century, this program was carried out with considerable success, each new experimental discovery being carried over to the theoretical plane either by introducing new variables and new equations, or by modifying the old equations, thereby allowing the newly observed phenomenon to be incorporated into the general scheme. During that entire period no experimental fact, no discovery led to any doubt concerning the soundness of the program itself. On the contrary, Classical Physics constantly progressed toward greater simplicity and greater unity. This happy evolution continued until about 1900; subsequently, as our knowledge of phenomena on the microscopic scale [1]) becomes more precise, Classical Theory runs

[1]) It is important to define the terms "microscopic" and "macroscopic" of which we shall make frequent use throughout this book. We define the "microscopic" scale as the one of atomic or subatomic phenomena, where the lengths which enter into consideration are at most of the order of several ångstroms (1 Å = 10^{-8} cm). The "macroscopic" scale is the one of phenomena observable with the naked eye or with the ordinary microscope, i.e. a resolution of the order of one micron (10^{-4} cm) at best.

into more and more difficulties and contradictions. It rapidly becomes evident that phenomena on the atomic and subatomic scale do not fit into the framework of classical doctrine itself, and that their explanation must be based upon entirely new principles. The discovery of these new principles will occur in stages, at the expense of numerous groping attempts; only around 1925, with the founding of Quantum Mechanics, will we have at our disposal a coherent theory of microscopic phenomena. The origins of this theory constitute the subject of the present chapter.

After sketching an overall picture of Classical Theoretical Physics, we shall discuss the main phenomena which justify the abandonment of the classical ideas. The phenomena are supposed familiar to the reader [1]); we shall therefore merely recall their essential features, emphasizing above all the points of contradiction with Classical Theory. The end of the chapter is devoted to a brief discussion of the first attempts at explaining these phenomena, known as the Old Quantum Theory.

I. THE END OF THE CLASSICAL PERIOD

2. Classical Theoretical Physics

At the end of the classical period, the various branches of physics are integrated in a general and coherent theoretical construct whose main features are as follows. In the universe, one distinguishes two categories of objects, *matter* and *radiation*. Matter is made up of perfectly localizable corpuscles subject to Newton's laws of Rational Mechanics; the state of each corpuscle is defined at any instant by its position and its velocity (or its momentum), that is six dynamical variables in all. Radiation obeys Maxwell's laws of electromagnetism; its dynamical variables – infinite in number – are the components of the electric and magnetic fields at each point of space. In contrast to matter, it is not possible to split radiation into corpuscles which can be localized in space and maintain this localized character during their evolution in the course of time; quite to the contrary, it exhibits a wave-like behavior which manifests itself particularly in the well-known phenomena of interference and diffraction.

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[1]) One may find a detailed discussion of these phenomena in the works dealing with Atomic Physics, for instance: M. Born, *Atomic Physics*, 6th ed. (Blackie, Glasgow, 1957).

The *corpuscular theory of matter* continues to develop during the course of the 19th century. While limited at first to the mechanics of heavenly bodies and of solid bodies of macroscopic dimensions, it emerges more and more as the basic theory governing the evolution of matter on the microscopic scale to the extent that the atomic hypothesis, proposed by the chemists, is confirmed. Without being able to verify this hypothesis directly by isolating the molecules and studying their mutual interactions, one can justify it indirectly by showing that the macroscopic properties of material bodies derive from the laws of motion of the molecules of which they are composed. Mathematically, we are dealing with a very complex problem. Under this hypothesis, in fact, macroscopic quantities appear as the mean values of certain dynamical variables of a system having a very large number of degrees of freedom [1]); there is no hope of solving the equations of evolution of such a system exactly, and one must have recourse to statistical methods of investigation. Thus a new discipline originated and developed, Statistical Mechanics, whose results, particularly in the study of gases (Kinetic Theory of Gases) and in Thermodynamics (Statistical Thermodynamics) enable us to verify qualitatively, and within the limits set by the possibilities of calculation, quantitatively, the foundation of a corpuscular theory of matter [2]).

At the same time, the *wave theory of radiation* becomes solidly established. In the field of optics, the old controversy on the wave nature or corpuscle nature of light is cut short in the first half of the 19th century, when decisive progress in the handling of problems of wave propagation (Fresnel) permits the exploration of all the consequences of the wave hypothesis. All the known light phenomena, including geometrical optics can now be based on this hypothesis. Meanwhile, the study of electric and magnetic phenomena develops rapidly. The decisive step forward is taken by Maxwell when he establishes, in 1855, the fundamental electromagnetic equations.

[1]) We recall that the number N of molecules per mole (Avogadro's number) is $N = 6.02 \times 10^{23}$. The first precise determination of N, due to Loschmidt (1865), was based on the kinetic theory of gases.

[2]) It is well to note that in all reasoning of Statistical Mechanics, there underlies a hypothesis of a statistical nature, the hypothesis of molecular chaos, from which one cannot escape without renouncing the statistical method itself. Although this hypothesis seems intuitively correct, its rigorous justification (ergodic theorem) turned out to be particularly delicate and is still the subject of controversy.