Applications of Optical Fourier Transforms

Edited by *HENRY STARK*

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DEPARTMENT OF ELECTRICAL, COMPUTER, AND SYSTEMS ENGINEERING
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List of Contributors

Numbers in parentheses indicate the pages on which the authors' contributions begin.

SILVERIO P. ALMEIDA (*41*), Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

WALLACE L. ANDERSON (*89*), Electrical Engineering Department, University of Houston, Houston, Texas 77004

F. M. M. AYUB[1] (*289*), Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, New York 12181

H. BARTELT (*499*), Physikalisches Institut der Universität Erlangen-Nürnberg, Federal Republic of Germany

S. K. CASE (*499*), Department of Electrical Engineering, University of Minnesota, Minneapolis, Minnesota 55455

P. S. CHEATHAM (*253*), Pacific-Sierra Research Corporation, Santa Monica, California 90404

P. DAS (*289*), Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, New York 12181

R. HAUCK (*499*), FB7, Universität Essen GHS, Essen, Federal Republic of Germany

GUY INDEBETOUW (*41*), Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

MARVIN KING (*209*), Riverside Research Institute, New York, New York 10023

SING H. LEE (*131*), Department of Electrical Engineering and Computer Sciences, University of California, San Diego, La Jolla, California 92093

[1] Present address: Advanced Technology Center, Bendix Corporation, Columbia, Maryland 21045.

JAMES R. LEGER[2] (*131*), Department of Electrical Engineering and Computer Sciences, University of California, San Diego, La Jolla, California 92093

ROBERT O'TOOLE (*465*), Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, New York 12181

WILLIAM T. RHODES (*333*), School of Electrical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332

BAHAA E. A. SALEH (*431*), Department of Electrical and Computer Engineering, University of Wisconsin, Madison, Wisconsin 53706

A. A. SAWCHUK (*371*), Image Processing Institute, Department of Electrical Engineering, University of Southern California, Los Angeles, California 90007

R. R. SHANNON (*253*), Optical Sciences Center, University of Arizona, Tucson, Arizona 85721

HENRY STARK (*1*, *465*), Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, New York 12181

T. C. STRAND (*371*), Image Processing Institute, Department of Electrical Engineering, University of Southern California, Los Angeles, California 90007

[2] Present address: 3M Company, St. Paul, Minnesota 55144.

Preface

Several years ago I was attending an image processing conference near Monterey and, being in the enviable position of not having to give a paper, I could both concentrate on what was being said and enjoy the magnificent Monterey scenery without feeling the trace of anxiety that normally goes with giving a paper. In those days there was quite a debate raging on whether image processing should be done with optics or computers. One evening, while thinking about the papers that I had heard, it occurred to me that if the opticists had, so to speak, an ace in the hole it was their ability to generate a high resolution, two-dimensional Fourier transform using their own version of an FFT computer, namely a lens. Moreover, this "computer" could do it at the speed of light, at extremely high resolution, and do it as quickly for a 10^6-pixel image as for a 100-pixel one. It was then that I became interested in producing a book that would stress the applications of optical Fourier transforms. In retrospect, perhaps the Monterey scenery and the fine California wine combined to generate uncharacteristic euphoria and ambition in me. Frankly, I don't know. But I did raise the issue with colleagues and they thought such a book a fine idea. Academic Press showed interest and, perhaps even before the effect of the last bottle of the Cabernet Sauvignon wore off, an agreement was signed and there I was, an editor.

Leaving myself out, I do feel that these pages are the work of an unusually talented group of co-authors who are among the most active contributors to the field. Readers will find in this book some of the extraordinary achievements of Fourier optics. Not all, perhaps not even all of the most important applications are to be found here. Unfortunately, time and circumstances did not permit the inclusion of applications of Fourier optics to medical imaging, diffraction-limited imaging through a turbulent atmosphere, and other timely topics. Nevertheless, what is included is impressive and furnishes convincing evidence that Fourier optical systems, in their broadest sense, and expecially when backed up by computers, constitute an elegant and useful technology.

Now I would like to go over briefly the contents and organization of this book. In Chapter 1: *Theory and Measurement of the Optical Fourier Transform*, I review the Fourier transform property of a lens and discuss some of the care that must be taken if the light observed in the focal plane of a lens is indeed to represent the spectrum of the input. Optical power spectral estimation is discussed along with conventional and some unconventional "windowing" considerations required for stable and accurate spectral estimates. In Chapter 2: *Pattern Recognition via Complex Spatial Filtering*, S. P. Almeida and G. Indebetouw discuss the theory and applications of complex spatial filters: how they can be made scale and rotation invariant; and how they can be applied, in signal detection, character recognition, water pollution monitoring, and other pattern recognition problems. Over 280 references are given.

The theme of pattern recognition is continued by W. L. Anderson in Chapter 3: *Particle Identification and Counting by Fourier-Optical Pattern Recognition*. After discussing the theoretical foundations of the method, Anderson furnishes a careful computation of the statistical characteristics of the Fourier irradiance pattern and shows how the inverse scattering problem, that is, obtaining the particle distribution from the spectrum, can be solved with the help of the celebrated Gauss–Markov Theorem. Hybrid methods for realizing the technique are thoroughly reviewed.

J. R. Leger and S. H. Lee continue the discussion of pattern recognition as well as the more general signal processing problem in Chapter 4: *Signal Processing Using Hybrid Systems*. The emphasis is on hybrid systems, that is, those eclectic systems that combine the best of optics, analog electronics, and digital computers to solve problems. Included here is an attempt to answer the question, "Which parts of an information processing computation are best done by optical systems, and which parts should be done by electronic ones?" Also included here are two topics of particular interest in hybrid system design: sampling theory with attendant aliasing considerations, and optical–electrical interconnections. Examples of working hybrid systems are given to illustrate the design techniques unique to this approach to signal processing.

Perhaps one of the earliest and most successful applications of Fourier optics was to the problem of radar signal processing. M. King, who is associated with a laboratory that pioneered in the use of coherent optical systems to extract range-Doppler information from radar echoes, reviews that activity in Chapter 5: *Fourier Optics and Radar Signal Processing*. Included here is a discussion of pulse-Doppler and chirp signals and of how these can be processed optically, optical processing for synthetic aperture radar, and recent progress in the field.

Despite progress with other media, the single most important medium for storing signals in optics is still photographic film. In Chapter 6: *Application of Optical Power Spectra to Photographic Image Measurement*, R. R. Shannon and P. S. Cheatham discuss their experimental investigation that involves photographic image evaluation by measuring the optical power spectra of signal and noise. Their work shows that the signal-to-noise power spectrum is a useful and reliable measure of the information content of photographic film and is useful in image quality determinations.

A newer technology, whose integration with Fourier optics can enable all kinds of useful operations such as light modulation, spectrum analysis, correlation and convolution, and others, is the phenomenon of ultrasound with surface confinement, or SAW (for *s*urface *a*coustic *w*aves). A thorough discussion of SAW devices that includes their underlying physics, modes of operations, and their integration and application in Fourier optics is furnished by P. Das and F. M. M. Ayub in Chapter 7: *Fourier Optics and SAW Devices*.

The signal processing systems described in Chapters 1–7 share the property of space invariance. Most signal processing systems are in fact, by design, space invariant. But not all systems share this property; for example, space invariance is not a property of ultrawide-angle lens systems or lenses exhibiting coma. For such systems, a lateral shift anywhere in the input plane does not result in a corresponding lateral shift anywhere in the output plane. In Chapter 8: *Space-Variant Optical Systems and Processing*, William T. Rhodes discusses such systems and shows how space-variant systems can be designed to carry out useful functions, for example, the realization of an operation such as the superposition integral.

Broadening the arena of Fourier optics still further, A. A. Sawchuk and T. C. Strand, in Chapter 9: *Fourier Optics in Nonlinear Signal Processing*, discuss how nonlinear systems can usefully be applied in Fourier optics. How do Fourier transforms enter into nonlinear systems? In some systems the Fourier transform is integral to the operation of the nonlinearity (for example, halftone techniques). In other systems, the Fourier transform is an important adjunct in a composite nonlinear system (for example, a system with input–output nonlinearities suffering from intermodulation noise). The authors describe a number of nonlinear systems and components (for example, the variable grating mode device) and show how they can be used in many applications, including digital logic.

Is there a link between optical Fourier transforms and the human visual system? This is the subject of Chapter 10: *Optical Information Processing and the Human Visual System*, by B. E. A. Saleh. The author discusses how Fourier methods are used to study the transmission of spatial information through the human visual system and reviews how coherent techniques are used in vision research, for example, the use of speckle to measure the dioptics of the eye and the application of spatial filtering techniques to pictures used in psychophysical studies.

In Chapter 11: *Statistical Pattern Recognition Using Optical Fourier Transforms*, R. K. O'Toole and I again pick up the theme of pattern recognition first introduced in Chapter 2. We consider two well-known pattern recognition problems that have received wide attention in the digital signal processing community. We concentrate on two questions: (i) can features based only on the optical Fourier transforms lead to a high probability of correct classification? and (ii) can a hybrid, that is, optical–digital, pattern recognizer, using only the Fourier irradiance spectrum, do as well as sophisticated all-digital routines based on co-occurrence gray scale statistics? As readers will find out, the answer to both questions is yes.

A discussion of Fourier optics presupposes coherent illumination, but does it need to? This is what H. Bartelt, S. K. Case, and R. Hauck (BCH) set out to explore in the last chapter: *Incoherent Optical Processing*. This chapter, like the ones by Rhodes and by Sawchuk and Strand (RSS) expands the realm of Fourier optics. But whereas RSS expanded the class of spatial systems, BCH expand in the time domain to include incoherent illumination. Starting with a telecentric optical system illuminated by an arbitrary source, they furnish a general theoretical model of the propagation of light fields from input to output. The model is then used to compute the fields when the source is restricted in size or bandwidth. They show that spatially incoherent optical systems can perform convolutions linear in intensity, that spatially coherent but temporally incoherent systems can be used in scale-multiplexed matched filtering (useful when the shape of the object is known but the size is not), and that such a system can be used in numerous other applications such as pseudocolor encoding and signal-to-noise improvement.

HENRY STARK
Troy, New York

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In my role as editor of this book, I wish first of all to thank my co-authors who did such an excellent job with a minimum of exhortation. So many people and agencies are responsible for creating a book such as this that it is really impossible to thank all of them individually. However, I do wish to single out individuals and organizations who played a vital, continuing role. Special thanks are due to the following: Rosan Laviolette for her typing and cooperation; the administration of Rensselaer Polytechnic Institute for permission to take the time to complete this project; the various funding agencies, especially the National Science Foundation, the Army Research Office, the Air Force Office of Scientific Research, and the Office of Naval Research for their continued support; the highly competent staff at Academic Press; and finally my wife Alice whose patience and support were much appreciated.

Contents

Chapter 3 **Particle Identification and Counting by Fourier-Optical Pattern Recognition**

WALLACE L. ANDERSON

Chapter 4 **Signal Processing Using Hybrid Systems**

JAMES R. LEGER AND SING H. LEE

Chapter 5 **Fourier Optics and Radar Signal Processing**

MARVIN KING

Chapter 6 **Application of Optical Power Spectra to Photographic Image Measurement**

R. R. SHANNON AND P. S. CHEATHAM

Chapter 7 **Fourier Optics and SAW Devices**

P. DAS AND F. M. M. AYUB

Chapter 8 **Space-Variant Optical Systems and Processing**

WILLIAM T. RHODES

Chapter 9 **Fourier Optics in Nonlinear Signal Processing**

A. A. SAWCHUK AND T. C. STRAND

Chapter 10 **Optical Information Processing and the Human Visual System**

BAHAA E. A. SALEH

Chapter 11 **Statistical Pattern Recognition Using Optical Fourier Transform Features**

HENRY STARK AND ROBERT O'TOOLE

Chapter 12 **Incoherent Optical Processing**

H. BARTELT, S. K. CASE, AND R. HAUCK

Chapter 1

Theory and Measurement of the Optical Fourier Transform

HENRY STARK

DEPARTMENT OF ELECTRICAL, COMPUTER,
AND SYSTEMS ENGINEERING
RENSSELAER POLYTECHNIC INSTITUTE
TROY, NEW YORK

In a book devoted to applications of optical Fourier transforms (FTs), it somehow seems fitting that we discuss the celebrated Fourier transform property (FTP) of a lens in the first chapter. The material is covered in several places (see, for example, Refs. [1–3]), so that we shall not give a very detailed and rigorous treatment here. As we shall see, the FTP of a lens is easily established from the diffraction integral that describes the propagation of monochromatic light in free space. The diffraction integral is the single most important object in the study of Fourier optics, and virtually all optical phenomena can be explained mathematically in terms of it. Surprisingly the diffraction integral can be easily derived from Fourier transform considerations alone plus certain notions such as plane waves and complex amplitudes.

The FTP of a lens is usually derived assuming an idealized lens between object and back focal plane (BFP) and—although sometimes not stated—the validity of the stationary phase approximation for the Fourier transform configuration. The diffraction integral is then applied twice—from the object to the lens and from the lens to the BFP. When vignetting† is ignored, the light amplitude in the BFP is then shown to be, up to a constant and a quadratic phase factor, the classical two-dimensional Fourier transform (2-DFT) of the transmittance of the diffracting object. The validity of this

† The phenomenon of vignetting is discussed in Section 1.3.

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result depends on the object being illuminated by a plane wave, although it is more generally true that—subject to the constraints listed above—there exists a Fourier transform relation between the light amplitude adjacent to the object and the light amplitude in the BFP. In the following few sections we shall derive the FTP of a lens and see under what conditions a true Fourier transform is obtained.

1.1 PLANE WAVES

In scalar diffraction theory, we are concerned with solutions to the scalar wave equation

$$\nabla^2 u(\mathbf{r}, t) = \frac{1}{c^2} \frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2}, \tag{1.1-1}$$

where ∇^2 is the Laplacian operator, $u(\mathbf{r}, t)$ is the scalar field (or a single component of a vector field), $\mathbf{r}$ is the position vector, t is time, and c is the speed of light. For monochromatic illumination, a solution of Eq. (1.1-1) is

$$u(\mathbf{r}, t) = a(\mathbf{r}) \cos[2\pi\nu t + \phi(\mathbf{r})], \tag{1.1-2}$$

where $a(\mathbf{r})$ is the time-independent amplitude, ν is the frequency in hertz (Hz), and $\phi(\mathbf{r})$ is a time-independent phase. It is customary to write Eq. (1.1-2) as

$$u(\mathbf{r}, t) = \mathrm{Re}[a(\mathbf{r})e^{-j\phi(\mathbf{r})}e^{-j2\pi\nu t}] \equiv \mathrm{Re}[U(\mathbf{r})e^{-j2\pi\nu t}], \tag{1.1-3}$$

where $U(\mathbf{r}) \equiv a(\mathbf{r}) \exp[-j\phi(\mathbf{r})]$, is commonly called the complex amplitude and satisfies the Helmholtz equation

$$(\nabla^2 + k^2)U(\mathbf{r}) = 0. \tag{1.1-4}$$

The constant k in Eq. (1.1-4) is called the wave number and is related to the free-space wavelength λ according to

$$k = 2\pi/\lambda = 2\pi\nu/c. \tag{1.1-5}$$

All linear operations involving $u(\mathbf{r}, t)$ can be done using only $U(\mathbf{r})$; real solutions, i.e., those whose form is as in Eq. (1.1-2) or linear combinations thereof, can always be obtained from $U(\mathbf{r})$ by the formalism of Eq. (1.1-3).

From now on, unless otherwise stated, we shall assume that all representations of the field $u(\mathbf{r}, t)$ will be through the artifice of the complex amplitude.

A. Propagation of Plane Waves

A unit amplitude plane with wave vector $\mathbf{k}$ is described by a complex amplitude

$$B(\mathbf{r}) = e^{j\mathbf{k}\cdot\mathbf{r}} \tag{1.1-6}$$

and corresponds to a real wave of the form

$$b(\mathbf{r}, t) = \cos(2\pi\nu t - \mathbf{k}\cdot\mathbf{r}). \qquad (1.1\text{-}7)$$

In a linear homogeneous medium the wave vector has magnitude $|\mathbf{k}| = k$, and its direction is in the direction of propagation of the wave. The complex amplitude is constant over points $\mathbf{r}$ such that $\mathbf{k}\cdot\mathbf{r} = c_0$, a constant. But $\mathbf{k}\cdot\mathbf{r} = c_0$ is the equation of a plane—hence the term plane wave. The surface of constant phase is normal to k, the direction of propagation. Letting $\mathbf{k} = k_x\mathbf{i}_x + k_y\mathbf{i}_y + k_z\mathbf{i}_z$, where $\mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z$ are unit vectors parallel to the Cartesian coordinate axes, we can define a set of direction cosines α, β, γ and a set of angles $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ according to

$$\begin{aligned} \alpha &\equiv \cos\hat{\alpha} = (\mathbf{k}\cdot\mathbf{i}_x)/k = k_x/k, \\ \beta &\equiv \cos\hat{\beta} = (\mathbf{k}\cdot\mathbf{i}_y)/k = k_y/k, \\ \gamma &\equiv \cos\hat{\gamma} = (\mathbf{k}\cdot\mathbf{i}_z)/k = k_z/k. \end{aligned} \qquad (1.1\text{-}8)$$

We note from Fig. 1.1-1 that $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ are the angles between $\mathbf{k}$ and the Cartesian unit vectors $\mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z$, respectively. Since $\mathbf{k}\cdot\mathbf{k}/k^2 = 1$, $\alpha^2 + \beta^2 + \gamma^2 = 1$ and we can write Eq. (1.1-6) as

$$B(x, y, z) = e^{j(k_x x + k_y y + k_z z)} = e^{jk(\alpha x + \beta y)} e^{jk(1-\alpha^2-\beta^2)^{1/2} z}. \qquad (1.1\text{-}9)$$

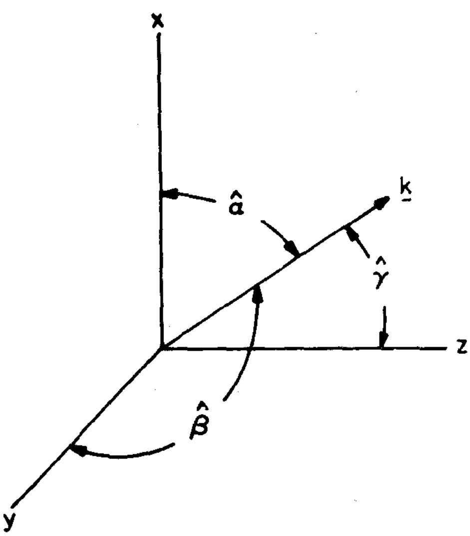

FIG. 1.1-1. The angles $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$.