

Mathematics

WITH APPLICATIONS
IN MANAGEMENT
AND ECONOMICS

BOWEN
PRICHETT
SABER

SIXTH
EDITION



Mathematics

with Applications in Management and Economics

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TO:

Steven and Pamela

Jill, Reid, Trevor, and Glenn

Carol, John, Stephen, and Nicholas

Preface

This text was written intending to explain the value of mathematics as a tool in solving actual problems. Earl Bowen's objectives in the First Edition were to present mathematics at a level appropriate to student preparation and directed specifically toward applications in management and economics. This Sixth Edition is motivated by those same objectives.

The systematic presentation in each chapter—explanation, example, answered exercise—which students and teachers have enthusiastically approved through five previous editions is maintained and amplified in this edition. This approach provides a text easily adapted to self-study.

Many of the algebraic details omitted in most texts at this level have been included within the text to help students learn or recall the arithmetic and algebra needed to solve problems. In addition, to limit the prerequisite for study of the text to no more than one year of secondary school algebra, we have included appendixes on the elements of mathematics for study or reference.

Our main concern in preparing this revision has been: *Can students read and understand the text?* The following features have been added or retained primarily with this objective in mind.

Examples. Several hundred worked examples are included in this text to demonstrate applications and techniques in problem solving.

Exercises. Each section contains answered exercises to allow students to reconfirm their understanding of the material while reading the text.

Applications. A greater number and selection of applications in management and economics than is customary is included in each chapter. Many chapters contain special application sections which are optional.

Calculator applications. Calculator applications are presented where appropriate. Examples and exercises using both standard features and pre-programmed functions are explained in detail.

Computer applications. Computer programs in both BASIC and PASCAL

and the use of computer packages such as LINDO and Minitab are included where appropriate. Computer exercises have also been added. A disc with all the programs used in the text is available for the Apple IIe and the IBM PC.

Theorems and definitions. All theorems and definitions are boxed and highlighted to emphasize their importance. Care has been taken to state theorems and definitions simply but accurately.

Problems. The text contains over 1,500 numbered problems, with answers, and over 750 additional review problems that can serve as a basis for examination and lecture purposes. Solutions to the review problems, worked out in detail, are included in the Instructor's Manual. In addition, there is a supplemental Student's Manual which contains worked-out solutions to selected problems.

Overhead transparencies. Transparency masters for the text material are available for the instructor. These allow the careful planning and preparation of each class session.

Chapter 1 begins with a review of distance, slope, and straight lines with applications in cost-output analysis, break-even analysis, and linear demand. Chapter 2 starts with solutions of 2 by 2 linear systems by the Elimination Procedure together with applications in supply and demand. This is generalized to 3 by 3 systems before discussing linear inequalities. Chapter 3 is an introduction to linear programming with special attention paid to graphical analysis for 2-variable problems and formulation of general problems. Chapter 4 discusses vectors, matrices, and the summation operation. Here, the Gauss-Jordan Method for solution of an m by n system of linear equations is developed simply as a generalization of the Elimination Procedure with the aid of matrix techniques. Chapter 5 presents the Simplex Method solution for linear programming problems with \leq constraints. Here, the concepts of sensitivity analysis and shadow prices are also introduced. Chapter 6 demonstrates the computer solution of linear programming problems and a complete summary of the entire Simplex Method in an eye-catching flowchart. Chapter 7 is a short treatment of exponential and logarithmic functions, followed by a fairly detailed presentation of the Mathematics of Finance in Chapter 8. At this point, we introduce elementary Probability and Statistics in Chapter 9 and illustrate the use of computer packages. The next four chapters give a detailed presentation of differential and integral calculus. Differential calculus is thoroughly developed in Chapters 10 through 12 with special emphasis on optimization problems and curve sketching for both one- and two-independent variables. Chapter 13 presents integral calculus including area applications, numerical integration, and differential equations. Chapter 14 introduces probability in the continuous case and

discusses means, expected values, variance and standard deviation, and the normal probability distribution. Finally, there are three appendixes to help students review the fundamentals of sets, algebra, and graphing that provide the background material for the text.

If the entire text including the appendixes is to be completed, there is sufficient material for a three-semester sequence of courses. However, the book is structured so that it is adaptable to a variety of courses, from one quarter to three semesters in duration by making parts of chapters, whole chapters, and groups independent. This structure permits omissions to be made without loss of continuity or prerequisite topics. A few possible plans are included in the following table:

Course Title	Time Required	Material to Be Covered
Finite Mathematics	3–4 semester hours	Chapters 1–8 (9 optional)
Calculus with Applications	3–4 semester hours	Chapters 1, 7, 8, 10–13
Basic Mathematics for Business Students	9–12 semester hours	Entire book
Introduction to Linear Programming	2–3 semester hours	Chapters 1–6

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Earl K. Bowen
Gordon D. Prichett
John C. Saber

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