Electric Motor Handbook

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McGRAW-HILL Book Company (UK) Limited

London · New York · St Louis · San Francisco · Auckland · Beirut
Bogotá · Düsseldorf · Johannesburg · Lisbon · Lucerne · Madrid
Mexico · Montreal · New Delhi · Panama · Paris · San Juan
São Paulo · Singapore · Sydney · Tokyo · Toronto

Published by
McGRAW-HILL Book Company (UK) Limited
MAIDENHEAD · BERKSHIRE · ENGLAND

British Library Cataloguing in Publication Data

Electric motor handbook.
1. Electric motors – Design and construction
2. Electric motors – Maintenance and repair
I. Werninck, E H
621.46'2 TK2511 77–30474

ISBN 0–07–084488–7

12345 JWA 80798

PRINTED AND BOUND IN GREAT BRITAIN

Preface

Men have said that they climb mountains, or cross seas and continents because they are there, presenting a challenge. The motivation to edit a Handbook of the nature of this one is that it is *not* there. However, editing a handbook cannot be said to require the same personal attributes and the only courage required is to publish and face the consequences.

In the very early days of my career, the then current edition of Knowlton's *Standard Handbook for Electrical Engineers* was of considerable help to me. General reference books, though uniquely useful particularly to the young and still inexperienced electrical engineer, cannot treat specific subjects in depth, or from all aspects, and so later it was a more specialized Handbook, *Truxall's Control Engineer's Handbook*, which gave me an insight into control engineering and the special requirements for electric motors in this field. It also helped me to understand better the designer's need to express performance mathematically, and the usefulness of transfer functions. These two examples from my personal experience illustrate the evolution of technical handbooks in the last three decades and the development of the engineering sciences which resulted in an information explosion and successively more specialized generations of books.

Thus, the second generation of handbooks dealt with specific subjects, such as electronics, process-control and instrumentation, radio, maintenance engineering, and so on. Another specialization which developed was aimed at engineers working in specific branches of their profession. This third generation was more selective and catered for industrial electronic control, industrial power systems, radio engineering, as well as many other subjects within the more general generic classification.

This *Handbook* I suggest is a member of a fourth generation, in that it deals with a specific type of electrical machine and is directed at those engineers who have to apply, use, or maintain it. Aspects of design are only covered to the extent that they may give a better understanding of characteristics, particularly under abnormal conditions.

Optimization of electric motor applications requires not only the matching of load, motor, and control gear, but such considerations as capital and operational costs, the need to increase productivity as well as the interaction with the environment. Thus it was considered essential to include gearing, couplings, brakes, and clutches, as well as a short section on noise. Many features of electrical equipment are easily changed in theory to meet particular system requirements exactly. In practice, economic production of such equipment demands considerable standardization which, however, still leaves many options. The special design approach can be thought of as 'synthesis', whereas the examination of the considerable number of systems which can be made up from standard motors, transmission, and control gear require analysis and decision-making skills.

Many engineers still underestimate the wealth of information and expert assistance which can be theirs for the asking, and consider analysis and evaluation of the alternatives offered a far inferior task to designing and developing at great cost their own special solution. Let me hasten to add that this generalization is, like many others, confirmed by many excellent exceptions.

In the early days of handbooks they were often the only sources of technical information for engineers remote from libraries or immediate advice. Today, it is often more the wealth of information and conflicting advice which can make decisions difficult. So many developments and solutions are due to particular circumstances or even industries with special conditions. It would be hopeless to try to cover the millions of permutations and combinations of electric motor types, sizes, and applications, but it is at this stage that I wish to thank all my contributors for their efforts. If they feel that my editing has at times been heavy, I ask their indulgence on the grounds that I have made an earnest attempt to bring together and summarize as much relevant information as possible without duplication.

The complexities of practical engineering problems are such that relatively few decisions can be supported by calculations based on fundamental principles. Similar applications are generally studied, scaled, modified and generally used as criteria for such performance requirements as reliability, quiet running and transient overloads. An attempt to define a 'good engineer' could easily take up the next few hundred pages and unleash fierce arguments. It can be stated with conviction and little fear of contradiction that an innovatory approach tempered by experience is the most likely to produce a satisfactory solution.

Every effort has been made to acknowledge the sources from which I have drawn and where diagrams have been modified, or scales changed, the object has been to unify and standardize the information. Here it is appropriate also to thank my wife for so patiently retyping much of the manuscript time and time again.

Acknowledgements

The editor, the contributors and the publishers are indebted to a number of individuals and establishments for help in the preparation of this handbook. Certain illustrations are acknowledged individually, and valuable information has been made available by Demag of Hamburg, Eberhard Bauer of Esslingen-Neckar, Elektra-Faurndau, and *Philips Technical Review*. The tables from British Standards are included by permission of BSI, 2 Park Street, London W1A 2BS from whom complete copies of the Standards can be obtained.

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1 Units, Mathematics, and Formulae

1.1 Systems of units

The introduction of a metric system for practical measurements is said to have been sponsored by the French politician, Talleyrand, at the beginning of the nineteenth century on the advice of the scientists of the day. The system took its name from the proposed unit of length, the 'metre', the magnitude of which was to be one ten millionth of the distance between the North Pole and the equator measured along a meridian passing through Paris.

In 1873 the British Association for the Advancement of Science adopted the centimetre and the gramme as the basic units of measurement. Engineers used and continued to use units based on the pound and the foot, that is the FPS system, while scientists worked in the CGS system. In 1875 the Conference Générale des Poids et Mesures (CGPM) was established as the International Authority on the metric system, and with the Bureaux International des Poids et Mesures (BIPM) set up the basic standards at Sèvres. The standard metre was defined by the distance between two marks on a standard bar and the kilogramme in terms of a prototype mass. These units were of a more convenient practical magnitude than the centimetre and the gramme, and were adopted as two of the fundamental units of yet another system, namely the MKS system of units. Other national laboratories such as the National Physical Laboratory in London (NPL) and the Physikalische Technische Bundesanstalt in Berlin which had collaborated in the setting up of standards became custodians of their own secondary standards.

From the basic dimensions of mass, length, and time other units were derived and some of these required the use of the 'constants' such as gravitational acceleration (g), permeability (μ_0), and permittivity of free space (ε_0) to link them into a compatible, practical system. To take account of these complications and 'rationalize' units the International Electrotechnical Commission (IEC) in 1950 accepted a suggestion made some 50 years earlier by the Italian Professor Giorgi. The MKSA system designated the ampere as a fourth basic unit and was named in his honour the 'Giorgi' system. Units for this system were defined as 'international' units. To simplify the system of practical units even more the SI Units now include absolute temperature (kelvin), luminous intensity (candela), amount of substance (mole).

Before these now universally accepted units were adopted, science and engineering students had to familiarize themselves with electrostatic, electromagnetic, and fundamental practical units. Some of these units, such as the slug and the poundal, which were introduced for mechanical calculations in the FPS system, were occasionally used for mass and force respectively. This confusion has now disappeared for at its tenth meeting in 1954 the CGPM adopted a rationalized and coherent system of metric units which, in 1960, was given the title 'Système International d'Unités' and is now universally designated as 'SI'.

Advances in science have led to the metre being defined more fundamentally, accurately, and conveniently as 1 650 763·73 times the wavelength in vacuum of the orange light of krypton 86 on an interference comparator. The convenience here is one reserved for scientists in the national laboratories who, with the necessary equipment can calibrate and certify sub- or secondary standards and measuring instruments. When linear dimensions must be maintained with tolerances of only a few hundredths of a millimetre then temperature and surface finish become significant and must be specified to ensure agreement between supplier and buyer.

The number of basic quantities on which the SI system has been based has recently been increased and though not all are directly applicable to electrical machines they have been included here.

Table 1.1 Basic SI units

length	metre	m
mass	kilogramme	kg
time	second	s
current	ampere	A
light	candela	cd
temperature	kelvin	K
substance	mole	mol

The *metre* as a unit of length has been defined above.

The *kilogramme* will continue to be defined by the mass of the platinum-iridium prototype in Sevres until scientists find a satisfactory more fundamental and readily reproducible measure of mass.

Table 1.2 Derived SI units

Physical quantity	Symbol	Unit	Definition	Definition in terms of basic units
Frequency	f	hertz Hz	s^{-1}	s^{-1}
Force	F	newton N	$kgms^{-2}$	$kgms^{-2}$
Work or Energy	W	joule J	Nm	kgm^2s^{-2}
Power	P	watt W	Js^{-1}	kgm^2s^{-3}
Pressure	P	pascal Pa	Nm^{-2}	$kgm^{-1}s^{-2}$
Electric Charge	Q	coulomb C	As	As
Potential difference	V	volt V	JC^{-1}	$kgm^2s^{-3}A^{-1}$
Capacitance	C	farad F	F	$A^2s^4kg^{-1}m^{-2}$
Resistance	R	ohm Ω	VA^{-1}	$kgm^2s^{-3}A^{-2}$
Conductance	G	siemens S		$kg^{-1}m^{-2}s^3A^2$
Flux Density	B	tesla T	N m	$kgs^{-2}A^{-1}$
Magnetic flux	Φ	weber Wb	Tm^2	kgm^2
Inductance	L or M	henry H	Wb/A = Vs/A	kgm

The *second* is the duration of 9 192 631 770 periods of a specified radiation from an atom of caesium-133.
The *ampere* is that constant electric current which, maintained in two infinitely long conductors 1 metre apart causes each to exert a force of 2×10^{-7} N per metre length on the other.
The *kelvin* is the 1/273·16 part of the thermodynamic temperature of the triple point of water.
The *candela* is the unit of luminous intensity.
The *mole* is the amount of substance of a system which contains as many elementary units as there are carbon atoms in 12×10^{-3} kg of carbon-12. Thus a mole contains $6{\cdot}022169 \times 10^{23}$ specified elementary units, such as atoms, molecules, ions, electrons, etc. From this follows that a mole of electrons has a charge of $-9{\cdot}6 \times 10^{4}$ C (coulomb), a quantity also known as the Faraday constant.

From these basic units the other SI units are derived as shown in table 1.2.

1.2 Metric Multiples and Submultiples

Table 1.3

T	Tera	10^{12}	p	Pico	10^{-12}
G	Giga	10^{9}	n	Nano	10^{-9}
M	Mega	10^{6}	μ	Micro	10^{-6}
k	Kilo	10^{3}	m	Milli	10^{-3}
h	Hecto	10^{2}	c	Centi	10^{-2}
da	Deca	10	d	Deci	10^{-1}

NOTE: 10^{18} = Trillion, 10^{12} = Billion (except in USA, France, Spain, and Italy where 1 billion is 10^{9} and 10^{12} is then called a trillion).

Double prefixes, even if more easily understood, may not be used.
For example:

the tonne (t) = 1000 kg = 10^{6} g or 1 Mg and *not* 1 kkg.
Millimicro is to be replaced by *nano-*, and
Micro-micro is *pico-*.

When using decimal multiples of units in formulae care must be taken to ensure that they remain coherent; thus Ohm's law for example is coherent for:

a) volts, amperes, and ohms
b) milli-volts (mV), milli-amperes (mA), and ohms (Ω)
c) kilo-volts (kV), milli-amperes (mA), and mega-ohms (MΩ)

In practice the only safe method of ensuring the correct order of magnitude is to always work in the basic units for which the SI formulae apply.

1.3 Units of Length

Table 1.4

	m	cm	mm	µm	yd	ft	in	µ in
1 m	1	100	1000	1×10^{6}	1·0936	3·2808	39·37	—
1 cm	0·01	1	10	$1\cdot10^{4}$	—	—	0·3937	—
1 mm	0·001	0·1	1	1000	—	—	0·0394	$3\cdot937\times10^{-4}$
1 µm	1×10^{-6}	1×10^{-4}	0·001	1	—	—	—	39·37
1 yd	0·9144	91·44	914·4	—	1	3	36	—
1 ft	0·3048	30·48	304·8	—	$\frac{1}{3}$	1	12	—
1 in	0·0254	2·54	25·4	—	$\frac{1}{36}$	$\frac{1}{12}$	1	1×10^{6}
1 µin	$2\cdot54\times10^{-6}$	$2\cdot54\times10^{-4}$	$2\cdot54\times10^{-3}$	0·0254	—	—	1×10^{-6}	1

1 dm = 10 cm = 0·1 m
1 mil (USA) = 1 thou (UK) = 0·001 in = 0·0254 mm
1 statute mile = 1760 yards = 5280 ft = 1·6093 km
1 (nautical) mile = 6080 ft = 1·8532 km
1 fathom = 6 ft = 1·828 m
1 µm 'micron' is strictly 1 micrometre

1.4 Units of Area

Table 1.5

	m^2	cm^2	mm^2	sq. yd	sq. ft	sq. in
1 m^2	1	1×10^{4}	1×10^{6}	1·196	10·764	—
1 cm^2	1×10^{-4}	1	100	—	—	0·1550
1 mm^2	1×10^{-6}	0·01	1	—	—	0·0016
1 sq. yd	0·8136	8361	$8\cdot36\times10^{5}$	1	9	—
1 sq. ft	0·0929	929·03	$9\cdot29\times10^{4}$	$\frac{1}{9}$	1	144
1 sq. in	$6\cdot45\times10^{-4}$	6·4516	645·16		$\frac{1}{144}$	1

1 circular mil = area of circle 0·001 in diameter $=\pi/4$ sq. mil $=5\cdot076.10^{-4}$ mm^2
1 ha (hectare) = 100 ar = 10 000 m^2 = 2·471 acres
1 acre = 0·4047 ha
1 sq. mile = 640 acres = 2·590 km^2 = 259 ha

1.5 Units of Volume

Table 1.6

	m^3	dm^3 (l)	cm^3 (ml)	imp. gal	imp. pt	cu. yd	cu. ft	cu. in
1 m^3	1	1000	$1\cdot10^{6}$	264·2	—	1·3079	35·31	$6\cdot1\times10^{4}$
1 dm^3 (l)	0·001	1	0·2642	0·2642	2·1136	—	—	61·02
1 cm^3 (ml)	1×10^{-6}	0·001	1	—	—	—	—	0·061
1 gal imp	$4\cdot55\times10^{-3}$	4·546	4546	1	8	$5\cdot91.10^{-3}$	0·165	—
1 pint imp	—	0·5682	—	—	1	—	—	34·67
1 cu. yd	0·7645	764·55	$7\cdot65\times10^{5}$	168·2	—	1	27	46656
1 cu. ft	0·0283	28·317	$2\cdot832\times10^{4}$	6·232	49·856	$\frac{1}{27}$	1	1728
1 cu. in	—	0·0164	16·3871	—	—	—	—	1

1 US gallon = 4 liquid quarts = 8 liquid pints = 3·7856 litres
1 US fluid oz = 8 US fl. drams = 29·57 ml, 1 minim = 0·0616 ml (cm^3)
1 imp. fluid oz = 8 imp. fl. drachms = 28·41 ml, 1 imp. minim = 0·0592 cm^3
1 barrel (petrochem./oil) = 42 gallons = 158·9952 litres
1 bushel (UK) = 4 pecks = 8 gallons = 36·37 litres

1.6 Units of Mass, Weight and Force

The unit of mass in the SI system is the kilogramme (kg) and Newton's law connects this with the unit of force as follows:

> a force of 1 newton acting on a mass of 1 kilogramme produces an acceleration of 1 m/s^2

$$F(\mathrm{N}) = \text{mass (kg)} \times \text{acceleration } (\mathrm{m/s^2})$$

Weight is the special force a body exerts on its support due to gravity. The earth's gravitational field varies from about 9·832 m/s^2 at the poles to 9·780 m/s^2 at the equator and for all but the most exact calculations it is sufficient to substitute 9·81 m/s^2 for gravitational acceleration (g). The confusion between mass and weight which has been mentioned in section 1.1 is recognized, and laws are being passed to make such technical units as the kilopond disappear, at least from legal documents, before the 1980s. The kilogramme force (kgf), or as it is known in some European countries the kilopond (kp), is defined as the force which gives a mass of one kilogramme an acceleration of 9·8066 metres per second per second, but as it does not fit into the coherent SI system must eventually disappear.

The measurement of weight must eventually be only in newtons and may either be made by a spring balance, or by comparison on some form of beam balance. The spring balance, calibrated in newtons, will at balance register the 'apparent' weight not corrected for any variation of the earth's gravitational field, wherever it is used. Beam balances compare masses by comparing their weight, and the true weight in SI units must be calculated from Newton's equation. The ballistic balance which is sometimes used for the measurement of impact compares inertial masses using the earth's gravitational pull.

Table 1.7

	N	kgf (kp)	dynes	(UK) tonf	lbf	poundal
1 N	1	0·1020	1×10^5	$1{\cdot}004\times10^{-4}$	0·2248	7·2464
1 kgf (kp)	9·807	1	$9{\cdot}81\times10^5$	$9{\cdot}84\times10^{-4}$	2·205	71·0672
1 dyne	1×10^{-5}	$1{\cdot}02\times10^{-6}$	1	$1{\cdot}10^{-9}$	$2{\cdot}25\times10^{-6}$	—
1 (UK) tonf	9964	1016	9964×10^5	1	2240	—
1 lbf	4·448	0·4536	4448×10^2	$4{\cdot}46\times10^{-4}$	1	32·23
1 poundal	0·138	0·0141	$1{\cdot}38\times10^4$	—	—	1

Imperial system

1 ozf = 28·35 gf (p), 16 ozf = 1 lbf = 453·6 gf (p),
112 lbf = 1 cwt = 50·8024 kgf (kp), 20 cwt = 1 (UK) ton
1 stone = 14 lbf = 6·35 kgf (kp)
Newton's law: Force (lbf) = mass (slugs) × acceleration (ft/s^2) therefore 1 slug = 32·2 lbf = 14·594 kgf (kp)
gravitational force (average value) 32·2 ft/s^2
1 long (UK) ton = 1·016 t
1 short (US) ton = 0·907 t = 2000 lb

1.7 Units of Torque

1 Nm = 0·10197 kgf m = 0·73756 lbf ft = 8·8507 lbf in
1 kgf (kp) m = $9{\cdot}81\times10^{-7}$ Nm = 0·0139 ozf in
1 ozf in = 72·01 cpm (gf cm) = $7{\cdot}062\times10^{-3}$ Nm
1 lbf in = 0·1129 Nm = 1152 gf cm (cpm)
1 lbf ft = 1·356 Nm = 0·1383 kgf (kpm)

1.8 Units of Power, Work and Energy

Power, which in the SI system is Joules/s or watts can be thought of as the capability of transferring energy.

In rotating electrical machines Power (W) = 2π(rev/s)T(N/m) as long as the torque T and the speed n remain constant.

Similarly for linear motion Power = force × speed

Table 1.8

	kW	H.P.	PS	kpm/s	ftlbf/s	kcal/s	Btu/s
1 kW =		1·3410	1·3596	102·0	737·6	0·2388	0·9478
1 HP =	0·7457	1	1·014	76·04	550·0	0·1781	0·7068
1 PS =	0·7355	0·9863	1	75·00	542·5	0·1757	0·6971
1 kp/s =	$9{\cdot}81\times10^{-3}$	$1{\cdot}315\times10^{-2}$	$1{\cdot}333\times10^{-2}$	1	7·233	$2{\cdot}342\times10^{-3}$	$9{\cdot}295\times10^{-3}$
1 ftlbf/s =	$1{\cdot}36\times10^{-3}$	$1{\cdot}360\times10^{-3}$	$1{\cdot}84\times10^{-3}$	0·1383	1	$0{\cdot}324\times10^{-3}$	$1{\cdot}185\times10^{-3}$
1 kcal/s =	4·1868	5·615	5·692	426·9	3088	1	3·968
1 Btu/s =	1·055	1·415	1·435	107·6	778·2	0·2520	1

Work done, or the total power over a specific distance moved in the direction of the applied force, or interval of time is a useful concept for the solution of certain drive problems. The SI unit of work is the joule, i.e., a force of 1 N having moved its point of application through 1 m.

Energy, or the capacity of doing work has the same units as work and not only appears in many forms but is subject to transformation from one form to another. Engineers considering electric motors are particularly concerned with the transformation of electrical into mechanical energy, and in this process most losses appear in the form of heat energy. In some applications the stored mechanical-kinetic energy will be converted back to electrical energy (regenerative braking).

1.9 Units of Temperature

The SI unit of thermodynamic temperature is the kelvin, which, as a measure of temperature differences, is equal in magnitude to the degree Celsius (Centigrade). The difference is in the zero point, which, for the kelvin, is based on the absolute or lowest theoretically obtainable temperature.

Some typical calibration points are as follows:

Table 1.9 Calibration points for thermometers

Calibration points	Kelvin	Celsius	Fahrenheit
Absolute zero	0	−273·15	−459·67
Liquid oxygen boils	90·18	−182·97	
Ice point	273·15	0	32
Steam point	373·15	100	212
Freezing point of zinc	692·7	419·55	
Freezing point of silver	1234·0	960·85	
Freezing point of gold	1336·2	1063·05	

Conversion: $t_C = \frac{5}{9}(t_F - 32)$
$t_F = 1{\cdot}8 t_C + 32$

1 calorie = heat energy required to raise 1 gramme of water from 14·5 to 15·5°C = 4·187 J

Thermal constants

$$1 \text{ Btu/sq in} = 391 \text{ kcal/m}^2 = 1{\cdot}64 \text{ MJ/m}^2$$

$$1 \text{ Btu/sq ft} = 2{\cdot}71 \text{ kcal/m}^2 = 11{\cdot}35 \text{ kJ/m}^2$$

$$1 \text{ Btu/cu ft} = 8{\cdot}90 \text{ kcal/m}^3 = 37{\cdot}26 \text{ kJ/m}^2$$

$$1 \text{ Btu/lb} = 0{\cdot}556 \text{ kcal/kg} = 2{\cdot}2328 \text{ kJ/kg}$$

1.10 Units of Pressure

When specifying pressure it must be made clear if it is absolute, or above or below atmospheric, and if the latter the value of atmospheric pressure at the time of measurement must also be specified.

Table 1.10

	N/m^2	m bar	kgf/m^2 = mm WG	kp/cm^2 = 1 at	Torr = mm Hg	atm	lbf/sq.ft	lbf/sq.in (p.s.i.)
1 N/m^2 (Pascal)	1	0·01	1·02	$1{\cdot}02 \times 10^{-5}$	0·0075	9×10^{-5}	0·0289	—
1 m bar	100	1	10·2	—	0·7501	—	2·089	0·0145
1 kgf/m^2 = mm WG	9·807	—	1	0·0001	—	—	0·2048	—
1 kgf/cm^2 = 1 at.	98·067	980·07	10 000	1	735·6	0·9678	2048	14·22
1 Torr = mm Hg	133·32	1·333	13·6	0·00136	1	—	2·785	0·01934
1 atm	101 325	1013	10 332	1·033	760	1	2116	14·7
1 lb/sq. ft (p.s.f.)	47·88	0·4788	4·882	—	0·3591	—	1	—
1 lb/sq. in (p.s.i.)	6894·8	68·95	703·1	0·07031	51·71	0·068	144	—

1.11 Units of Viscosity

Viscosity is defined as the internal friction between different layers of a fluid moving with different velocities.

Dynamic viscosity is the force in dynes necessary to move a layer of liquid 1 cm high and area 1 cm^2 with a speed of 1 cm/s.

Unit 1 P (Poise) = 100 cP (centipoises).

Kinematic viscosity = Dynamic viscosity/density (g/cm^2)

Unit: Stokes = 100 cST (centistokes)

Most common liquids

Frictional force, $F = nA$ newton

A = area in m^2

n = coefficient of viscosity (frictional force/unit area of a liquid when it is in a region of unit velocity gradient (i.e. 1 m/s),

i.e. Ns/m^2 dekapoise

1.12 Trigonometric Functions and Equations

Figure 1.1 treats the trigonometric ratios from a geometrical aspect. Circular functions can also be expressed as series by using Maclaurin's theorem: i.e.

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0)\ldots\frac{x^n}{n!}f^{n'}(0)\ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\cdots = \frac{e^{jx} + e^{-jx}}{2j}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\cdots = \frac{e^{jx} + e^{-jx}}{2}$$

Hyberbolic functions $\sinh x$ and $\cosh x$ relate to a hyperbola in the same way as $\sin x$ and $\cos x$ relate to a circle. Thus

$$\sinh x = \frac{e^x - e^{-x}}{2} = -j \sin jx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

and

$$\cosh x = \frac{e^x + e^{-x}}{2} = \cos jx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

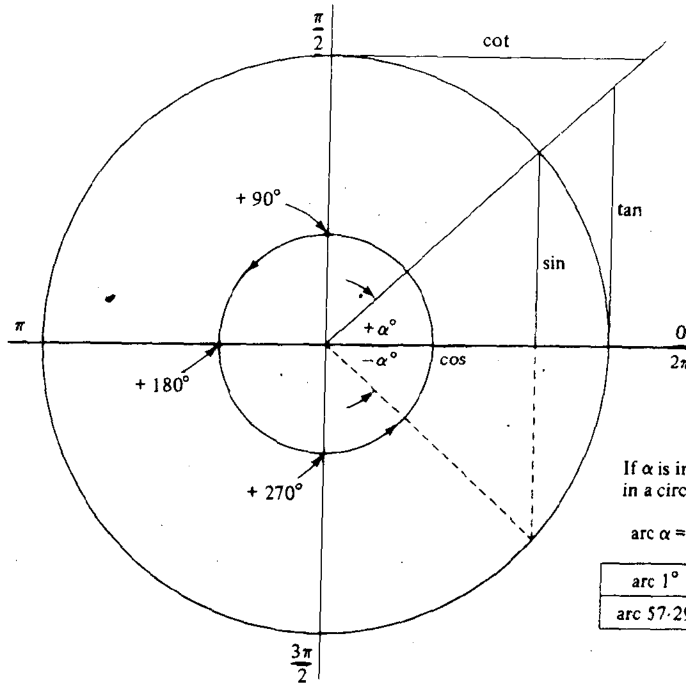
1.12.1 *Equations for triangles*

In any triangle such as those shown in Fig. 1.2 the sum of the internal angles is 180 degrees. Thus $\alpha + \beta + \gamma = 180$.

The most suitable equation for solving triangles depends on the known parameters.

Sine rule

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



If α is in degrees then in a circle radius r

$$\text{arc } \alpha = \alpha \frac{\pi}{180} \times r$$

arc 1° = 0·017453
arc 57·29578° = 1 rad

$\theta =$	$\pm\alpha$	$90 \pm \alpha$	$180 \pm \alpha$	$270 \pm \alpha$	0° / 2π	30° / $\frac{\pi}{6}$	45° / $\frac{\pi}{4}$	60° / $\frac{\pi}{3}$	90° / $\frac{\pi}{2}$
$\sin\theta =$	$\pm\sin\alpha$	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta =$	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta =$	$\pm\tan\alpha$	$\mp\cot\alpha$	$\pm\tan\alpha$	$\mp\cot\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot\theta =$	$\pm\cot\alpha$	$\mp\tan\alpha$	$\pm\cot\alpha$	$\mp\tan\alpha$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{2}$	0

$\sec\alpha = \frac{1}{\cos\alpha}$	$\operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\cot\alpha}$
$\cos^2\alpha + \sin^2\alpha = 1$	$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\sin 3\alpha = 3\sin\alpha - 4\sin^2\alpha$
	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$	$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\sin\alpha\sin\beta = 2\sin\frac{\alpha\pm\beta}{2}\cos\frac{\alpha\mp\beta}{2}$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \cos\alpha\sin\beta$	$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
	$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$	$\tan\alpha \pm \tan\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$
$\cot(\alpha \pm \beta) = \frac{\cot\alpha\cot\beta \mp 1}{\cot \pm \cot\alpha}$	$\cot\alpha\cot\beta = \frac{\sin(\beta\pm\alpha)}{\sin\alpha\sin\beta}$

More accurate values: π = 3·141 592 65 1 rad = 57·295 7·79 5.3 degrees
e = 2·718 281 828 5

Fig. 1.1 Trigonometric functions (on circle of unit radius)

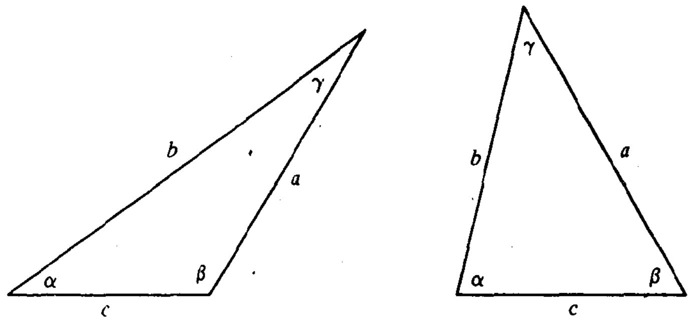


Fig. 1.2

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$
$$b^2 = c^2 + a^2 - 2ca \cos \beta$$
$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

If one of the angles is a right angle (90 degrees or $\pi/2$ radians), then the formulae simplify since $\cos 90 = 0$ and become

$$a^2 = b^2 + c^2$$
$$b^2 = c^2 + a^2$$
$$c^2 = a^2 + b^2$$

usually known as Pythagoras' theorem.

1.12.2 *Equations*

Straight line (Fig. 1.3)

$$y = mx + c$$

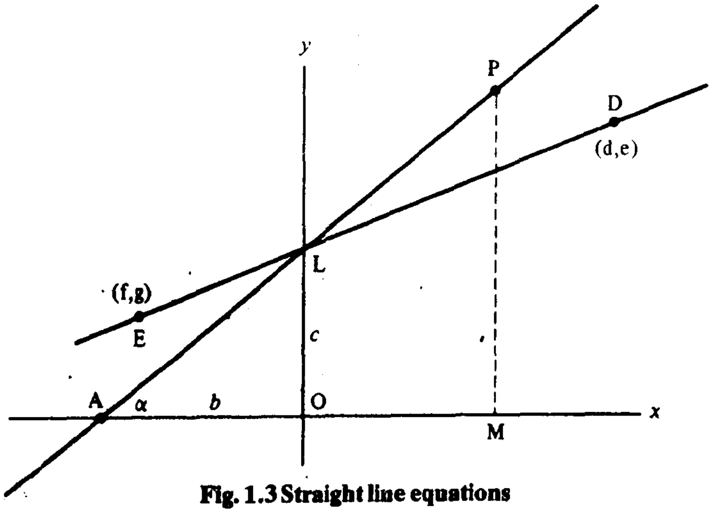


Fig. 1.3 Straight line equations