Multirate and Wavelet Signal Processing

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Multirate and Wavelet Signal Processing

Wavelet Analysis and Its Applications

The subject of wavelet analysis has recently drawn a great deal of attention from mathematical scientists in various disciplines. It is creating a common link between mathematicians, physicists, and electrical engineers. This book series will consist of both monographs and edited volumes on the theory and applications of this rapidly developing subject. Its objective is to meet the needs of academic, industrial, and governmental researchers, as well as to provide instructional material for teaching at both the undergraduate and graduate levels.

Among the attractive features of wavelet analysis is the computational aspect of the subject. In particular, computation of the discrete wavelet transform can be accomplished by filter bank algorithms in subband coding. This eighth volume of the series is an elementary treatise of the subject of multirate, including the detailed discussion of filter banks and their lattice structures, as well as an application of multirate to wavelet implementation.

The series editor would like to congratulate the author for an insightful presentation of an important area in wavelet analysis.

This is a volume in
WAVELET ANALYSIS AND ITS APPLICATIONS

CHARLES K. CHUI, SERIES EDITOR

A list of titles in this series appears at the end of this volume.

This book was only possible through the
patience, understanding, encouragement,
and support of my wife Debbie.
As such, I dedicate this book to her.

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Preface

The field of multirate and wavelet signal processing finds applications in speech and image compression, the digital audio and digital video industries, adaptive signal processing, and in many other applications.

The utilization of multirate techniques is becoming an indispensable tool of the electrical engineering profession. This point can be illustrated in three ways. First, if a performance specification is controlling the design of a particular system, that is, the performance specification exceeds the current state-of-art, then by converting the system to a multirate system, the overall system specification can be met with slower components. Secondly, if the dollar cost specification is controlling the design of a particular system, that is, the design of a competitive commercial system where bottom line cost is most important, then by converting the system to a multirate system, the overall system cost will be reduced through the utilization of slower, cheaper devices. Thirdly, if power consumption is controlling the design of a particular system, that is, the design of a hand-held system powered by a couple AA batteries, or possibly a satellite system, then by converting the system to a multirate system will reduce power consumption through the utilization of devices with slower switching speed, and as a result, lower power dissipation.

Wavelet transforms are closely related to filter banks. As such, a background in filter banks will make it easier for the reader to understand, design, and implement wavelet transforms.

Many of the most important applications, such as video compression, and many challenging research problems are in the area of multidimensional multirate. As such, multidimensional multirate is integrated throughout the book.

The focus of this book is to present a sound theoretical foundation by emphasizing the general principles of multirate. This book is self-contained for readers who have some prior exposure to linear algebra (at the level of Horn and Johnson's *Matrix Analysis*) and multidimensional signal processing (at the level of Lim's *Two-Dimensional Signal and Image Processing* or Dudgeon and Mersereau's *Multidimensional Digital Signal Processing*).

Moreover, this text will bring the reader to a point where he/she can read, understand, and appreciate the vast multirate literature.

The organization of this book is as follows. The first two chapters are devoted to basic multirate ideas including decimators, expanders, polyphase notation, etc. This presentation is first given for one-dimensional signals in Chapter 1 and then generalized to multidimensional signals in Chapter 2. The next two chapters deal with filter banks. Chapter 3 presents the theory of filter banks for both one-dimensional and multidimensional signals. Chapter 4 deals with lattice structures, an efficient implementation strategy for filter banks. Chapter 5 highlights an important application of multirate — the implementation of wavelets.

I would also like to take this opportunity to thank Professor Charles Chui for his enthusiasm about this project and for including this text in his distinguished wavelet series. The following people have provided very useful feedback during the writing of this book. They include: Bill Cowan, Tom Foltz, Jerry Gerace, Ying Huang, You Jang, Matt Kabrisky, Mark Oxley, Robert Parks, Juan Vasquez, and Dan Zahirniak.

Fairborn, Ohio
February 9, 1997

Bruce W. Suter

Multirate and Wavelet Signal Processing

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Chapter 1

Multirate Signal Processing

1.1 Introduction

This chapter provides the basic concepts used in the study of multirate and wavelet signal processing. Some of the earliest contributions to the study of the fundamentals of multirate were due to Schafer and Rabiner[40], Meyer and Burrus[32], Oetken *et al.*[37], and Crochiere and Rabiner[10]. The idea of polyphase representation is a key concept throughout the development of this book. This nontrivial idea was first articulated by Bellanger *et al.*[3]. Much more recently, Evangalista[17] carefully examined another important idea - digital comb filters.

Many of the concepts developed in this chapter are also discussed in the other multirate texts by Crochiere and Rabiner[11], Fliege[19], Strang and Nguyen[46] and Vaidyanathan[49].

Section 1.2 presents a framework for multirate and it introduces two important representations for discrete signals. Section 1.3 introduces the basic building blocks. Section 1.4 provides ways to interchange the basic building blocks. Section 1.5 presents a filter bank example.

1.2 Foundations of multirate

First we will examine some sampling considerations and then present some basic transforms for analyzing signals.

1.2.1 Sampling considerations

Multirate is the study of time-varying systems. As such, the sampling rate will change at various points in time in an implementation. This will require us to vary the gain (magnitude) of filters in series with the time-varying building blocks so that the resulting gain is consistent with what one would expect if the sampling interval after the time-varying block had been the original sampled frequency. Towards this end, let us analyze a train of impulses.

Theorem 1.2.1.1. $\sum_{k=-\infty}^{\infty} \delta(t-kT) = \frac{1}{T}\sum_{m=-\infty}^{\infty} \exp(\frac{j2\pi mt}{T})$.

Proof: Let us expand $\sum_{k=-\infty}^{\infty} \delta(t-kT)$ in a Fourier series. So,

$$\sum_{k=-\infty}^{\infty} \delta(t-kT) = \sum_{m=-\infty}^{\infty} a(m) \exp\left(\frac{j2\pi mt}{T}\right)$$

where,

$$a(m) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\sum_{k=-\infty}^{\infty} \delta(t-kT)\right] \exp\left(-\frac{j2\pi mt}{T}\right) dt$$

or equivalently,

$$a(m) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t-kT) \exp\left(-\frac{j2\pi mt}{T}\right) dt.$$

Let $\tau = t - kT$. Then,

$$a(m) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\int_{-\frac{T}{2}-kT}^{\frac{T}{2}-kT} \delta(\tau) \exp\left(-\frac{j2\pi m(\tau+kT)}{T}\right) d\tau.$$

We recognize this as a sum of integrals with adjoining limits and simplify to

$$a(m) = \frac{1}{T}.$$

Hence,

$$\sum_{k=-\infty}^{\infty} \delta(t-kT) = \frac{1}{T}\sum_{m=-\infty}^{\infty} \exp\left(\frac{j2\pi mt}{T}\right).$$

■

Let $\mathcal{F}$ denote the Fourier transform. So that if $x(t)$ is a signal, then

$$\mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)dt.$$

Let us examine the Fourier transform of an impulse train.

Theorem 1.2.1.2. $\mathcal{F}\left[\sum_{k=-\infty}^{\infty} \delta(t-kT)\right] = \frac{1}{T}\sum_{m=-\infty}^{\infty} \delta(f-\frac{m}{T})$.

Proof: From the previous theorem,

$$\sum_{k=-\infty}^{\infty} \delta(t-kT) = \frac{1}{T}\sum_{m=-\infty}^{\infty} \exp\left(\frac{j2\pi mt}{T}\right).$$

Therefore,

$$\begin{aligned}
\mathcal{F}\left[\sum_{k=-\infty}^{\infty}\delta(t-kT)\right] &= \mathcal{F}\left[\tfrac{1}{T}\sum_{m=-\infty}^{\infty}\exp(\tfrac{j2\pi mt}{T})\right] \\
&= \tfrac{1}{T}\int_{-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\exp(\tfrac{j2\pi mt}{T})\exp(-j2\pi tf)dt \\
&= \tfrac{1}{T}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-j2\pi t\left(f-\tfrac{m}{T}\right)\right]dt \\
&= \tfrac{1}{T}\sum_{m=-\infty}^{\infty}\delta(f-\tfrac{m}{T}).
\end{aligned}$$

■

Now, performing the Fourier transform of a sum of the product of the input $x(t)$ and Dirac delta functions, which can be expressed as the convolution of the corresponding functions, produces

$$\begin{aligned}
\mathcal{F}\left[\sum_{k=-\infty}^{\infty}x(t)\delta(t-kT)\right] &= \int_{-\infty}^{\infty}X(f-f')\tfrac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f'-\tfrac{k}{T}\right)df' \\
&= \tfrac{1}{T}\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}X(f-f')\,\delta\left(f'-\tfrac{k}{T}\right)df' \\
&= \tfrac{1}{T}\sum_{k=-\infty}^{\infty}X\left(f-\tfrac{k}{T}\right).
\end{aligned}$$

At this point, we will interpret these results for linear time-varying systems. If the sampling interval is increased by an integer factor of M, where $M > 1$, then the magnitude of the Fourier transform will need to be decreased by a factor of $\frac{1}{M}$ to reconstruct the original system, that is

$$\mathcal{F}\left[\sum_{k=-\infty}^{\infty}x(t)\delta(t-kMT)\right] = \frac{1}{MT}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{MT}\right).$$

Similarly, if the sampling interval is decreased by an integer factor of L, where $L > 1$, then the magnitude of the Fourier transform will need to be increased by a factor of L to reconstruct the original signal, that is

$$\mathcal{F}\left[\sum_{k=-\infty}^{\infty}x(t)\delta\left(t-\frac{kT}{L}\right)\right] = \frac{L}{T}\sum_{k=-\infty}^{\infty}X\left(f-\frac{kL}{T}\right).$$

1.2.2 Sampled signals

For completeness, the definition of z-transforms and the discrete Fourier transforms will be presented. Then, we will present two sampled signal representations: the modulation representation and the polyphase representation. The theory of multirate and wavelet signal processing utilizes both of these representations

1.2.2.1 z-Transforms

Definition 1.2.2.1. *The z-transform of a sequence $x(n)$ is defined by*

$$X(z) = \mathcal{Z}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)z^{-n}.$$

An important property of z-transforms is the following scaling theorem.

Theorem 1.2.2.1. *If the z-transform of x exists and α is a scalar, then*

$$\mathcal{Z}\left[\alpha^{-n}x(n)\right] = X(\alpha z).$$

Proof: By definition,

$$\mathcal{Z}\left[\alpha^{-n}x(n)\right] = \sum_{n=-\infty}^{\infty} \alpha^{-n}x(n)z^{-n}$$

or equivalently,

$$\mathcal{Z}\left[\alpha^{-n}x(n)\right] = \sum_{n=-\infty}^{\infty} x(n)(\alpha z)^{-n}.$$

Hence,

$$\mathcal{Z}\left[\alpha^{-n}x(n)\right] = X(\alpha z).$$

■

If the z-transform converges for all z of the form $z = \exp(j\omega)$ for real ω, then the z-transform can be represented as the sum of harmonically related sinusoids, *i.e.*

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\exp(-j\omega n),$$

which is sometimes called the discrete-time Fourier transform.

1.2.2.2 Discrete Fourier transform

Definition 1.2.2.2. *The discrete Fourier transform of a periodic sequence $x(n)$ of length N is given by*

$$X(k) = \sum_{n=0}^{N-1} x(n)\exp\left(-\frac{j2\pi kn}{N}\right); k = 0, \ldots, N-1$$

and the corresponding inverse discrete Fourier transform is given by

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\exp\left(\frac{j2\pi kn}{N}\right); n = 0, \ldots, N-1.$$

Let $W_N = \exp\left(\frac{-j2\pi}{N}\right)$, the (principal) Mth root of unity. Then, the discrete Fourier transform matrix, denoted $\mathbf{W}_N$, is an N x N matrix, defined by

$$[\mathbf{W}_N]_{k,n} = W_N^{kn} = \exp\left(\frac{-j2\pi kn}{N}\right).$$

1.2.2.3 Modulation representation

Definition 1.2.2.3. *Given a sequence $x(n)$ and a positive integer M, then the components of the modulation representation of the z-transform of $x(n)$ are defined $X(zW_M^k)$, $k = 0, 1, \ldots, M-1$.*

If $M = 2$, then the components of the modulation representation are $X(z)$ and $X(-z)$. The term modulation representation can be most easily visualized in the time domain. Using the scaling theorem of z-transforms, we obtain

$$\begin{aligned} \mathcal{Z}^{-1}[X(zW_M^k)] &= \left(W_M^{-k}\right)^n x(n) \\ &= \exp\left(\frac{j2\pi kn}{M}\right) x(n). \end{aligned}$$

It is interesting to note that the components of the modulation representation can be combined pairwise to form a real signal, that is

$$\begin{aligned} \mathcal{Z}^{-1}[X(zW_M^k) + X(zW_M^{M-k})] &= W_M^{-kn}x(n) + W_M^{-Mn+kn}x(n) \\ &= 2\cos\left(\frac{2\pi kn}{M}\right)x(n). \end{aligned}$$

1.2.2.4 Polyphase representation

Definition 1.2.2.4. *Given a sequence $x(n)$ and a positive integer M, then the Type-I polyphase components of $x(n)$ are defined as $x_k(n) = x(Mn+k)$, $k = 0, 1, \ldots, M-1$.*

If $M = 2$, then $x_0(n)$ would be the even-numbered samples and $x_1(n)$ would be the odd-numbered samples. Now, let us investigate the z-transform of the Type-I polyphase components, that is

$$X(z) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{M-1} x(Mn+k)\, z^{-(Mn+k)}$$

or equivalently,

$$X(z) = \sum_{k=0}^{M-1} z^{-k} \sum_{n=-\infty}^{\infty} x(Mn+k)(z^M)^{-n}.$$

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