

MODERN ANTENNA DESIGN

THOMAS A. MILLIGAN



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PREFACE

I wrote this book, from my perspective as a designer, for users and designers of antennas. The two groups have similar needs. Each must thoroughly understand the properties of antennas and the limits of design. The initial part of design is the selection of the proper antenna type. To enable a quick selection of antenna type and approximate size without performing a detailed design, the book stresses the limitations and summarizes the characteristics of antennas. The use of this material will minimize the costly misdirection of effort. Beyond providing that help in selecting the design approach, the book reduces antenna theory to a few basic ideas and presents concepts that I myself have found useful when designing and troubleshooting antenna systems.

Because antenna theory is a mature subject, it is easy to become lost in the mathematics and lose sight of what is happening. Ideas are presented with a minimum of mathematical development except in areas where it is necessary for understanding. Expositions of analytical techniques, such as the method of moments and the geometric theory of diffraction, have been excluded to maximize the space for design methods. The results of analytical techniques are included, but only the basic ideas of the methods are given. The book presents many design methods that start with the usual specifications and end with dimensions. Methods that are applicable to all antennas do not exist, but the basic concepts developed in the first few chapters will help with starting any antenna design. Because there is always a place for empirically derived designs, dimensions of successful designs are listed.

The book arose from a set of notes used in a three-semester in-house continuing education course given a number of times to combinations of designers, users, recent graduates, and other interested persons. Each group had special needs, but all wanted a basic understanding and a unified approach useful for meeting real-world problems. The variety of interests,

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questions, and initial misconceptions of the members of the classes helped define the book.

Although the material was initially presented at a variety of levels, the following knowledge is needed to achieve full benefit from the material: an understanding of electromagnetics in the relation of fields and boundary conditions, an ability to add signals expressed as amplitude and phase, and some understanding of the vector nature of fields. If a section is beyond your mathematical level, skip it. You can continue without it because there are no long mathematical developments which build on earlier sections. Most of the design methods use only algebra; the exceptions are the design of shaped reflectors and two-surface lenses, which require a knowledge of integral and differential equations. The material is suitable for a graduate-level course stressing the practical aspects of design, but it will then require the addition of supplemental problems.

Many of the results are presented in tables. Graphical presentations give broad trends, but tables contain better information. The tables are ideal in computer programs when combined with an interpolation algorithm. Simple programs can be written on small machines to produce designs quickly. Explicit computer routines have not been included because I feel that each person should understand the design method and write his or her own program in a language which fits the machine in use.

The first few chapters comprise the introduction. Chapter 1 lists basic definitions and approaches the subject in terms of a communication link, that is, from the user's point of view. Chapter 2 touches lightly on electromagnetics by describing the three basic radiation sources: electric currents, magnetic currents developed from duality, and the combination of the two in the radiation from apertures. The ideas exceed the importance of the mathematics. The second part of Chap. 2 summarizes antenna types and gives methods for simple analyses of expected performance as a starting point for antenna selection. Chapter 3, on arrays, introduces the idea of radiation from an extended source as the summation of phasors from a finite number of radiation points. Array synthesis is not covered until after aperture distributions have been discussed because many large arrays are designed by sampling a continuous distribution. Chapter 4 is a transition between introductory material and the design of particular antennas. It develops the duality of dipoles and slots. Both the importance and the design descriptions of baluns are presented so the reader will understand the problems of feed line radiation and stray currents on support structures.

The remaining chapters either discuss design methods of particular antennas or develop general ideas, such as aperture distributions and array synthesis, more thoroughly. The choice of topics reflects the author's experience with microwave antennas, although many of the concepts apply to lower-frequency antennas. The chapters may be considered in almost any

order. Although some topics build on preceding ideas, quick reference to earlier sections will let the reader jump around. Sections 6-2, on amplitude taper and phase error efficiencies, and 6-16, on quadratic phase error, are exceptions. The sections following those two make extensive use of the idea of the separation of directivity into distribution losses (Sec. 6-2) and quadratic phase error (Sec. 6-16). On first reading, the rest of Chap. 6, on aperture distributions, can be skipped.

The chapters on the design of particular antennas are summarized as follows. Chapter 5, on conformal antennas, presents the practical aspects of microstrip and stripline antennas, together with methods for quick design of these thin convenient antennas, and tables of limitations allow rapid determination of their possible uses. The general aperture distributions of Chap. 6 can be applied to many antennas without undertaking detailed analysis. Chapter 7, on horns, not only covers analysis techniques but gives design methods that produce dimensions from specifications. Chapter 8, on reflectors, covers all aspects of design and analysis extensively and includes examples to demonstrate design methods from both mechanical and electrical performance points of view. Chapter 9, on lenses, stresses the increasing importance of lens antennas above microwave frequencies, although lower-frequency types such as metal plate and bootlace designs are covered. Chapter 10 discusses the usual Chebyshev methods for equal sidelobe array synthesis, shaped beam methods using Fourier series and Woodward methods, and planar array synthesis through convolution of simple arrays. Chapter 11, on traveling wave antennas, uses both aperture theory and specific antennas, such as Yagi-Uda, helical wire, and dielectric antennas, to demonstrate design methods and a generalized approach to surface wave and leaky-wave antennas. Chapter 12, on self-scaling antennas, combines a discussion of structural requirements for broadband antennas with design details of log-periodic and spiral antennas.

My special thanks go to Manuel R. Moreno, who not only reviewed the entire manuscript and made many suggestions on its unclear sections but also encouraged me while I was writing it. I would also like to acknowledge the useful criticisms of Loren K. DeSize, who suggested changes in emphasis. My students provided invaluable help through their questions, which arose not only from misconceptions but from experience in the design of particular antennas and which identified areas of possible misunderstanding. In conclusion, I must express my gratitude to Mary Wright Milligan, who supported and encouraged me throughout this project and who read and proofread the various manuscripts and detected many wording problems. I have enjoyed writing this book, which clarified my own thinking, and I hope the book will be useful to the reader.

Thomas A. Milligan

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PROPERTIES OF ANTENNAS

An antenna converts bound circuit fields into propagating electromagnetic waves and, by reciprocity, removes power from passing electromagnetic waves. Maxwell's equations predict that any time-varying electric or magnetic field produces the other field and forms an electromagnetic wave. In an electromagnetic wave, the pair of fields are orthogonally placed and propagate in the direction of the normal to the plane defined by the perpendicular electric and magnetic fields. The electric field, the magnetic field, and the direction of propagation form a right-hand coordinate system. The propagating wave field intensity decreases by $1/R$ away from the source, whereas a static field drops off by $1/R^2$. Any circuit with time-varying fields has the capability of radiating to some degree. Many circuit designers have discovered this radiation by running their fingers around the circuit while observing the response. Antenna designers are also fond of running their hands over an antenna while monitoring the input match, but we are looking for changes in the response indicating radiation. How do antennas radiate and how do we keep circuits from radiating?

The retarded potential concept explains radiation. Simply stated, a change in a circuit is not detected until it has had time to propagate to the detection point. The propagating velocity is the speed of light in the medium, a consequence of the theory of special relativity limiting the speed of

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information propagation. We will only consider time-harmonic fields and will use the phasor notation with the time dependence $e^{j\omega t}$. An outward-propagating wave is given by

$$e^{-j(kR - \omega t)}$$

where k , the wave number, is given by $2\pi/\lambda$. λ is the wavelength of the wave given by c/f , where c is the velocity of light (3×10^8 m/s in free space) and f is the frequency. Increasing the distance from the source decreases the phase of the wave.

Consider a two-wire transmission line with fields bound to it. The currents on one wire will radiate; but as long as the ground return is near, its radiation will nearly cancel the other line's radiation because the two are 180° out of phase and the waves travel about the same distance. As the lines become farther and farther apart, in terms of wavelengths, the fields produced by the two currents will no longer cancel in all directions. The phase delay is different for each field, and power escapes from the line. We keep circuits from radiating by providing close ground returns. Hence, high-speed logic requires ground planes to reduce radiation and its unwanted cross talk.

Consider a conical transmission line made from two cones with a common vertex and different cone angles. The characteristic impedance between the cones is constant. If the line is fed from the center, the power will spread out in spherical waves between the cones. The waves will reach the ends of the cones and be reflected by the open circuit. Because the distance between the cones has grown to be a significant part of a wavelength, the field halfway between the cones cannot know until sometime later (retarded potential) that the transmission line is open-circuited. We could say that the power just flies off into space because it forgot it was part of the transmission line mode. This is not very rigorous, but it has a good physical feel to it. The power reflected by the open boundary is out of phase with the fields reflected by the ends because of the retardation time finding out about the open circuit. Since this center reflection is out of phase, the total reflected field is reduced and the difference is radiated. Dipoles and horns can also be considered as diverging transmission lines radiating for the same reason as the biconical horn. The impedance along the biconical horn is constant, but the dipole transmission line impedance rises rapidly [1].*

1-1 Antenna Radiation

Antennas radiate spherical waves propagating in the radial direction with the center of the coordinate system on the antenna. At large distances spherical waves can be approximated by plane waves. Plane waves are useful because they simplify the problem. They are not physical, however, because they require infinite energy.

*Numbers in brackets indicate references listed at ends of chapters.

The Poynting vector describes both the direction of propagation and the power density of the electromagnetic wave. It is found from the vector cross product of the electric and magnetic fields and is denoted S .

$$S = E \times H^* \quad \text{W/m}^2$$

Root mean square (rms) values are used to express the magnitude of the fields. H^* is the complex conjugate of the magnetic field phasor. The magnetic field is proportional to the electric field in the far field. The constant of proportion is η , the impedance of free space (376.7Ω).

$$|S| = S = \frac{|E|^2}{\eta} \quad \text{W/m}^2 \quad (1-1)$$

Because the Poynting vector is the vector product of the two fields, it is orthogonal to both fields and the triplet defines a right-hand coordinate system: (E, H, S):

Consider a pair of concentric spheres centered on the antenna. The fields around the antenna decrease as $1/R$, $1/R^2$, $1/R^3$, etc. Constant-order terms would require that the power radiated grow with distance and power would not be conserved. The power density, due to field terms proportional to $1/R^2$, $1/R^3$, and higher, decreases with distance faster than the area increases. The energy on the inner sphere is larger than that on the outer sphere. The energies are not radiated but are instead concentrated around the antenna; they are near-field terms. Only the $1/R^2$ term of the Poynting vector ($1/R$ field terms) represents radiated power because the sphere area grows as R^2 and gives a constant product. All the radiated power on the inner sphere will propagate to the outer sphere.

The sign of the input reactance depends on the near-field predominance of field type: electric (capacitive) or magnetic (inductive). At resonance (zero reactance) the stored energies due to the near fields are equal. Increasing the stored fields increases the circuit Q and narrows the impedance bandwidth.

Far enough from the antenna we consider only the radiated fields and power density. The power is the same through each sphere.

$$4\pi R_1^2 S_{1, \text{avg}} = 4\pi R_2^2 S_{2, \text{avg}}$$

The average power density is proportional to $1/R^2$. Consider differential areas on the two spheres at the same coordinate angles. The antenna radiates only in the radial direction; therefore, no power may travel in the θ or ϕ direction. Power travels in flux tubes between areas, and it follows that not only the average Poynting vector but also every part of the power density is proportional to $1/R^2$.

$$S_1 R_1^2 \sin \theta d\theta d\phi = S_2 R_2^2 \sin \theta d\theta d\phi$$

Since in a radiated wave S is proportional to $1/R^2$, E is proportional to $1/R$.

It is convenient to define radiation intensity to remove the $1/R^2$ dependence

$$U(\theta, \phi) = S(R, \theta, \phi)R^2 \quad \text{W/solid angle}$$

Radiation intensity depends only on the direction of radiation and makes the pattern the same at all distances. A probe antenna measures the relative radiation intensity (pattern) by rotating in a circle (constant R) around the antenna. Often, of course, the antenna rotates and the probe is stationary.

Some patterns have established names. Patterns along constant angles of the spherical coordinates are called either conical (constant θ) or great circle (constant ϕ). The great circle cuts when $\phi = 0^\circ$ or $\phi = 90^\circ$ are the principal plane patterns. Other cuts also are used, but their names are dependent on the measurement positioners and cause confusion. Annotate these patterns carefully to avoid confusion between people measuring patterns on different positioners. Patterns are measured by using three scales: (1) linear (power), (2) square root (field intensity), and (3) decibels. The dB scale is used most because it reveals more of the low-level responses (sidelobes).

Figure 1-1 demonstrates many characteristics of patterns. The half-power beamwidth is sometimes called just the beamwidth. The tenth-power and null beamwidths are used in some applications. The pattern comes from a parabolic reflector whose feed is not located on the axis. The vestigial lobe occurs when the first sidelobe becomes joined to the main beam and forms a shoulder. When the feed is located on the axis of the parabola, the first sidelobes will be equal.

1-2 Gain

Gain is a measure of the ability of the antenna to direct the power delivered to the input into radiation in a particular direction. It is measured at the peak radiation intensity. Consider the power density radiated by an isotropic antenna at a distance R and input power P_0 :

$$S = \frac{P_0}{4\pi R^2}$$

An isotropic antenna radiates equally in all directions, and the power density S is found by dividing the radiated power by the area of the sphere with radius R . The isotropic radiator is considered to be 100 percent efficient. The gain of a real antenna increases the power density in the direction of the peak radiation.

$$S = \frac{P_0 g}{4\pi R^2}$$

Gain is achieved by directing the radiation so that other parts of the radiation

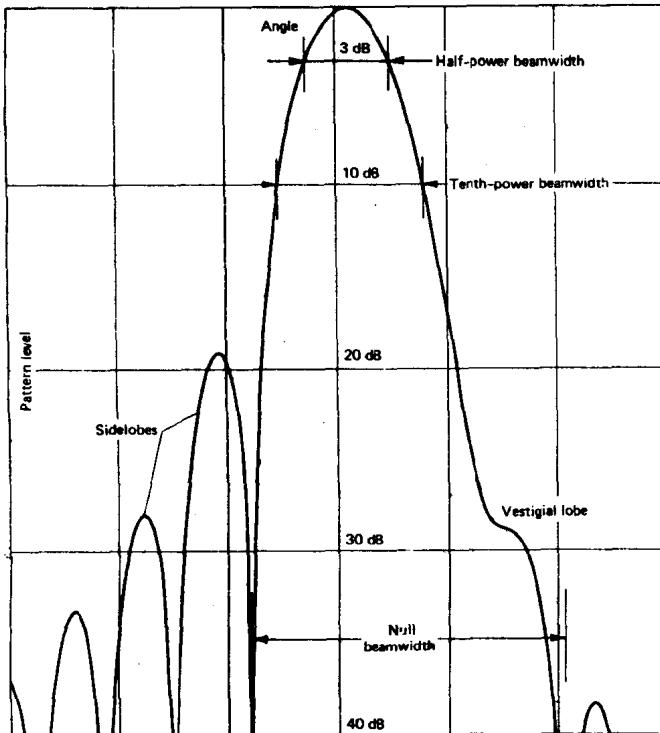


Figure 1-1 Antenna pattern characteristics.

sphere will have fields less than the isotropic radiator. In general, gain is defined as the gain-biased pattern of the antenna

$$S(\theta, \phi) = \frac{P_0 g(\theta, \phi)}{4\pi R^2} \quad \text{power density} \quad (1-2)$$

$$U(\theta, \phi) = \frac{P_0 g(\theta, \phi)}{4\pi} \quad \text{radiation intensity}$$

The surface integral of the radiation intensity over the radiation sphere divided by the input power P_0 is a measure of the relative power radiated by the antenna, or the antenna efficiency.

$$\frac{P_r}{P_0} = \int_0^{2\pi} \int_0^\pi \frac{g(\theta, \phi)}{4\pi} \sin \theta d\theta d\phi = \eta_e \quad \text{efficiency}$$

where P_r is the radiated power. Material losses in the antenna reduce the radiated power.

In a system the transmitter output impedance or the receiver input impedance may not conjugate-match the antenna. Peak gain occurs for a

conjugate match. Precision gain measurements require a tuner between the antenna and receiver to conjugate-match the two together. Alternatively, the mismatch loss must be removed by calculation after the measurement. The effects of mismatches are considered separately for a given system. Many antennas are measured into the system impedance, and mismatch loss is considered to be part of the efficiency.

Example Find the peak power density at 10 km of an antenna with an input power of 3 W and a gain of 15 dB.

First convert dB gain to a ratio.

$$g = 10^{15/10} = 31.62$$

The area of the sphere with radius 10 km is $4\pi(10^4)^2 \text{ m}^2$. The power density is

$$S = \frac{(3 \text{ W})(31.62)}{4\pi \times 10^8 \text{ m}^2} = 75.5 \text{ nW/m}^2$$

We find the electric field intensity by rearranging Eq. (1-1).

$$|E| = \sqrt{S\eta} = \sqrt{(75.5 \times 10^{-9})(376.7)} = 5333 \text{ } \mu\text{V/m}$$

Gain is relative to an isotropic antenna. Some antenna gains are with reference to a half-wavelength dipole, which has a gain of 2.14 dB.

1-3 Effective Area

Antennas capture power from passing waves and deliver some of it to the terminals. Given the power density of the incident wave and the effective area of the antenna, the power delivered to the terminals is the product.

$$P_d = SA_{\text{eff}} \quad (1-3)$$

For an aperture antenna such as a horn, parabolic reflector, or flat-plate array, the effective area relates to the physical area by the aperture efficiency. Antennas with infinitesimal physical areas, such as dipoles, have effective areas because they remove power from passing waves. Losses due to material, distribution, and mismatch reduce the ratio of the effective area to the physical area. A typical value for a parabolic reflector is 55 percent aperture efficiency.

1-4 Path Loss

We combine the gain of the transmitting antenna with the effective area of the receiving antenna to find the delivered power and path loss. The power density at the receiving antenna is given by Eq. (1-2), and the received power is given by Eq. (1-3). By combining the two, we obtain the path loss: