GUIDED WEAPON CONTROL SYSTEMS

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GUIDED WEAPON CONTROL SYSTEMS

PREFACE

During the last twenty five years a large number of textbooks have been written on the subject of automatic control. Many of these books are introductory student texts concerned with basic theory and the examples used for illustration are often, of necessity, so simplified or contrived that the embryo control engineer gains little or no appreciation of the true nature of the problems that will be encountered in practice. The present authors believe that this text, which concentrates in depth on the use of closed loop control theory in one particularly rich and varied field of application, viz the design of guided weapon systems, will be a useful supplement to the aforementioned introductory texts and will provide further insight into the uses and limitations of the basic theory without being constrained to "examination" type problems and their solutions. We hope, therefore, that this book will be of interest to lecturers and postgraduate students taking control and systems orientated courses as well as to those engineers actively involved in guided weapon design who require a compact reference book directly related to their own field. The only essential prerequisite is a basic knowledge of classical control theory and familiarity with such terms as bandwidth, damping ratio, phase margin, steady state gain etc.

The subject matter of this book is based on lecture notes given to the Guided Weapon Systems (M.Sc.) Course at the Royal Military College of Science; this course is the only one of its type in this country and has been running continuously for twenty six years.

Finally, it is a pleasure to acknowledge the suggestions and discussions with our many colleagues, both from industry and the government establishments, and we regret that they are too numerous to mention individually. Also, we are grateful to Mrs A Hare who typed the manuscript and Mrs H Killeen and Mrs S Greener who prepared the drawings.

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August 1976

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CHAPTER 1

THE PERFORMANCE OF TARGET TRACKERS

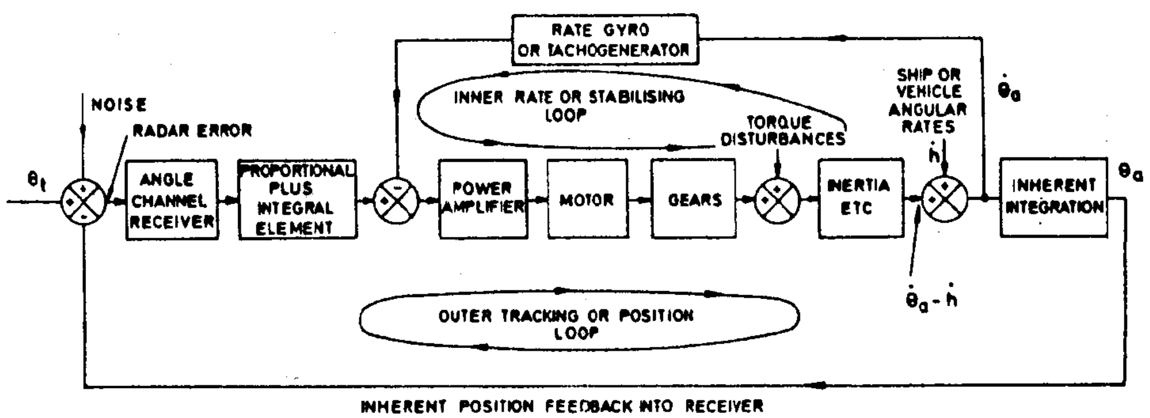
1.1 INTRODUCTION

A guided missile is one which is usually fired in a direction approximately towards the target and subsequently receives steering commands from the guidance system to improve its accuracy. Inertial guidance is often used in medium and long range missiles (over 40 km say) when the intention is to hit a given map reference. The techniques used in such systems are quite different from those used in most short and medium range systems; moreover they have been adequately described elsewhere (1), (2), (3). The guidance-control systems covered in this book are command systems and homing systems. There is much in common between these two systems; for instance one has to track the target in both systems. In command systems the tracker is usually stationary or moving slowly (e.g. the target tracker could be on a ship). In homing systems the target tracker is in the missile and in such a case it is the relative movement of target and missile which is relevant. The special tracking problems associated with homing are considered in chapters 8 and 9; so in this chapter we assume that the tracker speed is small enough not to influence the kinematics of the engagement seriously.

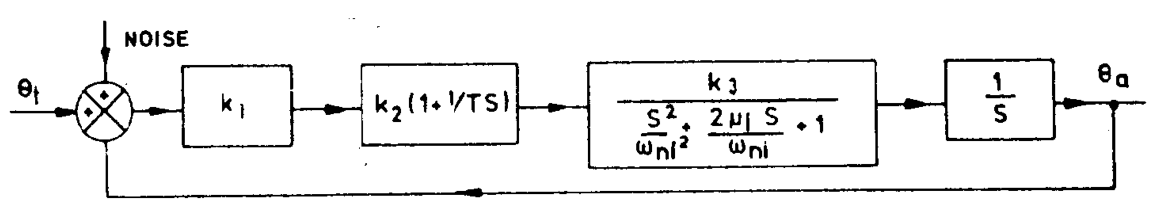
1.2 A TRACKER SERVO

A target tracker attempts to align its electrical null axis (or "boresight" as it is often called) in elevation and azimuth with a line joining the tracker and target called the line of sight (LOS). There are two identical servo systems to do this, so only one will be considered. The first and essential requirement is for a device which produces two signals (one up-down and the other left-right) proportional to the misalignment between the LOS and the boresight. The angular error detecting mechanism associated with a tracker is either a radar receiver or an optical (and this includes infra-red) signal processing system (4).

Most of these error detectors are very non-linear for large misalignments but are regarded as essentially linear for small misalignments of about 1° or less; tracking errors are rarely as large as this. The receiver may well provide range and range rate information as well but since we are concerned only with angle measurement we will call this particular aspect of the receiver the "angle channel receiver". A typical target tracking servo loop is shown in Fig 1.2-1(a). The angle channel receiver produces a signal proportional to the misalignment between target and its own boresight, $\theta_t - \theta_a$. Since it is a linear device, and no reckonable time lag is associated with it, its transfer function is a simple gain k_1 volts/radian (misalignment). This error signal is fed to a proportional plus integral amplifier whose transfer function is $k_2\ (1 + 1/Ts)$. The usual servo components now follow, a power amplifier, motor (electric or hydraulic) and a gear speed reducer, together with the lumped inertia and viscous friction, if any. If the tracker is immobile some rate feedback is usually supplied by a tachogenerator. Typical transfer functions are shown in Fig 1.2-1(b). Provided sufficient power is available in the amplifier and motor, the effect of moderate



(a) Block diagram



(b) Transfer functions

FIG 1.2-1 Target tracker

or high gain in this rate loop is to reduce the lags in these components. If the tracker is on a moving vehicle, rate feedback is provided by a rate gyro (details in chapter 5). This has the additional effect of stabilising the antenna to a large degree against base motion; call this base or hull motion $\dot{h}$. Since the servo drives relative to the hull it will produce a speed relative to the hull $\dot{\theta}_a - \dot{h}$. If the open loop transfer function of this inner loop is $K' G(s)$ where K' is the d.c. gain, in the absence of an error signal from the radar we can write

$$(\dot{\theta}_a - \dot{h}) = - K' \, G(s) \, \dot{\theta}_a$$

Rearranging this is

$$\frac{\dot{\theta}_a}{\dot{h}} = \frac{1}{1 + K' \, G(s)} \qquad (1.2\text{-}1)$$

Since $G(s) \to 1$ at low frequencies and $\to 0$ at high frequencies this equation is saying that if K' is large, say 100 or more, base motion isolation is very good at low frequencies. High gains are difficult to achieve in really heavy equipments due to the resilience of the gear train, but even tank gun and turret stabilisers using rate gyros achieve open loop gains well in excess of 100.

We now consider the closed loop transfer function of the whole servo. If there are two effective lags in the rate loop and its steady state gain is k_3 rad/sec/volt its response is completely defined by an undamped natural frequency ω_{ni} and a damping ratio μ_i. Since there is the usual inherent integration from antenna speed to position the closed loop transfer function is easily shown to be

$$\frac{\theta_a}{\theta_t} = \frac{Ts + 1}{\dfrac{s^4}{c^2 \omega_{no}^4} + \dfrac{2\mu_i s^3}{c\omega_{no}^3} + \dfrac{s^2}{\omega_{no}^2} + Ts + 1} \qquad (1.2\text{-}2)$$

where $\omega_{no}^2 = k_1 k_2 k_3 / T$ and represents the undamped natural frequency of the system if the rate loop lag is negligible and $c = \omega_{ni}/\omega_{no}$; $\mu_o = \sqrt{Tk_1 k_2 k_3}/2$. Since the coefficient of "s" in the numerator is the same as that in the denominator the system exhibits zero velocity lag, and has a steady state error to a constant input acceleration α of α/ω_{no}^2. This is easily seen when one visualises a constant error θ_ε. Since there is an integrator between this signal and the output speed the *slope* of the output speed is $\theta_\varepsilon k_1 k_2 k_3 / T = \theta_\varepsilon \, \omega_{no}^2$; and the slope of the speed output is the acceleration α.

1.3 TRACKING ACCURACY IN THE ABSENCE OF NOISE

In order to define the performance of any system one must first define the input. In practice one specifies a maximum speed of the target, a minimum tracking range and possibly a minimum "crossing range". This latter range is defined, on the assumption that the target continues to fly straight, as the shortest distance that it can be from the tracker. When it is at this point it is "passing" the tracker and is at the "point of closest approach". It is important to note that "crossing range" means slant range. If the point of closest approach in ground range is 4 km and the target height is 3 km the crossing range is 5 km. A target may well manoeuvre for a short time before attacking but during most or all of the time targets fly straight with constant velocity. We will therefore take such a target as the standard; surprisingly enough moderate target manoeuvres do not add greatly to the tracking task. In Fig 1.3-1 the crossing range is d and the corresponding ground range is d'. The instantaneous slant range is r and the corresponding ground range is r'.

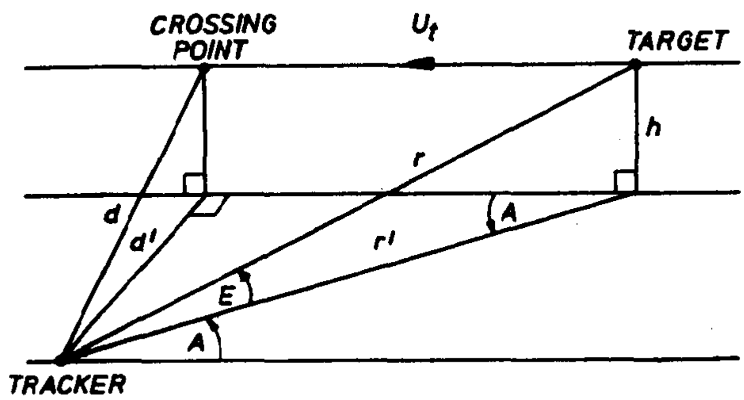


FIG 1.3-1 Angles and distances associated with tracking a target flying at constant altitude

In the following analysis it is assumed that the target speed U_t is constant.

$$\dot{A} = U_t \sin A/r' = U_t \sin^2 A/d' \tag{1.3-1}$$

$$\ddot{A} = \frac{d}{dA} \cdot \frac{dA}{dt} \cdot \frac{dA}{dt} = \frac{2U_t}{d'} \cos A \sin A \frac{U_t \sin^2 A}{d'}$$

$$= \frac{U_t^2}{(d')^2} \sin 2A \sin^2 A = \frac{U_t^2 \sin 2A}{(r')^2} = \frac{U_t^2 \sin 2A}{r^2 \cos^2 E} \tag{1.3-2}$$

Hence if R_{min} is the minimum specified slant range the maximum angular acceleration for a given E occurs when $A = 45^0$ and is given by

$$\ddot{A}_{max} = U_t^2/R_{min}^2 \cos^2 E \tag{1.3-3}$$

It should be noted that angular accelerations can be very large in azimuth if the angle of elevation is large; in practice the maximum angle is often limited to about 55^0.

Similarly, expressions for $\dot{E}$ and $\ddot{E}$ can be obtained:

$$\dot{E} = \frac{U_t}{r} \sin E \sin A \tag{1.3-4}$$

$$\text{and } \ddot{E} = -\frac{U_t^{\,2}}{r^2} \tan E \,\{1 - \cos^2 A\, (1 + 2\cos^2 E)\} \qquad (1.3\text{-}5)$$

If now $A = 90^0$ (i.e. target is crossing)

$$\ddot{E}_{max} = -\frac{U_t^{\,2}}{R_{min}^{\;2}} \tan E \qquad (1.3\text{-}6)$$

and if $A = 0^0$ (i.e. target directly ahead)

$$\ddot{E}_{max} = \frac{U_t^{\,2}}{R_{min}^{\;2}} \sin 2E \qquad (1.3\text{-}7)$$

It is seen therefore that, even for the apparently straightforward case of a constant speed target flying at constant height, angular rates and accelerations are not constant. We can if necessary obtain expressions for the third and higher derivatives. Had the angular acceleration been *constant* we could have calculated the constant following error by the method discussed in section 1.2; in practice the tracking error will vary with time. The graphs drawn in Figs 1.3-2 and 1.3-3 show the ratio of the actual following error θ_e to the approximate value given by the instantaneous angular acceleration divided by the loop gain. The graphs are applicable to motion in azimuth when the angle of elevation is small or to motion in elevation when A is small; the generalised angle θ has been used instead of E or A since the results are then applicable to both channels. It is seen that when the acceleration is increasing ($\theta < 60^0$) the following error is less than the approximate value, and when it is decreasing the error is greater. This is explained when one remembers that displacements are not accelerations and that a following error in *position* takes time to integrate up. When one takes the rather more complicated inputs (e.g. motion in azimuth when the angle of elevation is not small) one arrives at a similar conclusion: the actual following error to a good first approximation for a type 2 servo is the *actual* input acceleration divided by the loop gain. This assumption is made in the following design example.

Suppose therefore that the problem is to track targets flying up to 600 m/sec as accurately as possible at ranges between 4 and 32 km. The design aim is, under the worst conditions, for the r.m.s. tracking error not to exceed 0.3 milliradians. If we concern ourselves with the elevation channel only and assume the tracking elevation will not exceed 45^0, equation 1.3-6 or 1.3-7 can be used to compute $\ddot{E}$. This occurs when $E = 45^0$ and is given by

$$\ddot{E}_{max} = 600^2/16 \times 10^6 = 2.25 \times 10^{-2} \text{ rad/sec}^2$$

The minimum open loop gain for θ_e to be 3×10^{-4} radians is $2.25 \times 10^2/3 = 75$ rad/sec^2/rad. The minimum outer loop natural frequency ω_{no} is therefore $\sqrt{75} = 8.7$ rad/sec.

The damping ratios of the outer and inner loops and the ratio of the bandwidths of these two loops have little effect on the tracking accuracy, but they will, of course, affect the stability margin of the system. Some indication of the optimum values for these parameters can be obtained by considering the performance in the presence of noise.

1.4 THE EFFECT OF THERMAL NOISE

Noise is present in all receivers and the reader is referred to Skolnik (4) for a discussion on the main sources of noise in radar receivers. Our problem stems from the fact that in closed loop systems high amplification of error signals is needed to obtain good system accuracy; nevertheless amplification of signals amplifies noise as well, so that some form of compromise is necessary. The main source of noise in radar receivers is "thermal noise" because electrons in any conductor at a temperature other than absolute zero are always in random motion. This motion gives rise to an electrical noise voltage which is essentially "white" i.e. its spectrum is independent of frequency from d.c. to a frequency far in excess of any servo tracker bandwidth. There are many other sources of noise associated with receivers including environmental background noise but in practice it is found that if receiver noise is significant it is largely due to thermal noise, and therefore is sensibly constant for a given receiver. The actual noise output expressed as a mean square voltage however is not constant. If the incoming signal is strong (e.g. a large target at short range) an automatic gain control reduces the gain of the IF amplifier, in order to keep the output independent of the signal power and this effectively reduces the noise output. The result is that the signal-to-noise ratio varies. If one is illuminating and tracking a target the received power will vary inversely as the (range)^4, all other things being equal. An assumption that is usually made therefore is that the mean squared noise output from a receiver for a given target, range and atmospheric conditions varies inversely as the signal-to-noise ratio and is proportional to (range)^4. Since all angle channel receivers are designed to produce a voltage proportional to the angular misalignment it follows that the mean square noise output can be regarded as a mean square angle. Hence, thermal noise is often referred to

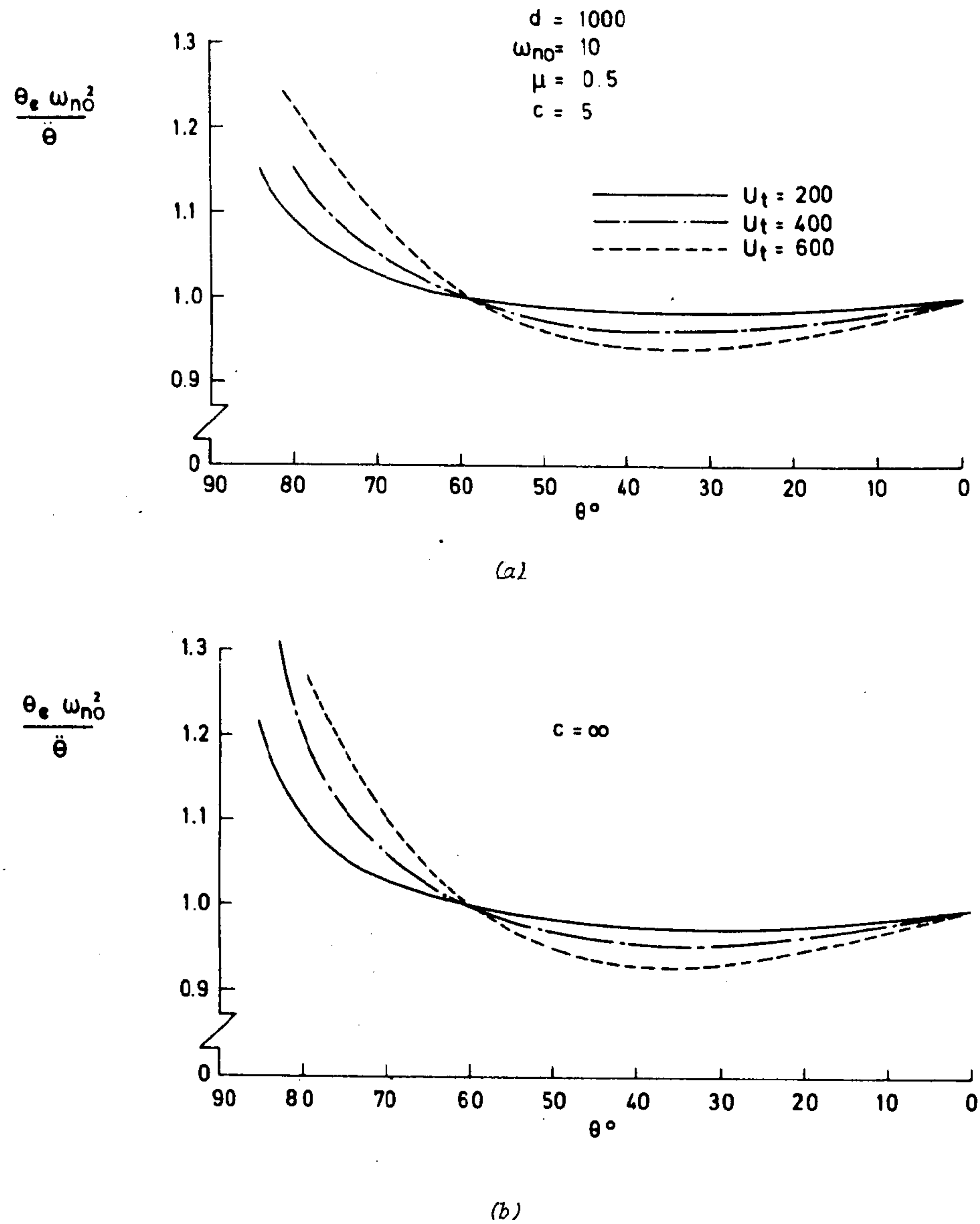
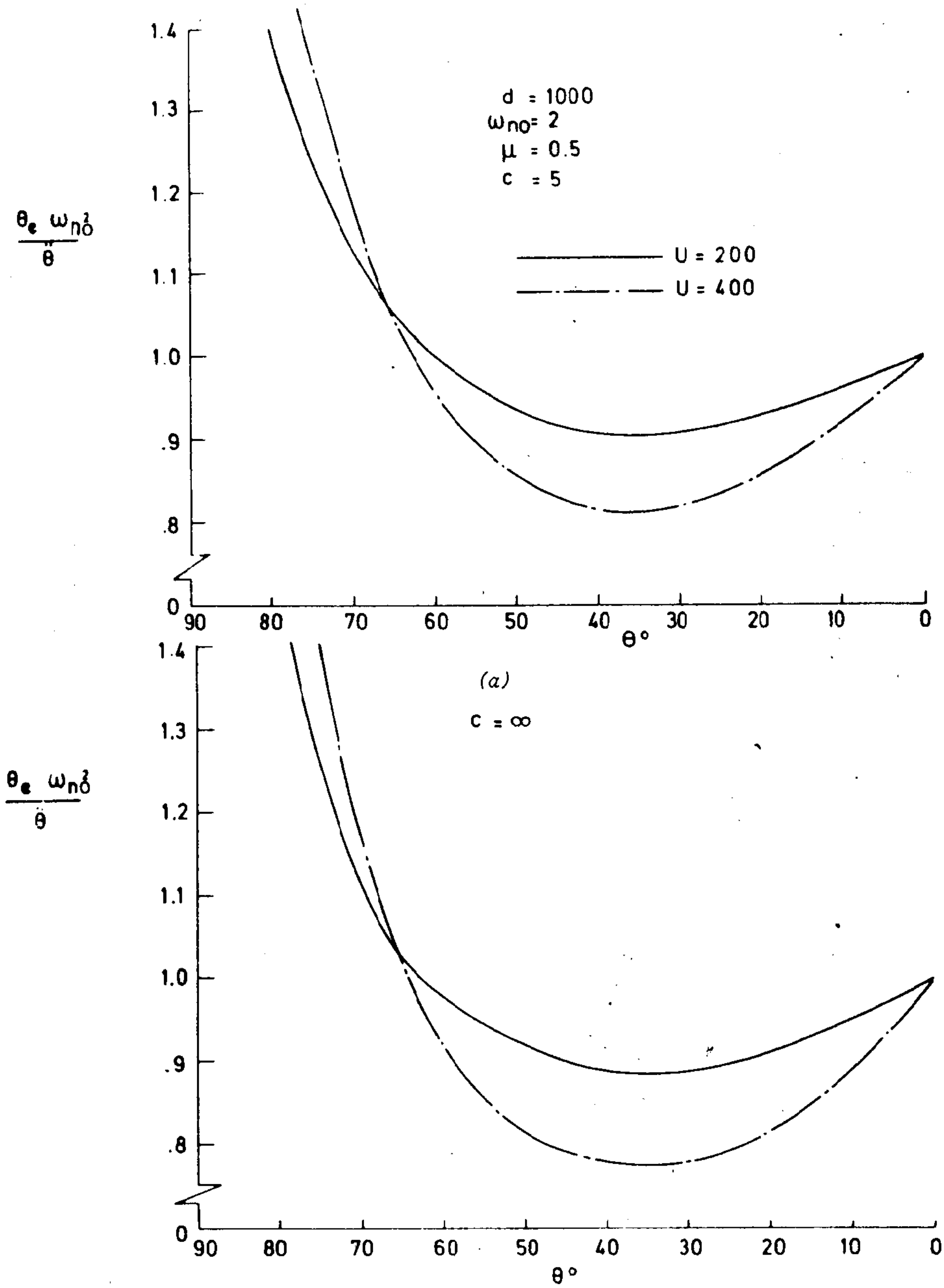


FIG 1.3-2 Following error for a type 2 servo



(b)

FIG 1.3-3 Following error for a type 2 servo as "angular" noise. Let the spectrum of this noise be defined as

$\Phi_a(\omega) = K_a{}^2$ rad^2/rad/sec, where K_a is a constant.

The problem now is to determine the mean square value (and hence r.m.s. value) of the antenna "jitter" due to this noise. It is most important to note at this stage that, ideally we require a servo to have zero bandwidth so that all this noise is filtered out. Consider now the mean square output σ^2 of a filter whose transfer function in frequency form is $1/1 + j\omega T$. The input is white noise whose spectrum is constant up to a frequency ω_b. The mean square output is therefore

$$\sigma^2 = K_a{}^2 \int_0^{\omega_b} \frac{d\omega}{(\sqrt{1 + \omega^2 T^2})^2} = K_a{}^2 \int_0^{\omega_b} \frac{d\omega}{1 + \omega^2 T^2}$$

$$= \frac{K_a^2}{T} \tan^{-1} \omega_b T$$

Provided $\omega_b T > 10$ say this approximates to $\pi K_a{}^2/2T$. This means therefore that the mean square output would be the same as that from another filter whose pass band was flat up to a frequency equal to $\pi/2T$ and then cut off completely at this frequency. In other words a simple first order lag has an effective bandwidth to white noise of $\pi/2$ times its own bandwidth of $1/T$. It is easily shown that the effective noise bandwidth of any linear filter is a function of the coefficients of the transfer function. In general a linear system will have a transfer function of the form

$$\frac{b_m s^m \ldots\ldots + b_1 s + b_o}{a_n s^n \ldots\ldots + a_1 s + a_o} = \frac{b(s)}{a(s)}$$

where the order of the numerator is at least one less than that of the denominator. Integrals of the form

$$I = \int_o^\infty \frac{|b(j\omega)|^2 \, d\omega}{|a(j\omega)|^2}$$

can be evaluated and Table 1.4-1 gives values for systems up to fourth order assuming as we always can that $a_o = 1$. We justify integrating to $\omega = \infty$ because the noise bandwidth of thermal noise is so high.