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Room: Donner Pass Posters Displayed: 9:30-4:00

Authors Available: 2:40-4:00

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| Double Slit Diffraction of a Localized Wave Packet I. M. Besieris,* A. M. Shaarawi, Virginia Polytechnic Institute, R. W. Ziolkowski, Lawrence Livermore National Laboratory | URSI |

SPHERICAL WAVEFUNCTION ANALYSIS OF RADIATION FROM THE PYRAMIDAL HORN

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University of Stellenbosch, 7600, South Africa

Introduction

The pyramidal horn is a simple and robust feed antenna. The exact analysis of this antenna is complex because the surfaces of the horn do not coincide with separable coordinate systems. The pyramidal horn may be approximated as a quasi-pyramidal horn, if the flare angles that describe the horn are measured from the same point, Fig. 1. The quasi-pyramidal horn is a finite length part of a radial waveguide that is described by surfaces of constant θ and ϕ spherical coordinates in the XYZ reference frame. The horn is defined by the radial length R and the flare angles θ_k and θ_N , defined in the XYZ reference frame. The ϕ -0 plane coincides with one side of the horn. θ_k is measured in the YZ plane. In this second reference frame the Z-axis defines the boresight of the horn, and the X-axis coincides with the -Z direction of the first reference frame. The aperture dimensions are defined by

$$\alpha^{\prime\prime} = 2R\sin(\theta_{H}/2)\cos(\theta_{E}/2), \quad b^{\prime\prime} = 2R\sin(\theta_{E}/2)\cos(\theta_{H}/2)$$

$$d = R\{1 - \cos(\theta_{E}/2)|\sin(\theta_{H}/2)\}.$$
(1)

Quasi-pyramidal horn model

The quasi-pyramidal horn model is proposed because it may be analysed analytically using spherical wavefunctions [1,pp 279-283]. Furthermore the properties of the guided waves in the throat of the pyramidal horn are closely related to the properties of the radial waveguide. In this paper only the dominant TE_{rol} mode is considered. This mode is transverse electric with respect to the radial direction. The indices 0 and 1 are associated with the separation constants for the ϕ and θ dependence respectively.

Assume that a source, to be defined, is placed in the waveguide at radius R, from the origin. The TE_{rol} mode will be dominant if kR, $<\nu$ for all other modes, with ν the values of the modal separation constants that satisfy the boundary condition on the surfaces of constant θ . This means that these modes are below the point of gradual cutoff. Thus the propagation constants are predominantly real and these modes are heavily attenuated. This condition may be physically realised if the source

- (a) E-field has even symmetry about the $\theta' = \pi/2$ and $\phi' = \theta_{\ell}/2$ planes and $\theta_{\ell} < \theta_{H}$ to ensure TE_{rol} dominance.
- (b) Is linearly polarized with E-field parallel to the γ -axis.

(c) Is placed in the vicinity of kR,≥v with v the smallest non-trivial value that satisfies [1,p 282]

$$t = \frac{dP_v(\cos\theta_2)}{d\theta_2} / \frac{dP_v(\cos\theta_1)}{d\theta_1} = \pm 1$$
 (2)

$$\theta_2 = \pi - \theta_1$$
, $\theta_1 = (\pi - \theta_H)/2$.

 $P_*(\cos\theta')$ is the Legendre function of the first kind.

Subject to these conditions the outgoing electric field, with ${\bf e}^{\prime {\bf w}\prime}$ time dependence, is

$$\vec{E}_{TE_{rol}} = h_{r}^{(2)}(kr) \frac{d}{d\theta} \left\{ t P_{r}(\cos\theta') - P_{r}(-\cos\theta') \right\} \hat{a}_{\phi'}, \tag{3}$$

with $h_{\ell}^{(2)}(kr)$ the spherical Hankel function of the second kind.

Radiation

Solutions for the fields in the quasi-pyramidal horn are strictly valid if the horn extends to infinity. The aperture field may however be approximated by the TE_{rol} mode field if $\beta, (kR) \rightarrow k$ with $\beta, (kR)$ the modal phase constant in the radial direction and k the free space wave number. It is assumed that end effects are negligible. The approximation is good if the radiated field is not needed close to the antenna. To simplify the boundary condition it is useful to consider the horn enclosed by a perfectly conducting sphere of radius R concentric to the apex of the horn. A finite spherical wave expansion (SWE) is used to describe the fields outside the sphere, with only outgoing waves needed.

$$\vec{E}(\vec{r}) = \sum_{s=1}^{2} \sum_{m=-M}^{M} \sum_{\substack{n=-\\ \text{max}(1,|m|)}}^{N} Q_{smn}^{(4)} \vec{F}_{smn}^{(4)}, \quad N = [kR_m] + n_o, \quad M \le N.$$
(4)

The notation of [2,pp 9-26] is used, with R_m the minimum sphere radius that encloses all the sources. The expansion coefficients are calculated by using the orthogonality properties of the wavefunctions and enforcing the boundary conditions over the sphere. Usually the SWE origin is chosen to minimize the number of waves needed. In this case it is advantageous to choose the XYZ coordinate system because the surface integral that describes $Q_{2m}^{(2)}$, simplifies to a one dimensional numerical integral. The coefficient $Q_{2m}^{(m)}$ is then transformed to the XYZ coordinate system with known rotation coefficients [2,Appendix A2] to get a better numerical representation for the field.

Results are shown in the XYZ reference frame, for $R=4.21\lambda$, $\theta_f=35.3^{\circ}$, $\theta_M=39.3^{\circ}$. From (1), the aperture dimensions are $\alpha^{(r)}=2.70\lambda$, $b^{(r)}=2.40\lambda$, $d=6.66\times10^{-2}\lambda$. The SWE is truncated at N=36, M=16. From (2) the solution of the separation constant is $\nu=4.1591466$. Furthermore $\beta_r(kR)=0.985k$. Thus the aperture field approximation is valid.

Conclusion

Fig.2 shows a comparison between the predicted and measured [3,p 57] far-field E-plane (ϕ -90°) pattern, with good agreement. Comparison with Jull's results [3,p 57] shows that the SWE method is more accurate in predicting the radiation pattern than the aperture integration (AI) method. The field may be calculated in any direction and Fig. 3 gives representative results. The coand cross-polarization E-field are defined by the third definition of Ludwig [4], with the reference vector parallel to the γ -axis. Furthermore the fields can easily be calculated at a finite distance from the origin. The pyramidal horn is modelled as a physically realisable structure in contrast to the usual AI method, which assumes the aperture placed in an infinite, flat conducting plane.

References

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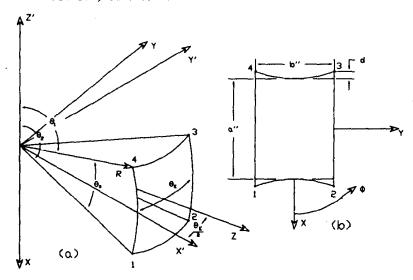


Fig. 1: (a) Quasi-pyramidal horn. (b) Intersection of horn with a plane surface defined by the points 1 2 3 4.

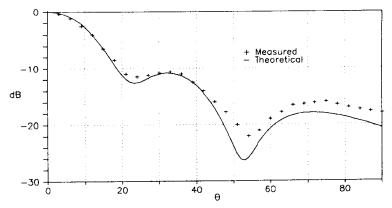


Fig. 2: Comparison between the theoretical and measured co-polarized far-field E-plane patterns.

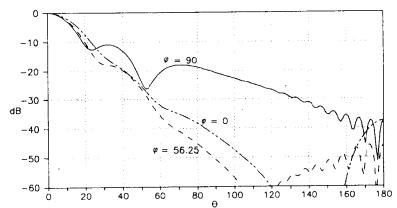


Fig. 3: (a) Co-polarized far-field patterns.

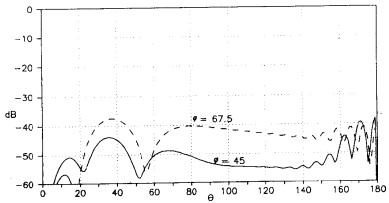


Fig. 3: (b) Cross-polarized far-field patterns.

TREATMENT OF VECTOR POTENTIAL IN A THREE-DIMENSIONAL LATTICE NETWORK OF SPATIAL NETWORK METHOD

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1. Introduction

In the analysis of electromagnetic field, the vector potential has important roles especially when sources exist. On the other hand, a physical existence of the magnetic vector potential is recently discussed by some experiments according to the A-B effect.

We have proposed the numerical vector analysis method of the electromagnetic field in three-dimensional space and time domain, which is named as the Spatial Network Method.(1),(2) The method is based on both the expression of the Maxwell's equation by the equivalent circuit in the three-dimensional space and the formulation by the Bergeron's method in time domain. The method nas many merits in analysis of dispersive medium involving anisotropic characteristics.(3)-(5)

In this paper, we propose the availability of applying the equivalent circuit for electromagnetic fields in the method to the vector potential fields in three-dimensional space and time domain. The electric vector potential is introduced as the dual quantity of the magnetic vector potential to satisfy the property of the equivalent circuit for the Bergeron's method. (6) The validity of the treatment is shown by computing the rotating magnetic field around the straight line current from the magnetic and electric vector potential.

2. The equivalent circuit of the vector potential We extend the lattice networks shown in Fig. 1 to the vector potential field. The magnetic vector potential A and the electric vector potential S are defined to satisfy the following equations.

$$\nabla \times \mathbf{S} = -\xi_0 \frac{\partial \mathbf{A}}{\partial t} \quad (= D) , \qquad (1a)$$

$$\nabla \times \mathbf{A} = \mu_0 \frac{\partial \mathbf{S}}{\partial t} \quad (= B) . \qquad (1b)$$

Here, the electric vector potential is supposed to have an inverse sign to the ordinary diffinition of it. These equations satisfy the ordinary definition of the magnetic vector potential and have similar forms to the Maxwell's equation except difference of the position of a negative sign. Table 1 presents the assignment of each component of these equations at each node and the correspondence between the circuit variables and the

potential quantities. The assignment of the magnetic potential to the voltage variable at the vector electric node of the equivalent circuit for electromagnetic field is introduced from the property of the 2nd term of Equ. (1a). The assignment of the electric vector potential to the magnetic node is introduced from Equ. (1b) as the same manner. The lattice network is interpreted as the equivalent circuit in which the line between nodes is a onedimensional transmission line and the node is a point at which the continuity law of currents occurs. The nodes are classified into two types as same as that in the electromagnetic field. One is the electric node at which each component of the magnetic vector potential is treated as a voltage variable and the other is a magnetic node at which each component of electric vector potential is treated as a voltage variable. All circuit variables at the magnetic nodes are also characterized by the symbol ' * ' because of the duality of their physical meaning, as compared with their interpretation at the electric nodes. In Fig. 1, the imaginary cubics, for example, plotted by the dashed lines or the chain lines is shown. In the cubics, the condition 'div A = 0' or 'div S = 0' is confirmed respectively. Then the network satisfies the wave equations when the space discretization \$\Delta d\$ is very smaller than the wave length as the same manner as that in the electromagnetic field.

The fundamental square lattice in which the content in Table 1 is illustrated is shown in Fig. 2. The symbol ' ' presents the direction of the propagation defined by P = S x A, which corresponds to the Poynting vector in the electromagnetic field. Also the gyrator ' is inserted in series with each magnetic node to show the duality of the physical meaning of the circuit variables of both nodes of each transmission line.

Analyzed Results

In Fig. 3, the analyzed model for a straight line current source with sinusoidal variation in time domain is shown. The steady state magnetic field distribution around the source in the central x-z plane are shown in Fig. 4. In this figure, (a) is calculated from rotation of the magnetic vector potential and (b) is computed from time derivative of the electric vector potential. Both distribution agree well with each other. Then the characteristics of this proposed method formulated by both the magnetic and electric vector potential is showm.

4. Conclusion

For the treatment of the vector potential, the gauge condition must be considered (7). The Coulomb Gauge is supposed in the above treatment. We are now studing the